Essays in Asset Pricing

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Abstract

In the first chapter, I build a New Keynesian asset pricing model with optimal monetary policy and Epstein-Zin preferences that accounts for some of the stylized facts concerning the term structures of equity and bond risk premia. The model-implied term structure of equity risk premia and its volatility are downward sloping, the term structure of bond risk premia is upward sloping, and the term structure of Sharpe ratios on dividend strips is downward sloping. Under Epstein-Zin preferences, the central bank amplifies short- and long-run productivity shocks to maximize surprise utility in an optimal monetary policy setting by making the output gap procyclical with respect to these shocks. The output gap gradually falls after a positive short- or long-run productivity shock so short-horizon output and dividends are more procyclical than medium-horizon output and dividends. Under the optimal monetary policy, the weight on the difference between inflation and its target in the loss function is large so inflation closely tracks the inflation target, which is persistent and responds negatively to long-run productivity shocks. This makes long-horizon price levels more countercyclical than short-horizon price levels with respect to the long-run productivity shock.

In the second chapter, I propose a model of sovereign credit risk within a monetary union and quantify the costs associated with entering such a union. The monetary authority sets the inflation rate for the monetary union to maximize an objective function consisting of the sum of the total values of each sovereign, while being constrained to keep the volatility of inflation low. Countercyclical monetary policy reduces the real value of debt in bad times through a higher inflation rate and increases it in good times through a lower inflation rate, allowing sovereigns a mechanism to hedge their nominal liabilities. The effectiveness of this mechanism is reduced in a monetary union as there is a single inflation rate for the entire union, and shocks to the real asset value of each sovereign are imperfectly correlated. Using data from the Eurozone, the calibration exercise determines the portion of credit spreads due to the loss of flexibility in monetary policy associated with joining a monetary union.

Additionally, the model generates economically significant increases in credit spreads and Arrow-Debreu prices of default for most countries and reductions in welfare for all countries in a monetary union when compared with the counterfactual of each sovereign conducting its own independent monetary policy.

In the third chapter, I present a heterogeneous-agent incomplete markets asset pricing model that accounts for many of the features of the nominal term structure of interest rates. There is a single state variable, termed household risk, that drives the conditional cross-sectional moments of household consumption growth and generates a countercyclical time-varying price of risk. Yields on nominal and real bonds are obtained in closed form and are affine in the state variable. Real yields are procyclical, nominal yields are countercyclical, the real term structure is downward sloping, and the nominal term structure is upward sloping. When calibrated to moments of consumption and dividend growth, the risk-free rate, market return, price-dividend ratio, and inflation, the model is able to produce realistic means and volatilities for nominal bond yields. The model is also able to account for the negative skewness and excess kurtosis of nominal bond yield changes and the failure of the expectations hypothesis with coefficients very similar to those in the data.

Contents

Chapter 1

Optimal Monetary Policy under Recursive Preferences and the Term Structure of Equity and Bond Risk Premia

1.1 Introduction

A number of papers have recently documented the fact that the term structure of equity risk premia is downward sloping. It has been well known in the literature for decades that the term structure of nominal bond yields and risk premia is upward sloping. Additionally, van Binsbergen and Koijen (2016) also find that the term structure of volatility of equity risk premia is downward sloping and the term structure of Sharpe ratios for dividend strips is downward sloping. Reconciling these stylized facts within a DSGE framework has been particularly challenging. To my knowledge, the only paper that addresses the downward sloping term structure of equity risk premium puzzle within such a framework has been Lopez, Lopez-Salido, and Vazquez-Grande (2015) who employ a New Keynesian habit-formation model in which countercyclical marginal costs make short-run dividends more procyclical after a technology shock and short-run inflation more countercyclical. They are able to achieve a downward sloping term structure of equity risk premia, an upward sloping term structure of nominal bond yields, and a upward sloping term structure of real bond yields. They cannot, however, reproduce the downward sloping term structure of Sharpe ratios on dividend strips. This paper is an attempt to address these puzzles in a New Keynesian framework with Epstein-Zin preferences, and optimal monetary policy. While the model cannot generate an upward sloping term structure of real bond risk premia, it can generate a downward sloping term structure of equity risk premia, an upward sloping term structure of nominal bond risk premia, a downward sloping term structure of Sharpe ratios on dividend strips, and a downward sloping term structure of volatility of equity risk premia.

In this paper, I propose a parsimonious New Keynesian asset pricing model in which the central bank minimizes a quadratic loss function which is consistent with the Epstein-Zin preferences of households. It is important to note that the quadratic loss function is not optimal in the Ramsey sense, but is optimal from the timeless perspective of Woodford (2003), allowing for time-invariant solutions for inflation and the output gap. In addition to containing the square of inflation as well as the square of the output gap, the optimal monetary policy loss function under Epstein-Zin preferences contains a new term: the square of utility surprises. Whether the central bank wants to maximize or minimize these utility surprises depends on the level of risk aversion of the representative household. I allow the weights to deviate from their optimal values, which are determined by the deep parameters of the model, in alternative calibrations in order to explore the macro and asset pricing implications of alternative monetary policy regimes in the spirit of the prevailing monetary policy literature. Under the baseline calibration, weights take on their optimal value. The addition of this term means that inflation and the output gap respond to short- and long-run productivity shocks. Optimal monetary policy under recursive preferences has significantly different implications for the term structure of equity risk premia when compared with the optimal policy under power utility preferences. The new term is responsible for generating a downward sloping term structure of equity risk premia through its amplification of short- and long-run productivity shocks on consumption. A negative weight on the square of utility surprises in the loss function induces the central bank to make the output gap procyclical with respect to short- and long-run productivity shocks. The output gap declines back to its steady-state value after a short- or long-run productivity shock making short-horizon output and dividends more procyclical than medium-horizon output and dividends. A standard monetary policy quadratic loss function containing only the square of inflation and the output gap cannot account for this stylized fact. Under the optimal policy, the difference between inflation and its target will be procyclical with respect to short- and long-run productivity shocks, but the large weight on inflation in the loss function means that inflation closely follows its target. The inflation target is persistent and countercyclical with respect to long-run productivity shocks to account for the upward sloping term structure of bond risk premia.

Epstein-Zin preferences allow for separate parameterization of risk aversion and intertemporal elasticity of substitution. I employ Epstein-Zin preferences with a logarithmic specification over consumption implying an intertemporal elasticity of substitution equal to 1 similar to Tallarini (2000) and more recently, Swanson (2015). The business cycle moments are only affected by the intertemporal elasticity of substitution, whereas risk premia are governed by the risk aversion parameter. With traditional power utility, a high risk aversion parameter is needed to match risk premia, which implies a low intertemporal elasticity of substitution, leading to counterfactual business cycle implications. These preferences have several convenient features from a computational perspective: they allow for the normalized value function to be solved for exactly, the optimal monetary policy loss function to be expressed in a simple quadratic form, and the solutions to bond and equity yields fit in the affine class of term structure models prevalent in the literature. The return on equity at different horizons is also affine in the state variables upon applying a log-linear approximation.

This paper lies at the intersection of several major strands of the finance literature. Term structure models of nominal bond yields that are built on macroeconomic foundations include Wachter (2006); Gallmeyer, Hollifield, Palomino, and Zin (2007); Bansal and Shaliastovich (2012); Piazzesi and Schneider (2006); Palomino (2012); Bekaert, Cho, and Moreno (2010); and Rudebusch and Swanson (2012). Wachter (2006) and Bansal and Shaliastovich (2012) study the term structure implications of Campbell and Cochrane's (1999) habit formation model and Bansal and Yaron's (2004) model with generalized recursive preferences and long run risk, respectively. Piazessi and Schneider (2006) look at the special case of generalized recursive preferences with a unit intertemporal elasticity of substitution. Gallmeyer, Hollifield, Palomino, and Zin (2007) allow for monetary policy via a Taylor rule to endogenously determine inflation and therefore, the term structure, in a model with recursive preferences. Palomino (2012); Tanaka (2012); Bekaert, Cho, and Moreno (2010); and Rudebusch and Swanson (2012) build New Keynesian asset pricing models to study the term structure of interest rates. Asset pricing models that provide a unified framework to fit the term structure of nominal bond yields and equity returns have emerged recently and include Burkhardt and Hasseltoft (2012); David and Veronesi (2013); Campbell, Pflueger, and Viceira (2014); Song (2016); and Swanson (2015). Burkhardt and Hasseltoft (2012), David and Veronesi

(2013); Campbell, Pflueger, and Viceira (2014); and Song (2016) are able to account for the changing correlation between stock and bond returns that has occurred over the past half century. Swanson (2015) shows that a New Keynesian asset pricing model with recursive preferences can account for nominal bond, real bond, equity, and defaultable bond data moments in a unified framework. However, none of the papers mentioned above address the term structure of equity risk premia. Lettau and Wachter (2011); Croce, Lettau, and Ludvigson (2015); Lopez, Lopez-Salido, and Vazquez-Grande (2015); Ai, Croce, Diercks and Li (2013); Belo, Collin-Dufresne and Goldstein (2015); Marfé (2015); Marfé (2017); and Doh and Wu (2016) address the downward sloping term structure of equity risk premia. Lettau and Wachter (2011) directly specify a stohastic discount factor and are able to match the upward sloping term structure of interest rates in addition to the downward sloping term structure of equity premia. Croce, Lettau, and Ludvigson (2014) introduce a bounded rationality limited information model in which the representative consumer is unable to distinguish between short-run and long-run consumption risks. Ai, Croce, Diercks and Li (2013) build a production-based asset pricing model with heterogeneous exposure to productivity shocks across capital vintages and an endogenous stock of growth options. Belo, Collin-Dufresne and Goldstein (2015) modify dividend dynamics so that leverage ratios are stationary. Hasler and Marfé (2015) build a model with rare disasters followed by recovery that generates higher risk premia for short horizon equity returns. Marfé (2017) considers a model in which labor rigidities affect dividend dynamics and the price of short-run risk. Doh and Wu (2016) employ a quadratic asset pricing model with long run risks in which processes for macro variables are endogenously determined functions of risk factors.

The remainder of this paper will proceed as follows. In section 2, I discuss the New Keynesian asset pricing model with Epstein-Zin preferences and optimal monetary policy. In section 3, I show how to solve for the dynamics of the output gap, inflation, and the normalized value function in this setting. In section 4, I derive nominal bond and dividend strip prices and the return on equity in this framework and provide details on the calibration of the model. In section 5, I document the sources of data used to calculate the empirical moments. In section 6 and 7, I discuss the calibration and results of the model, respectively. In section 8, I conclude the paper and give some possible directions for future work.

1.2 Model

1.2.1 Representative Household's Problem

Time in the model is discrete and continues forever. There is a representative household with recursive preferences as in Epstein and Zin (1989) and Weil (1989). The utility function of the representative household over consumption, C_t, and labor, N_t, can be expressed recursively in the following way:

$$U_t = \log(C_t) + \varphi_0 \log(1 - N_t) + \frac{\beta}{\sigma} \log(E_t[\exp(\sigma U_{t+1})]), \tag{1.1}$$

where φ_0 is the preference parameter for leisure, $\sigma = \frac{(1-\beta)(1-\varphi)}{1+\varphi_0}$, φ governs relative risk aversion, and β is the discount factor. The coefficient of relative risk aversion is $RRA = \frac{\varphi+\varphi_0}{1+\varphi_0}$. Consumption is measured in units of final goods. This specification of recursive preferences follows Tallarini (2000) and imposes an elasticity of intertemporal substitution of 1. As $\varphi \to 1$, the utility function specified above reduces to time-additive expected utility. For values of $\varphi > 1$, the representative household is more risk averse than in the expected utility case and the opposite is true for values of $\varphi < 1$.

As is standard in the New Keynesian literature, the aggregate consumption good, C_t, is defined as a CES aggregate of intermediate consumption goods:

$$C_t \equiv \left[\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.2}$$

where j is the index and $\epsilon > 1$ is the price elasticity of demand of each intermediate good. The household is endowed with a unit of time, which it can allocate to either leisure, L_t, or labor, N_t. The aggregate labor supply is used to produce intermediate goods, $N_t = \int_0^1 N_t(j)dj$, where $N_t(j)$ is the labor supply used to produce each individual intermediate good j. Therefore, it must be the case that $L_t + N_t = 1$. Markets are complete in the model and the representative household owns shares in the intermediate goods firms, so the budget constraint of the representative household is

$$C_t + E_t\left[Q_{t,t+1}\frac{B_{t+1}}{P_{t+1}}\right] \leq \frac{W_t N_t}{P_t} + \int_0^1 D_t(j)dj - T_t + \frac{B_t}{P_t} \tag{1.3}$$

where B_{t+1} is the portfolio of nominal state contingent claims in the complete contingent claims market, $Q_{t,t+1}$ is the real stochastic discount factor for calculating the real value at time t of one unit of consumption at $t+1$, W_t is the nominal wage rate, T_t is the real lump sum tax, and $D_t(j)$ is real dividend income from each intermediate goods firm j. The price index, P_t, is defined as

$$P_t \equiv \left[\int_0^1 P_t(j)^{1-\epsilon}dj\right]^{\frac{1}{1-\epsilon}} \tag{1.4}$$

and is derived from the profit maximization problem of the final goods firm. The intratemporal optimality condition that comes from equating the marginal rate of substitution between consumption and labor for the representative household with the real wage is

$$\frac{\varphi_0 C_t}{1-N_t} = \frac{W_t}{P_t}. \tag{1.5}$$

1.2.2 Intermediate Goods Firms' Problem

Each intermediate goods producer produces intermediate goods according to a constant returns to scale production function in labor with a common productivity shock, A_t:

$$Y_t(j) = A_t N_t(j). \tag{1.6}$$

The log of productivity growth, $\Delta a_t = a_t - a_{t-1}$, follows the following stochastic process:

$$\Delta a_t = z_{t-1} + \sigma_a \epsilon_t^a \tag{1.7}$$

where

$$z_t = \rho_z z_{t-1} + \sigma_z \epsilon_t^z. \tag{1.8}$$

Long-run risk in productivity growth is a relatively recent development in the asset pricing literature. Croce (2014) employs long-run risk in productivity to match the equity premium in a production-based model. Long-run risk in productivity will translate directly into long-run risk in consumption growth in my model. Intermediate goods producers face a common wage, W_t, and act to minimize total cost by choosing the quantity of labor, $N_t(j)$, subject to the constraint of producing enough to meet demand:

$$\min_{N_t(j)} W_t N_t(j) \tag{1.9}$$

subject to

$$A_t N_t(j) \geq \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t. \tag{1.10}$$

The nominal marginal cost derived from the first-order condition is

$$NMC_t = \frac{W_t}{A_t}. \tag{1.11}$$

The New Keynesian Phillips curve is derived from the optimization problem of intermediate goods firms facing Calvo (1983) staggered pricing frictions. The intermediate goods firm j solves the following maximization problem:

$$\max_{P_t(j)} E_t \left[\sum_{T=t}^{\infty} \omega^{T-t} Q_{t,T} \left(\frac{P_T(j)}{P_T} Y_T(j) - \frac{W_T N_T(j)}{P_T}\right)\right], \tag{1.12}$$

where ω is the probability that a firm cannot change its prices in a given period, independent of the last time when the firm last changed its price. The maximization is subject to the constraints of the demand function for its good, the production function, and real marginal cost, respectively:

$$Y_T(j) = \left(\frac{P_T(j)}{P_T}\right)^{-\epsilon} Y_T, \tag{1.13}$$

$$Y_T(j) = A_T N_T(j), \tag{1.14}$$

and

$$MC_T = \frac{W_T}{A_T P_T}. \tag{1.15}$$

Firms that cannot change their prices utilize an indexation scheme where the firms index according to a time-varying inflation target following a first-order autoregressive process set by the central bank each period:

$$\log P_T(j) = \log P_{T-1}(j) + \pi_T^*, \tag{1.16}$$

where

$$\pi_t^* = \pi^* + w_t \tag{1.17}$$

and

$$w_t = \rho_w w_{t-1} + \sigma_w \epsilon_t^w + \sigma_{w,z} \epsilon_t^z. \tag{1.18}$$

Similar to Dew-Becker (2011), the inflation target responds to innovations in expected productivity growth. This feature turns out to be essential in generating an upward sloping nominal term structure of interest rates and an upward sloping term structure of nominal bond risk premia. With $\sigma_{w,z} < 0$ and ρ_w close to 1, the central bank will drive the inflation target down persistently in response to a positive shock to long-run productivity growth, creating more countercyclical inflation at long horizons than at short horizons. Since the process for the inflation target is persistent and agents prefer early resolution of uncertainty under the baseline calibration, the inflation risk premium will be higher at longer horizons, leading to an upward sloping nominal term structure of interest rates. The solution to the maximization problem in equation 1.12 is

$$P_t(j) = \mu \frac{E_t\left[\sum_{T=t}^{\infty} \omega^{T-t} Q_{t,T} P_T^{\epsilon} e^{(1+\pi_{t+1}^*+\ldots+\pi_T^*)(-\epsilon)} MC_T Y_T\right]}{E_t\left[\sum_{T=t}^{\infty} \omega^{T-t} Q_{t,T} P_T^{\epsilon-1} e^{(1+\pi_{t+1}^*+\ldots+\pi_T^*)(1-\epsilon)} Y_T\right]}, \tag{1.19}$$

where $\mu = \frac{\epsilon}{\epsilon-1}$ is the monopolistic markup over marginal cost under no Calvo pricing frictions and MC_T is the real marginal cost. Since there is no capital accumulation, the representative household consumes all output produced by the final goods firm:

$$Y_t = C_t. \tag{1.20}$$

Therefore, equation 1.20 can be expressed in logs as

$$y_t = c_t, \tag{1.21}$$

which is the market clearing condition in the economy. Output can be calculated by utilizing the intratemporal optimality condition in equation 1.5, noting that $y_t = c_t$ and $MC_t = \frac{W_t}{P_t A_t}$, and log-linearizing:

$$y_t = -\log(1+\varphi_0\mu) + a_t + \frac{\varphi_0\mu}{1+\varphi_0\mu}[mc_t - \log(\mu^{-1})]. \tag{1.22}$$

Flexible price output, the output in the absence of Calvo pricing frictions, can be determined by solving the above maximization problem for intermediate goods firms with $\omega = 0$. Noticing that under flexible prices, all firms will change their prices in every period, implies $P_t(j) = P_t$, $\forall j$, so that the real marginal cost must equal μ^{-1}. Plugging $mc_t = \log(\mu^{-1})$ into the above expression for output yields

$$y_t^f = -\log(1+\varphi_0\mu) + a_t. \tag{1.23}$$

The log output gap, x_t, is defined as

$$x_t \equiv y_t - y_t^f. \tag{1.24}$$

The solution to the intermediate firms' maximization problem can be log-linearized and expressed in terms of the output gap as the New Keynesian Phillips Curve:

$$\pi_t - \pi_t^* = \kappa x_t + \beta E_t[\pi_{t+1} - \pi_{t+1}^*], \tag{1.25}$$

where $\kappa = \frac{(1-\omega\beta)(1-\omega)\left(\frac{1}{\mu}+\varphi_0\right)}{\omega\varphi_0}$. The New Keynesian Phillips Curve under Epstein-Zin preferences is identical to the standard one derived under power utility, as demonstrated by Levin (2008). See Appendix A.5 for a derivation of the New Keynesian Phillips curve.

According to the definition of the output gap and the market clearing condition,

$$\Delta c_t = \Delta a_t + \Delta x_t. \tag{1.26}$$

I follow Abel (1990) in letting dividend growth be a levered claim on consumption growth with leverage parameter δ. There is also risk in dividend growth, ϵ_{t+1}^d, that is uncorrelated with all other shocks in the model and therefore, uncorrelated with consumption growth. This source of risk is captured by the second term on the right hand side in the expression below:

$$\Delta d_{t+1} = \delta \Delta c_{t+1} + \sigma_d \epsilon_{t+1}^d. \tag{1.27}$$

1.2.3 Optimal Monetary Policy Problem

Deriving the loss function for the optimal monetary policy problem can be simplified by rewriting the utility function of the representative consumer as

$$U_t = \log(C_t) + \varphi_0 \log(1 - N_t) - \frac{\beta}{\sigma} \log(M_{t,t+1}) + \beta U_{t+1}, \tag{1.28}$$

where $M_{t,t+1} = \frac{\exp(\sigma U_{t+1})}{E_t[\exp(\sigma U_{t+1})]}$. The second-order approximation of $M_{t,t+1}$ is $M_{t,t+1} \approx 1 + \hat{M}_{t,t+1} + \frac{1}{2}\hat{M}^2_{t,t+1}$, where $\hat{M}_{t,t+1} = \sigma(U_{t+1} - U - E_t[U_{t+1} - U])$ is the log-deviation of $M_{t,t+1}$ about its steady-state value and U is the steady-state value of U_t. Expressing the lifetime utility function as an infinite sum starting at $t = 0$, substituting in the second-order approximation for $M_{t,t+1}$, and taking the expectation, I obtain

$$\sum_{t=0}^{\infty} \beta^t E_{-1}[\log(C_t) + \varphi_0 \log(1 - N_t)] - \frac{1}{\sigma} \sum_{t=0}^{\infty} \beta^{t+1} E_{-1}\left[\frac{1}{2}\hat{M}^2_{t,t+1}\right]. \tag{1.29}$$

Following Woodford (2003), the second-order approximation to the first sum can be expressed in terms of the square of inflation and the output gap, as well as a linear term in the output gap due to the distorted steady-state. I assume that the optimal monetary policy problem is solved under the timeless perspective of Woodford (2003), so there is a pre-commitment and I am able to obtain a time-invariant solution. Therefore, the second summation must be shifted back one time period:

$$-\frac{1}{\mu}\sum_{t=0}^{\infty} \beta^t E_{-1}\left[(1-\mu)\,x_t + \left(1 - \mu + \frac{1}{\varphi_0 \mu}\right)\frac{x_t^2}{2} + \frac{\epsilon}{\gamma}\frac{(\pi_t - \pi_t^*)^2}{2}\right] - \sigma \sum_{t=0}^{\infty} \beta^t E_{-1}\left[\frac{(U_t - E_{t-1}[U_t])^2}{2}\right], \tag{1.30}$$

where $\gamma = \frac{(1-\omega\beta)(1-\omega)}{\omega}$. See Appendix A.6 for a derivation of the welfare-based quadratic loss function. Since there is no subsidy to offset the distorted steady-state induced by monopolistic competition ($\mu > 1$), there is a linear term in x_t in the expression. The size of the steady-state distortion is measured by the parameter ν, which represents the wedge between the marginal product of labor and the marginal rate of substitution between consumption and hours evaluated at the steady-state. Formally, ν is defined as follows:

$$-MC = MPN(1 - \nu), \tag{1.31}$$

where MC and MPN are the steady-state values of the marginal rate of substitution between consumption and labor and the marginal product of labor, respectively. In this model, there is no subsidy to correct the distortion due to firms' market power in the final goods market so $\nu = 1 - \frac{1}{\mu}$. Since ν has the same order of magnitude as fluctuations in the output gap or inflation for the chosen value of ϵ, the steady-state distortion can be assumed to be "small" and the minimization problem can be solved directly without appealing to more complex methods. See Galí (2008) for details. Therefore, the steady-state distortion affects the first-order moments of inflation and the output gap but has no effect on higher-order moments.

I obtain a quadratic loss function in $x_t - x^*$, $\pi_t - \pi_t^*$, and $U_t - U^* - E_{t-1}[U_t - U^*]$. I rewrite equation 1.30 as a loss function with weights λ_x, λ_π, and λ_U on the square of the deviation of output gap from its target, inflation from its target, and utility surprises, respectively:

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_{-1}\left[\lambda_x(x_t - x^*)^2 + \lambda_\pi(\pi_t - \pi_t^*)^2 + \lambda_U(U_t - U^* - E_{t-1}[U_t - U^*])^2\right]. \qquad (1.32)$$

The welfare-based loss function has $\lambda_x = \frac{1}{\mu} - 1 + \frac{1}{\varphi_0\mu^2}$, $\lambda_\pi = \frac{\epsilon}{\gamma\mu}$, $\lambda_U = \sigma$, and $x^* = \frac{1-\frac{1}{\mu}}{2\lambda_x}$. The sign of λ_U in the welfare-based loss function will change based on whether the representative household is more risk averse than in the power utility case ($\sigma < 0$) or less risk averse ($\sigma > 0$). Setting arbitrary weights provides flexibility in fitting macro and asset pricing moments, and can provide insight into the relative importance of each of the monetary policy objectives in determining the slope of the term structure of interest rates and equity.

The first constraint for the optimal monetary policy problem is the New Keynesian Phillips Curve and the second is the log-linearized equation for lifetime utility:

$$U_t - U^* = a_t + \nu x_t + \beta E_t[U_{t+1} - U^*]. \qquad (1.33)$$

The details of this derivation are shown in Appendix A.7. If there is a subsidy that offsets the distorted steady-state such that $\mu = 1$, there will be no role for monetary policy in affecting the magnitude of utility surprises so the last term will drop out of the loss function. In the case of power utility, $\sigma = 0$, the representative household is indifferent to utility surprises and the last term again drops out of the loss function. $U_t - U^*$ can then be eliminated from the loss function so that the only endogenous variables to solve for are $\pi_t - \pi_t^*$ and x_t and the minimization problem can be solved with respect to a single constraint: the New Keynesian Phillips Curve. I demonstrate how this is done in Appendix A.8.

1.2.4 Mechanism Generating Downward Sloping Term Structure of Equity Risk Premia and Upward Sloping Term Structure of Bond Risk Premia

The mechanism that amplifies the procyclicality of short-duration dividends relative to medium-duration dividends can be seen intuitively by examining the expression for utility surprises:

$$U_t - U^* - E_{t-1}[U_t - U^*] = \nu \sum_{k=0}^{\infty} \beta^k \left\{ E_t[x_{t+k}] - E_{t-1}[x_{t+k}] \right\} + \frac{\sigma_a \epsilon_t^a}{1-\beta} + \frac{\beta \sigma_z \epsilon_t^z}{(1-\beta)(1-\beta\rho_z)}.$$

The first term on the right hand side is the impact of a time t output gap surprise on surprise utility. The time t output gap surprise is a linear combination of time t innovations to short- and long-run productivity. The weights are determined endogenously as part of the optimal monetary policy problem. The second and third terms are the impact of time t innovations to short-run productivity and long-run productivity on surprise utility, respectively. When $\lambda_U < 0$, the central bank likes surprise movements in utility, and will act to amplify these surprises. In the case of a positive shock to short- or long-run productivity, the central bank will amplify the shock with a positive shock to the output gap. By the definition of the output gap and the market clearing condition, $c_t = a_t + x_t$, up to a constant. Therefore, a positive shock to short- or long-run productivity leads to a positive shock to the output gap, ultimately leading to a positive shock to consumption. Dividends are a levered claim on consumption. Accordingly, a positive shock to short- or long-run productivity leads to a positive shock to dividends as well when $\lambda_U < 0$. Since under the optimal policy the output gap is stationary, it decreases sluggishly back to its steady-state value after a positive short- or long-run productivity shock. Therefore, medium-horizon dividend strips will be less procyclical than short-horizon strips, leading to an initially downward sloping term structure of equity risk premia. The persistent nature of the long-run productivity shock dominates at long horizons making long-horizon consumption more procyclical with respect to the long-run productivity shock than medium-horizon consumption. Since the difference between inflation and its target, $\pi_t - \pi_t^*$, is the discounted present value of future output gaps according to the New Keynesian Phillips Curve, there will be a positive shock to the difference between inflation and its target upon the realization of a positive shock to short- or long-run productivity. Therefore, since $\pi_t - \pi_t^*$ is stationary, this will have the effect of making long-horizon price levels minus target price levels, $P_t - P_t^*$, more procyclical than short-horizon price levels minus target price levels.

The opposite is true when $\lambda_U > 0$: the central bank dislikes surprise movements in utility, and will act to offset these surprises, so a positive shock to short- or long-run productivity results in a negative shock to the output gap and a negative shock to the difference between inflation and its target. Whether a positive shock to short- or long-run productivity results in a positive or a negative shock to consumption (or dividends), depends on if the positive impact of the shock on a_t is larger or if the negative impact of the shock on x_t is larger, respectively. For relatively small values of $\lambda_U > 0$ (the threshold values will be different depending on whether the short- or long-run shock is under consideration), the former effect dominates, and the latter effect dominates for relatively large values. For sufficiently large $\lambda_U > 0$, short-horizon dividend strips are more countercyclical than long-horizon strips, leading to a steeply upward sloping term structure of equity. Long-horizon price levels minus target price levels are more countercyclical than short-horizon price levels minus target price levels for $\lambda_U > 0$.

There are two important opposing forces driving the slope of the nominal term structure of interest rates. When $\lambda_U < 0$, the central bank drives $\pi_t - \pi_t^*$ and short- and long-run productivity shocks in the same direction, making $\pi_t - \pi_t^*$ procyclical with respect to these shocks. However, the downward movement of the inflation target upon a positive long-run productivity shock makes the inflation target countercyclical with respect to long-run productivity shocks. Under the optimal policy, the weight on $(\pi_t - \pi_t^*)^2$ is extremely large, so $\pi_t - \pi_t^*$ has very little exposure to short- or long-run productivity shocks, virtually negating the first force. The latter force is therefore dominant under an optimal policy. Another way of saying this is that inflation closely tracks its target under an optimal policy. Given the high persistence of the inflation target and the preference for the early resolution of uncertainty, long-horizon bond yields will be particularly sensitive to shocks to the inflation target. Since shocks to the inflation target are countercyclical with respect to consumption, long-horizon nominal bonds will have higher inflation risk premiums than short-horizon nominal bonds implying an upward sloping term structure of bond risk premia.

1.3 Solution

To my knowledge, there is no closed form solution available for the optimal monetary policy problem due to the addition of the utility surprise term in the quadratic loss function. Following the solution method in Debortoli, Maih, and Nunes (2012), the optimal monetary policy problem can be written in the form:

$$\tilde{s}_{-1}' V \tilde{s}_{-1}' + d = \min_{\{\tilde{s}_t\}_{t=0}^{\infty}} E_{-1} \sum_{t=0}^{\infty} \beta^t \tilde{s}_t' W \tilde{s}_t' \tag{1.34}$$

such that

$$A_{-1}\tilde{s}_{t-1} + A_0 \tilde{s}_t + A_1 E_t \tilde{s}_{t+1} + B\epsilon_t = 0. \tag{1.35}$$

The Langrangean for the optimal monetary policy problem is

$$\mathcal{L} \equiv E_{-1} \sum_{t=0}^{\infty} \beta^t [\tilde{s}_t' W \tilde{s}_t + \tilde{\lambda}_{t-1}' \beta^{-1} A_1 \tilde{s}_t + \tilde{\lambda}_t' (A_{-1} \tilde{s}_{t-1} + A_0 \tilde{s}_t + B\epsilon_t)], \tag{1.36}$$

where $\tilde{\lambda}_{-1} = 0$ and $\tilde{s}_{-1}$ is given. ϵ_t is an I.I.D. vector of standard normal random variables that are mutually uncorrelated. $\tilde{\lambda}_{-1}$ is set to 0 so that there is no time-inconsistency in the optimality conditions. The solution to the Lagrangean can be written recursively by expanding the state vector to include the Lagrange multiplier vector $\tilde{\lambda}_t$. The solution to the problem will then be

$$\tilde{\chi}_t = \begin{bmatrix} \tilde{s}_t \\ \tilde{\lambda}_t \end{bmatrix} = \begin{bmatrix} \tilde{H}_{\tilde{s}\tilde{s}} & \tilde{H}_{\tilde{s}\tilde{\lambda}} \\ \tilde{H}_{\tilde{\lambda}\tilde{s}} & \tilde{H}_{\tilde{\lambda}\tilde{\lambda}} \end{bmatrix} \begin{bmatrix} \tilde{s}_{t-1} \\ \tilde{\lambda}_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{G}_{\tilde{s}} \\ \tilde{G}_{\tilde{\lambda}} \end{bmatrix} \epsilon_t.$$

The first-order conditions for the Langrangean are

$$\frac{\partial \mathcal{L}}{\partial \tilde{\lambda}_t} = A_0 \tilde{s}_t + A_1 E_t \tilde{s}_{t+1} + A_{-1} \tilde{s}_{t-1} + B\epsilon_t = 0 \tag{1.37}$$

and

$$\frac{\partial \mathcal{L}}{\partial \tilde{s}_t} = 2W\tilde{s}_t + A_0' \tilde{\lambda}_t + \beta^{-1} A_1' \tilde{\lambda}_{t-1} + \beta A_{-1}' E_t \tilde{\lambda}_{t+1} = 0. \tag{1.38}$$

Upon plugging the conjectured solution in for the conditional expectations above, the linear rational expectations system defined by the two first-order conditions can be solved by the method of undetermined coefficients. The algorithm used is detailed in Appendix A.4.

Extracting the constant term from the state vector and removing from the state vector and Lagrange multiplier vector those variables which are redundant (facilitates computation of first and second moments of the expanded state vector since $\tilde{H}$ is not invertible), the solution can be written as

$$\chi_t = \begin{bmatrix} s_t \\ \lambda_t \end{bmatrix} = c + \begin{bmatrix} H_{ss} & H_{s\lambda} \\ H_{\lambda s} & H_{\lambda\lambda} \end{bmatrix} \begin{bmatrix} s_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \begin{bmatrix} G_s \\ G_\lambda \end{bmatrix} \epsilon_t.$$

The final expanded state vector is $\chi_t' = [s_t \ \lambda_t]' = [\Delta a_t \ \pi_t - \pi_t^* \ x_t \ w_t \ z_t \ \lambda_t^{PC}]'$, where λ_t^{PC} is the Lagrange multiplier on the Phillips Curve constraint.

Once the dynamics of the output gap are solved for, the normalized value function can be solved for exactly. Using the transformation $\log(V_t) = U_t(1-\beta)$, the utility function of the representative consumer can be written as

$$V_t = [C_t(1-N_t)^{\varphi_0}]^{1-\beta} E_t[V_{t+1}^{\sigma/(1-\beta)}]^{\beta(1-\beta)/\sigma}. \tag{1.39}$$

Letting $\Gamma_t = C_t(1-N_t)^{\varphi_0}$, dividing both sides by Γ_t, and taking logs, this can be written in a form that facilitates an exact calculation of the normalized value function by a guess and verify method:

$$v_t - \gamma_t = \frac{\beta(1-\beta)}{\sigma} \log E_t \left\{ \exp\left[(v_{t+1} - \gamma_{t+1} + \Delta\gamma_{t+1}) \frac{\sigma}{1-\beta} \right] \right\}. \tag{1.40}$$

I conjecture that $v_t - \gamma_t = F_0 + F_1'\chi_t$ and $\Delta\gamma_{t+1} = G_0 + G_1'\chi_{t+1} + G_2'\chi_t$ and solve for the coefficients by plugging into the above equation. The expressions for the coefficients can be found in Appendix A.1.

1.4 Prices of Zero-Coupon Nominal Bonds, Dividend Strips, and Return on Equity

The real pricing kernel in the economy is

$$Q_{t,t+1} = \beta \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{V_{t+1}}{E_t[V_{t+1}^{\sigma/(1-\beta)}]^{(1-\beta)/\sigma}} \right)^{\frac{\sigma}{1-\beta}}, \tag{1.41}$$

consistent with the preferences of the representative household. The log real pricing kernel is

$$q_{t,t+1} = \log(\beta) - \Delta c_{t+1} + \frac{\sigma}{1-\beta}(v_{t+1} - \gamma_{t+1} + \Delta\gamma_{t+1}) - \log E_t \left\{ \exp\left[\frac{\sigma}{1-\beta}(v_{t+1} - \gamma_{t+1} + \Delta\gamma_{t+1}) \right] \right\}. \tag{1.42}$$

The nominal pricing kernel is $Q^{\$}_{t,t+1} = Q_{t,t+1}\frac{1}{\Pi_{t+1}}$, so that $q^{\$}_{t,t+1} = q_{t,t+1} - \pi_{t+1}$. More concisely,

$$q_{t,t+1} = -\Omega_0 - \Omega_1' \chi_{t+1} - \Omega_2' \chi_t \tag{1.43}$$

and

$$q^{\$}_{t,t+1} = -\Omega^{\$}_0 - \Omega'^{\$}_1 \chi_{t+1} - \Omega'^{\$}_2 \chi_t. \tag{1.44}$$

The expressions for Ω_0, Ω_1, Ω_2, and their nominal counterparts can be found in Appendix A.2.

The model fits in the affine framework popular in the term structure of interest rates literature. I extend this framework to the term structure of equity as well. Let $P^{\$}_{n,t}$ be the time t price of a zero-coupon nominal bond that has a unit payoff (in nominal terms) in n periods. Conjecturing that the prices of nominal bonds are exponentially affine in the state variables, $P^{\$}_{n,t} = \exp(-A^{\$}_n - B'^{\$}_n \chi_t)$, the coefficients $A^{\$}_n$ and $B^{\$}_n$ can be calculated using a guess and verify method by plugging into the Euler equation

$$P^{\$}_{n,t} = E_t[Q^{\$}_{t,t+1} P^{\$}_{n-1,t+1}]. \tag{1.45}$$

The coefficients are defined recursively as

$$A_n^{\$} = \Omega_0^{\$} + A_{n-1}^{\$} + \Omega_1'^{\$} c - \frac{1}{2}(\Omega_1'^{\$} + B_{n-1}'^{\$})GG'(\Omega_1^{\$} + B_{n-1}^{\$}) + B_{n-1}'^{\$} c \tag{1.46}$$

and

$$B_n'^{\$} = B_{n-1}'^{\$} H + \Omega_1'^{\$} H + \Omega_2'^{\$}. \tag{1.47}$$

Accounting for the fact that $P_{0,t}^{\$} = 1$, $A_0^{\$} = B_0'^{\$} = 0$. The yield to maturity on a nominal bond is defined as

$$y_{n,t}^{\$} = -\frac{1}{n}\log P_{n,t}^{\$} = \frac{1}{n}(A_n^{\$} + B_n'^{\$}\chi_t). \tag{1.48}$$

The gross one-period holding period return on a nominal bond is

$$R_{n,t+1}^{\$} = \frac{P_{n-1,t+1}^{\$}}{P_{n,t}^{\$}}. \tag{1.49}$$

Let $P_{n,t}^{d}$ be the time t price of a dividend strip that pays the aggregate dividend, D_t, in n periods. In solving for the prices of dividend strips, it is beneficial from a computational perspective to scale the price by the aggregate dividend at time t to eliminate D_t as an additional state variable. Conjecturing that the prices of dividend strips scaled by the aggregate dividend are exponentially affine in the state variables, $\frac{P_{n,t}^{d}}{D_t} = \exp(-A_n^d - B_n'^d\chi_t)$, the coefficients A_n^d and B_n^d can be calculated using a guess and verify method by plugging into the Euler equation

$$\frac{P_{n,t}^{d}}{D_t} = E_t\left[Q_{t,t+1}\frac{P_{n-1,t+1}^{d}}{D_{t+1}}\frac{D_{t+1}}{D_t}\right]. \tag{1.50}$$

The coefficients are defined recursively as

$$A_n^d = \Omega_0 + A_{n-1}^d + [\Omega_1' - \delta(e_1' + e_3')]c - \frac{1}{2}[\Omega_1' - \delta(e_1' + e_3') + B_{n-1}'^d]GG'[\Omega_1 - \delta(e_1' + e_3') + B_{n-1}^d] + B_{n-1}'^d c - \frac{1}{2}\sigma_d^2 \tag{1.51}$$

and

$$B_n'^d = [B_{n-1}'^d + \Omega_1' - \delta(e_1' + e_3')]H + \Omega_2' + \delta e_3'. \tag{1.52}$$

The yield on equity is defined by van Binsbergen, et al. (2014) as

$$y^d_{n,t} = -\frac{1}{n}\log\frac{P^d_{n,t}}{D_t} = \frac{1}{n}(A^d_n + B'^d_n\chi_t). \tag{1.53}$$

A plot of mean equity yields over different horizons, n, is the equity yield curve, defined in an analagous way to the nominal bond yield curve. The gross one-period holding period return on a dividend strip is then

$$R^d_{n,t+1} = \frac{P^d_{n-1,t+1}}{P^d_{n,t}} = \frac{P^d_{n-1,t+1}/D_{t+1}}{P^d_{n,t}/D_t}\frac{D_{t+1}}{D_t}, \tag{1.54}$$

so the continuously compounded return on a dividend strip is $r^d_{n,t+1} = \log R^d_{n,t+1}$.

The gross return on the market is $R^d_{t+1} = \frac{P^d_{t+1}+D_{t+1}}{P^d_t}$. Log-linearizing the expression for the gross return, the continuously compounded return on the market can be expressed as

$$r^d_{t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \delta\Delta c_{t+1} + \sigma_d\epsilon^d_{t+1}, \tag{1.55}$$

where pd_t is the log price-dividend ratio and $\kappa_1 = \frac{\exp(\bar{pd})}{1+\exp(\bar{pd})}$ and $\kappa_0 = \log(1+\exp(\bar{pd})) - \kappa_1\bar{pd}$ are linearization constants that must be solved for endogenously. Guessing that $pd_t = \xi_0 + \xi_1'\chi_t$, the coefficients ξ_0 and ξ_1 can be solved for by plugging into the Euler equation

$$E_t[e^{q_{t,t+1}+r^d_{t+1}}] = 1. \tag{1.56}$$

The expressions for the coefficients can be found in Appendix A.3. The linearization constants κ_0 and κ_1 are still functions of $\bar{pd}$. $\bar{pd}$ can be determined by solving a fixed point problem, evaluating the expanded state vector at its unconditional mean.

Sharpe ratios of nominal bond and dividend strip returns for one-period holding period returns on the n-period security are defined as

$$SR_n^i = \frac{E[r_n^i - r^f]}{\sigma[r_n^i]} \tag{1.57}$$

for $i = \$, d$. A term structure of Sharpe ratios for nominal bonds or equity can then be defined by looking at Sharpe ratios over different horizons, n.

1.5 Data

Annual data from 1930-2016 on real personal consumption expenditures, the personal consumption expenditures price index, and net corporate profits before tax come from the St. Louis Federal Reserve FRED website. Real gross domestic product and real potential gross domestic product are only available over the time period from 1949-2016. Annual data from 1930-2016 on dividend yields and nominal returns on the S&P 500 index are taken from Robert Shiller's website. The annualized yield on the one month Treasury bill comes from CRSP. Consumption growth is the log first difference of real personal consumption expenditures and inflation is the log first difference of the personal consumption expenditures price index. The output gap is the log difference of real gross domestic product and real potential gross domestic product. The log price-dividend ratio is the log of the inverse of the dividend yield. The equity premium is the difference between the nominal return on the S&P 500 index and the return on the one month Treasury bill. The summary statistics for macro moments and asset pricing moments are given in Tables 1.1 and 1.2, respectively.

1.6 Calibration

Parameters are calibrated at the quarterly frequency. The parameters of the model are

$$\Theta = \{\beta, \varphi, \varphi_0, \epsilon, \omega, \delta, \sigma_a, \rho_z, \sigma_z, \pi^*, \rho_w, \sigma_w, \sigma_{w,z}\}. \tag{1.58}$$

The parameters $\{\beta, \varphi_0, \epsilon, \omega, \delta\}$ have values that are fairly uncontroversial in the asset pricing and New Keynesian literature. The discount factor, β, is set to 0.985 so the annualized real interest rate is 1.41%, well in line with previous literature. The preference parameter for leisure, φ_0, is set to 2.9, taken from Tanaka (2012). The price elasticity of demand, $\epsilon = 11$, implies a monopolistic markup, μ, for intermediate goods firms of 1.1. This value is consistent with estimates from Smets and Wouters (2007) and Altig et al. (2011). The Calvo parameter, ω is set to 0.7, which implies an average duration for prices of 3.33 quarters, slightly lower than the value of 0.75 in Christiano, Trabandt, and Walentin (2010). Consistent with Lettau et al. (2008), I calibrate the leverage parameter, δ, to be 4.5. This value is slightly higher than the value used by Bansal and Yaron (2004) of 3.

I set the persistence and the standard deviation of the long-run productivity shock to be $\rho_z = 0.99$, slightly higher than the value of 0.98 chosen by Diercks (2015). Following Croce (2014), the standard deviation of the long-run productivity shock is 0.25 times the standard deviation of the short-run productivity shock, $\sigma_z = 0.25 \times \sigma_a$. The standard deviation of the short-run productivity shock is chosen to match the volatility of consumption growth in the data. The standard deviation of the short-run shock is set to be $\sigma_a = 0.001244$. The standard deviation of the short-run dividend shock is set to $\sigma_d = 0.0642$. This parameter is calibrated to match the volatility of dividend growth. The equity premium is extremely sensitive to the the choice of ρ_z, since stocks prices reflect the price of a claim to an infinite stream of dividends, which are defined as a multiple of consumption in each period. In the model, $\Delta d_{t+1} = \delta(z_t + \sigma_a \epsilon^a_{t+1} + \Delta x_{t+1}) + \sigma_d \epsilon^d_{t+1}$, so the persistence of long-run productivity is very important for the magnitude of the equity premium.

Target inflation, π^*, is set to 0. The persistence of the inflation target, ρ_w, is set to 0.999. Many papers assume a unit root for the inflation target and my calibration is consistent with those studies. The exposure of the inflation target to long-run productivity shocks, $\sigma_{w,z}$, is set to equal the standard deviation of idiosyncratic inflation target shocks, σ_w, but have the opposite sign, consistent with Dew-Becker (2014). He estimates that half of the variance of innovations to the inflation target is accounted for by long-run productivity shocks. The exposure of the inflation target to long-run productivity shocks is negative in order to account for the upward sloping term structure of bond risk premia. These parameters are set to match the volatility of inflation in the data.

Tallarini (2000) shows that the relative risk aversion with this specification of Epstein-Zin utility is $RRA = \frac{\varphi+\varphi_0}{1+\varphi_0}$. As shown in Swanson (2013), the presence of labor in the utility function allows households to better insure themselves against shocks. A number of studies employing Epstein-Zin preferences with unit intertemporal elasticity of substitution in a DSGE framework have estimated and calibrated the level of risk aversion to match both the term premium and equity premium. Rudebusch and Swanson (2012) estimate a value of 110, Dew-Becker (2014) estimates a value of 23, Swanson (2015) uses a value of 60, Piazzesi and Schneider (2006) use a value of 57, and Tallarini (2000) uses a value of 50. I use a value for φ of 100, implying a coefficient of relative risk aversion of 26.38. The baseline calibration is given in Table 1.3. The model-implied macro and asset pricing moments are given in Tables 1.4 and 1.5, respectively.

The parameters λ_x, λ_π, and λ_U are functions of the deep parameters of the household's utility function, Calvo parameter, and monopolistic markup over marginal cost as shown earlier. I can tweak these parameters from their optimal values to examine the effects of various types of non-optimal monetary policy. In an optimal monetary policy setting for the given calibration of the model, the weights on inflation, the output gap, and surprise utility in the loss function are $\lambda_\pi^{optimal} = 75.15$, $\lambda_x^{optimal} = 0.19$, and $\lambda_U^{optimal} = -0.38$. These will be the weights used in the baseline calibration. The weight on inflation is two orders of magnitude larger than the weights on the output gap and surprise utility under the optimal policy . The first set of figures, Figures 1 to 5, plot the impulse response functions for macro variables to short- and long-run productivity shocks, excess returns on dividend strips and nominal bonds, the term structure of Sharpe ratios on dividend strips, and the term structure of volatilities of excess returns on dividend strips and nominal bonds under the optimal monetary policy

$(\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = \lambda_U^{optimal})$. The second set of figures, Figures 6-10, repeats the plots above for the case in which all deep parameters are kept the same as in the baseline calibration and the weights on inflation and the output gap are kept at their optimal levels $(\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal})$, but the weight on utility surprises is $\lambda_U = 0$. In the second set of figures, I explore the implications for the term structure of equity and bond risk premia of a policy in which central banks are only concerned with stabilizing inflation and the output gap and show why it is necessary to account for utility surprises in the loss function to account for the downward sloping term structure of equity risk premia. In this case, the divine coincidence result holds and inflation will equal its target and the output gap will equal its steady-state value as the central bank does not face a trade-off between the goals of inflation and output gap stabilization.

The properties of the term structure of consumption risk premia are explored in Appendix A.9.

1.7 Results

For the purposes of clarity, the impulse response functions for short- and long-run productivity shocks are normalized so the steady-state value of each variable is 0. The results from the impulse response functions for the short-run productivity shocks show that for $\lambda_U = \lambda_U^{optimal}$ and for $\lambda_U < 0$, more generally, there is a positive shock to the difference between inflation and its target, the output gap, and consumption upon a positive short-run productivity shock at $t = 0$ and each quantity eventually decreases until it reaches its steady-state value as $t \to \infty$. The price level is relatively insensitive to short-run productivity shocks. These plots show that in this case, short-horizon consumption is more procyclical with respect to short-run productivity shocks than long-horizon consumption. The same is true for the difference between inflation and its target. The inflation target does not respond to short-run productivity shocks so the impulse response function is flat. In the case of long-run productivity shocks, the situation is different. Upon a positive long-run productivity shock, there is a positive shock to the difference between inflation and its target, the output gap, and consumption at $t = 0$. The difference between inflation and its target and the output gap decrease until they reach their steady-state values as $t \to \infty$. The price level decreases in a monotone fashion upon a long-run productivity shocks so long horizon price levels are more countercyclical with respect to the long-run productivity shock than are short-horizon price levels. However, consumption initially decreases then starts increasing as $t \to \infty$. In this case, short-horizon and long-horizon consumption are more procyclical with respect to the long-run productivity shock than medium-horizon consumption. The inflation target decreases initially in response to the long-run productivity shock and increases back to its steady-state value very slowly as the persistence of the inflation target is very close to 1. The difference between inflation and its target is more procyclical at short horizons than long horizons with respect to long-run productivity shocks, mirroring the results for the short-run productivity shock.

For values of $\lambda_U < 0$ that are sufficiently negative, the short end of the term structure of equity slopes downward while the long end of the term structure slopes upward. This is similar to the U-shaped term structure of equity generated by models such as Ai, Croce, Diercks, and Li (2013); Marfé (2017); and Lopez, Lopez-Salido, and Vazquez-Grande (2015) to name a few. The non-monotone nature of the term structure of equity in my model is due to the long-run productivity shock. When $\lambda_U < 0$, the presence of the utility surprise term in the quadratic loss function induces the central bank to maximize utility surprises and expose the output gap positively to innovations in short- and long-run productivity. The output gap is procyclical with respect to short- and long-run productivity. This combined with the gradual fall of the output gap to its equilibrium value generates the initial downward sloping nature of the term structure by making shorter horizon dividend strips more procyclical than longer horizon dividend strips. The upward sloping long end of the term structure of equity is due to the persistent long-run productivity shock that generates long-run risk in long horizon dividend strips due to investors' preference for early resolution of uncertainty under a standard calibration of the model. Therefore, short-run productivity shocks induce a monotone decreasing term structure of equity and long-run productivity shocks induce a U-shaped term structure of equity for sufficiently negative values of λ_U. The larger the magnitude of $\lambda_U < 0$, the more pronounced the initial downward slope is and the more the long end flattens out. Alternatively, for values of $\lambda_U > 0$, the term structure of equity is positively sloped. For increasing values of $\lambda_U > 0$, the slope increases as the equity risk premia on short-horizon dividend strips decreases more rapidly than the risk premia on long-horizon dividend strips.

For $\lambda_U = 0$, the output gap and inflation do not respond to short- and long-run productivity shocks. Therefore, short-horizon consumption and long-horizon consumption are as procyclical as one another with respect to short-run productivity. However, short-horizon consumption is less procyclical than long-horizon consumption with respect to long-run productivity shocks due to the persistence of the long-run productivity shock generating long-run risk in long horizon dividend strips. In contrast to the situation for $\lambda_U < 0$, short-run productivity shocks induce a flat term structure of equity risk premia and long-run productivity shocks induce an upward sloping term structure of equity risk premia.

Rudebusch and Swanson (2012) generate a positive and time-varying term premium in a New Keynesian model with Epstein-Zin preferences and a Taylor rule with a time-varying inflation target by using a third-order approximation to the equilibrium conditions. Tanaka (2012) generates a positive term premium with Epstein-Zin preferences by employing Blanchard and Galí (2007) style wage rigidities which creates a new trade-off in the NKPC for a central bank conducting optimal monetary policy. By adding the utility surprise term to the quadratic loss function, my model could generate a positive term premium if $\lambda_U > 0$. However, as this is not consistent with optimal monetary policy or a downward sloping term structure of equities, it is necessary to add a time-varying inflation target to the model. This is not a panacea, though, as a negative λ_U can generate considerable procyclicality in the inflation process depending on the value of λ_π. With relatively large λ_π, the central bank places a high weight on insulating inflation from productivity shocks. In an optimal policy setting, the relative weight placed on inflation stabilization vs. output gap stabilization is approximately 400 given my calibration and is large for any reasonable calibration. Therefore, the only significant shock driving inflation in the model is the shock to the inflation target. Since the inflation target is very persistent and households have a preference for the early resolution of uncertainty, long horizon nominal bonds will be more risky than short horizon nominal bonds leading to an upward sloping term structure of excess returns on nominal bonds.

A novel contribution of my model is its ability to generate a downward sloping term structure of Sharpe ratios on dividend strips and nominal bonds when $\lambda_U = \lambda_U^{optimal}$. Van Binsbergen and Koijen (2016) document this phenomena and show that standard asset pricing models cannot reproduce this empirical regularity. The model-implied term structure of Sharpe ratios on dividend strips and nominal bonds for the cases in which $\lambda_U = \lambda_U^{optimal}$ and $\lambda_U = 0$ are shown in Figures 1.4 and 1.9, respectively. When $\lambda_U = 0$, the term structure of Sharpe ratios on dividend strips is upward sloping. The model also generates a downward sloping term structure of volatilities of excess returns on dividend strips when $\lambda_U = \lambda_U^{optimal}$. These results are shown for the cases in which $\lambda_U = \lambda_U^{optimal}$ and $\lambda_U = 0$ in Figures 1.5 and 1.10, respectively. When $\lambda_U = 0$, the term structure of volatilities on excess returns on dividend strips is upward sloping.

Given the parsimonious and highly stylized nature of the model, it does a good job in fitting macro, nominal bond, and equity moments. The volatilities of consumption growth, dividend growth, and inflation are matched with the data and the model-implied means and standard deviations of the equity premium and the log price-dividend ratio are in line with the data. The means and first-order autoregressive coefficients of consumption growth and dividend growth are not matched with the data. The average quarterly (non-annualized) Sharpe ratio of 0.16 under the baseline calibration matches up well with the value of 0.15 in

Uhlig (2007).

Since the model is calibrated at a quarterly frequency and the data is expressed on an annualized basis, the means of log growth rates and returns are multiplied by 4 and the standard deviations are multiplied by 2 to arrive at annualized figures.

Empirical evidence supports the fact that the output gap responds procyclically to total factor productivity (TFP) shocks and that inflation responds countercyclically to TFP shocks in a manner consistent with the behavior of the central bank under the optimal monetary policy in the model. The series for TFP is available annually over the time period from 1950-2014. TFP growth is defined as the log first difference of the series for TFP. The annual correlation between the output gap and TFP growth over this time period is 0.19 and the annual correlation between inflation and TFP growth -0.34.

1.8 Conclusion

In this paper I have shown that a downward sloping term structure of equity risk premia is an outcome of a central bank practicing optimal monetary policy in an Epstein-Zin setting. The central bank responds to short- and long-run productivity shocks, making the output gap procyclical with respect to these shocks. The procyclicality of the output gap makes short-horizon dividends more procyclical than medium-horizon dividends. In a standard monetary policy loss function in which the central bank is concerned with inflation and output gap stabilization, the output gap and inflation do not respond to short- and long-run productivity shocks. Therefore, a standard loss function cannot account for a downward sloping term structure of equity risk premia. The model is parsimonious and highly stylized but can account for a variety of other term structure regularities documented by van Binsbergen and Koijen (2016) including an upward sloping term structure of bond risk premia, a downward sloping term structure of Sharpe ratios on dividend strips, and a downward sloping term structure of volatility of excess returns on dividend strips. In future work, I would like to utilize a medium-scale New Keynesian model in the the vein of Smets and Wouters (2007) and do a structural estimation of the model.

Table 1.1: Summary Statistics for Macro Moments

	Inflation, π	Output Gap, x	Consumption Growth, Δc	Dividend Growth, Δd
Standard Deviation	3.64	2.09	2.94	18.46
AR(1) Coefficient	0.75	0.62	0.39	-0.21

Summary statistics for macro moments. Standard deviations and AR(1) coefficients are in annual terms and standard deviations are in percentage terms. The data for inflation, consumption growth, and dividend growth spans 1930-2016. The data for the output gap spans 1949-2016.

Table 1.2: Summary Statistics for Asset Pricing Moments

	$r_m - r_f$	pd
Mean	5.75	3.37
Standard Deviation	15.30	0.46

Summary statistics for asset pricing moments. Data on returns is in annualized and percentage terms. The log price-dividend ratio mean and standard deviation are in natural units. The data for the equity premium and price-dividend ratio spans 1930-2016.

Table 1.3: Baseline Calibration ($\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = \lambda_U^{optimal}$)

β (discount factor)	0.985
φ (risk aversion parameter) [implied RRA]	100 [26.38]
φ_0 (leisure preference parameter)	2.9
ϵ (price elasticity of demand)	11
ω (Calvo parameter)	0.7
δ (leverage)	4.5
σ_a (standard deviation of short-run productivity shock)	0.001242
ρ_z (persistence of long-run productivity shock)	0.99
σ_z (standard deviation of long-run productivity shock)	0.25×0.001244
σ_d (standard deviation of dividend growth shock)	0.0643
π^* (mean inflation target)	0
ρ_w (persistence of inflation target)	0.999
σ_w (standard deviation of inflation target shock)	0.000574
$\sigma_{w,z}$ (exposure of inflation target to long-run productivity shock)	-0.000574
λ_π (weight on inflation in loss function)	75.15
λ_x (weight on the output gap in loss function)	0.19
λ_U (weight on utility surprise in loss function)	-0.38

Table 1.4: Model-Implied Macro Moments

	Inflation, π	Output Gap, x	Consumption Growth, Δc	Dividend Growth, Δd
Standard Deviation	3.64	2.15	2.94	18.46
AR(1) Coefficient	0.96	2.80E-5	0.038	0.038

Model-implied macro moments. Standard deviations are annualized and in percentage terms and AR(1) coefficients are in annual terms.

Table 1.5: Model-Implied Asset Pricing Moments

	$r_m - r_f$	pd
Mean	5.38	2.68
Standard Deviation	16.52	0.29

Model-implied statistics for asset pricing moments. Data on returns is annual and in percentage terms. The log price-dividend ratio mean and standard deviation are annualized and in natural units.

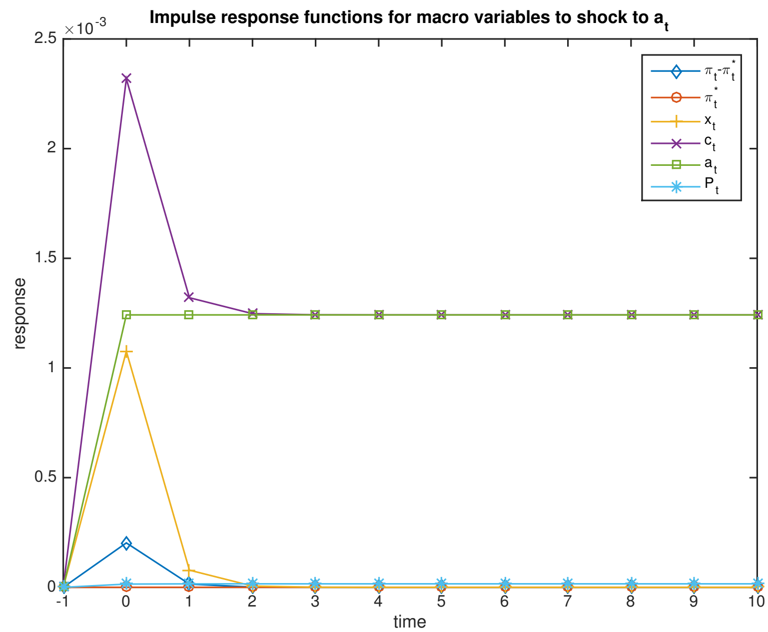

Figure 1.1: Impulse-response functions for macro variables to one standard deviation short-run productivity shock at $t = 0$. ($\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = \lambda_U^{optimal}$)

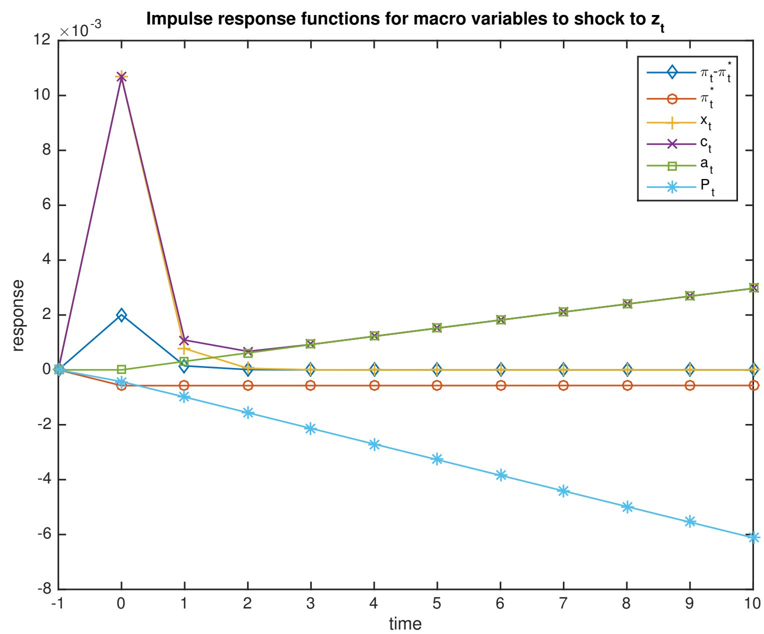

Figure 1.2: Impulse-response functions for macro variables to one standard deviation long-run productivity shock at $t = 0$. ($\lambda_\pi = \lambda_\pi^{optimal}$, $\lambda_x = \lambda_x^{optimal}$, $\lambda_U = \lambda_U^{optimal}$)

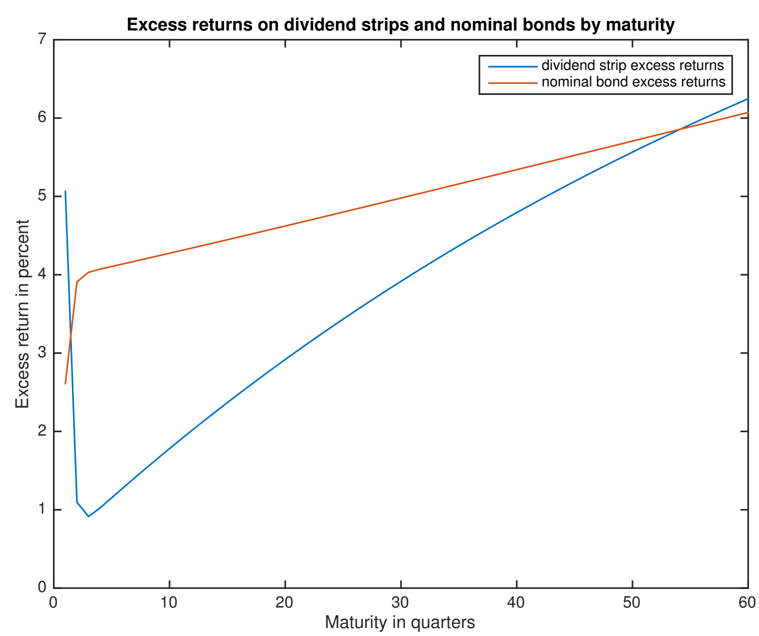

Figure 1.3: Model-implied term structure of expected excess returns on dividend strips, $E[r_n^d - r^f]$, and nominal bonds, $E[r_n^\$ - r^f]$. ($\lambda_\pi = \lambda_\pi^{optimal}$, $\lambda_x = \lambda_x^{optimal}$, $\lambda_U = \lambda_U^{optimal}$)

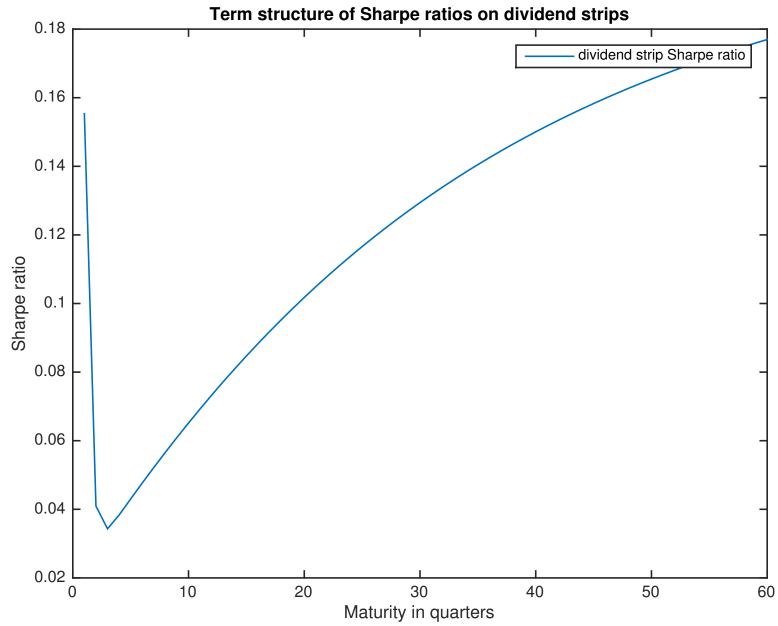

Figure 1.4: Model-implied term structure of Sharpe ratios on dividend strips, SR_n^d. ($\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = \lambda_U^{optimal}$)

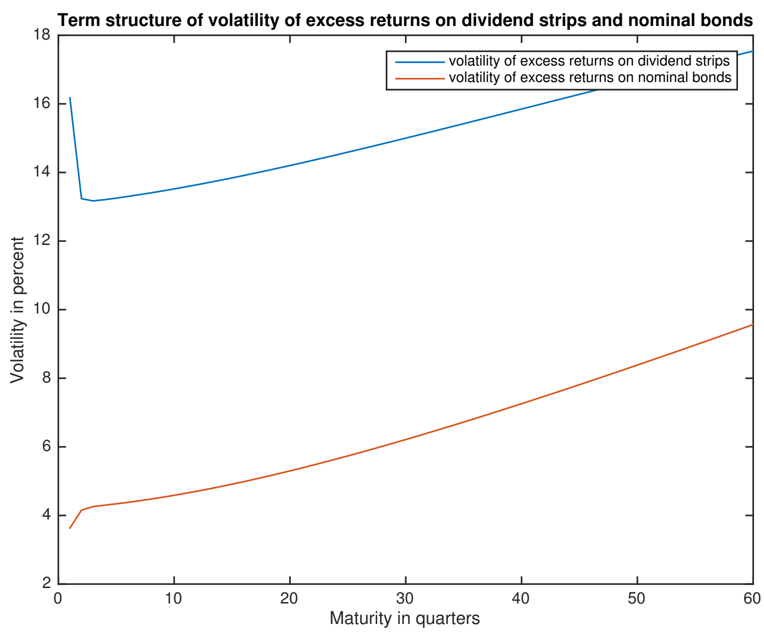

Figure 1.5: Model-implied term structure of volatilities of excess returns on dividend strips, $std(r_n^d - r^f)$, and on nominal bonds, $std(r_n^\$ - r^f)$. ($\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = \lambda_U^{optimal}$)

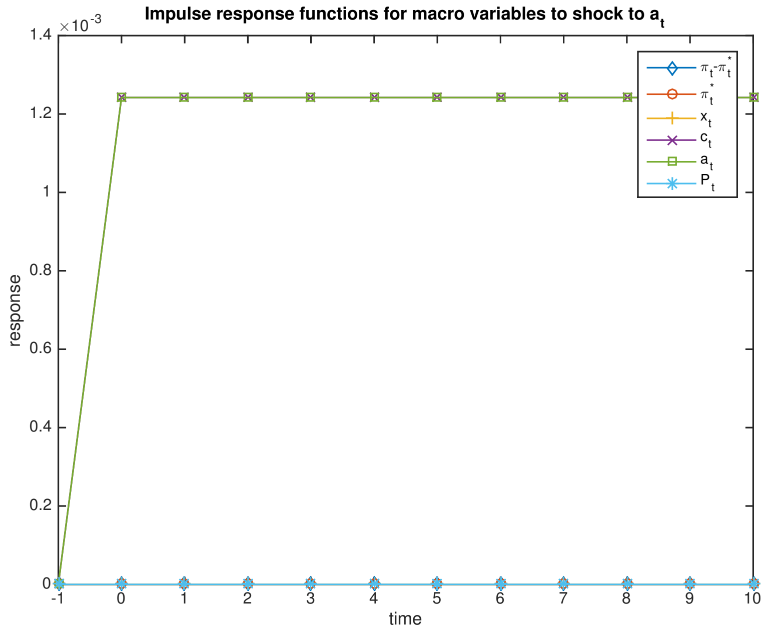

Figure 1.6: Impulse-response functions for macro variables to one standard deviation short-run productivity shock at $t = 0$. $(\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = 0)$

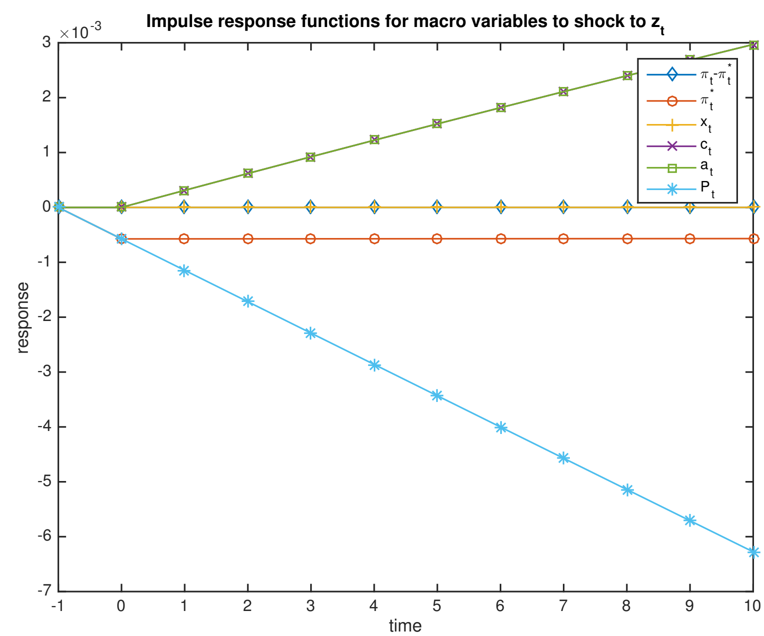

Figure 1.7: Impulse-response functions for macro variables to one standard deviation long-run productivity shock at $t = 0$. $(\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = 0)$

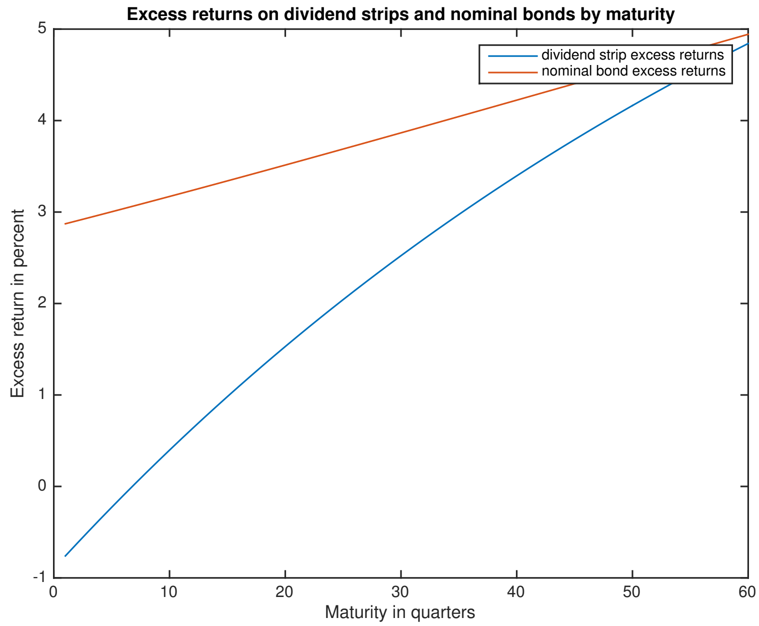

Figure 1.8: Model-implied term structure of expected excess returns on dividend strips, $E[r_n^d - r^f]$, and nominal bonds, $E[r_n^\$ - r^f]$. ($\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = 0$)

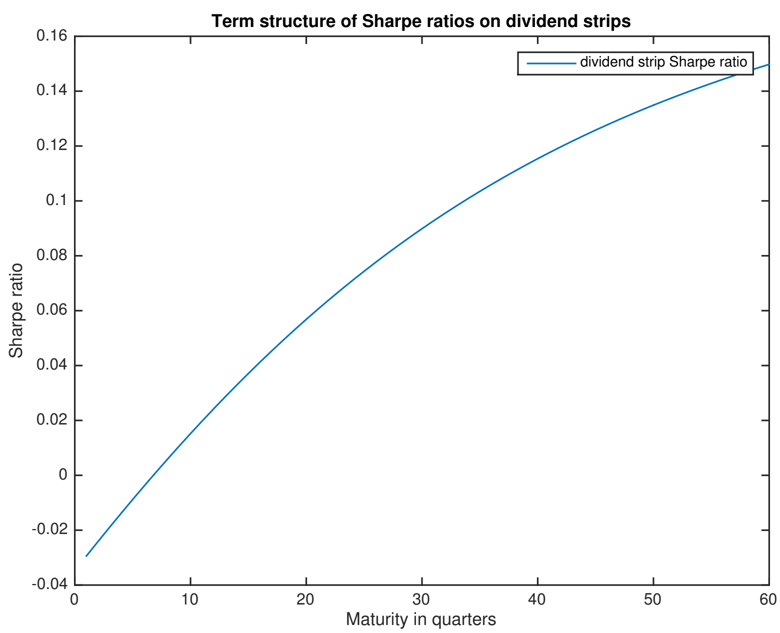

Figure 1.9: Model-implied term structure of Sharpe ratios on dividend strips, SR_n^d. ($\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = 0$)

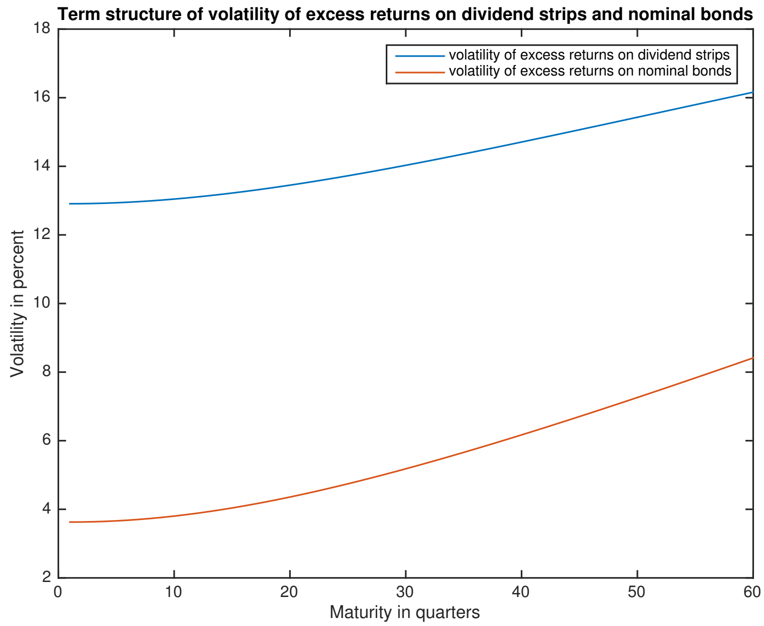

Figure 1.10: Model-implied term structure of volatilities of excess returns on dividend strips, $std(r_n^d - r^f)$, and on nominal bonds, $std(r_n^\$ - r^f)$. ($\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = 0$)

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Appendixes

A.1 Solve for Normalized Value Function

First, I solve for the coefficients G_0, G_1, and G_2 in the expression $\Delta\gamma_{t+1} = G_0 + G_1'\chi_{t+1} + G_2'\chi_t$:

$$\begin{aligned}
\Delta\gamma_{t+1} &= \Delta c_{t+1} + \varphi_0 \Delta l_{t+1} \\
&= \Delta a_{t+1} + \Delta x_{t+1} - \frac{\varphi_0 N}{1-N}\Delta n_{t+1} \\
&= \Delta a_{t+1} + \Delta x_{t+1} - \frac{\varphi_0 N}{1-N}\Delta x_{t+1} \\
&= \left[e_1' + \left(1 - \frac{\varphi_0 N}{1-N}\right) e_3'\right]\chi_{t+1} - \left(1 - \frac{\varphi_0 N}{1-N}\right) e_3'\chi_t,
\end{aligned}$$

implying that $G_0 = 0$, $G_1' = e_1' + \left(1 - \frac{\varphi_0 N}{1-N}\right) e_3'$, and $G_2' = -\left(1 - \frac{\varphi_0 N}{1-N}\right) e_3'$.

The coefficients F_0 and F_1 can be solved for by conjecturing that $v_t - \gamma_t = F_0 + F_1'\chi_t$ and plugging into equation 1.40:

$$\begin{aligned}
F_0 + F_1'\chi_t &= \frac{\beta(1-\beta)}{\sigma} E_t\left[\left(F_0 + F_1'\chi_{t+1} + G_0 + G_1'\chi_{t+1} + G_2'\chi_t\right)\frac{\sigma}{1-\beta}\right] \\
&+ \frac{\beta(1-\beta)}{\sigma}\frac{1}{2} var_t\left[\left(F_0 + F_1'\chi_{t+1} + G_0 + G_1'\chi_{t+1} + G_2'\chi_t\right)\frac{\sigma}{1-\beta}\right] \\
&= \frac{\beta(1-\beta)}{\sigma}\left[\left(F_0 + F_1'c + F_1'H\chi_t + G_0 + G_1'c + G_1'H\chi_t + G_2'\chi_t\right)\frac{\sigma}{1-\beta}\right] \\
&+ \frac{\beta(1-\beta)}{\sigma}\frac{1}{2}\left[\frac{\sigma^2}{(1-\beta)^2}(F_1' + G_1')GG'(F_1 + G_1)\right],
\end{aligned}$$

implying that $F_0 = \frac{\beta}{1-\beta}\left[G_0 + (F_1' + G_1')c + \frac{1}{2}\frac{\sigma}{1-\beta}(F_1' + G_1')GG'(F_1 + G_1)\right]$ and $F_1' = \beta(G_1'H + G_2')(I - \beta H)^{-1}$.

A.2 Real and Nominal Pricing Kernels

The real pricing kernel is

$$q_{t,t+1} = -\Omega_0 - \Omega_1'\chi_{t+1} - \Omega_2'\chi_t, \tag{A.59}$$

where $\Omega_0 = -\log(\beta) + \frac{1}{2}\frac{\sigma^2}{(1-\beta)^2}(F_1' + G_1')GG'(F_1 + G_1)$, $\Omega_1' = e_1' + e_3' - \frac{\sigma}{1-\beta}(F_1' + G_1')$, and $\Omega_2' = -e_3' + \frac{\sigma}{1-\beta}(F_1' + G_1')G$. The nominal pricing kernel is defined as $q_{t,t+1}^{\$} = q_{t,t+1} - \pi_{t+1}$:

$$q_{t,t+1}^{\$} = -\Omega_0^{\$} - \Omega_1'^{\$}\chi_{t+1} - \Omega_2'^{\$}\chi_t, \tag{A.60}$$

where $\Omega_0^{\$} = -\log(\beta)+\pi^*+\frac{1}{2}\frac{\sigma^2}{(1-\beta)^2}(F_1'+G_1')GG'(F_1+G_1)$, $\Omega_1'^{\$} = e_1'+e_3'+e_2'+e_4'-\frac{\sigma}{1-\beta}(F_1'+G_1')$, and $\Omega_2'^{\$} = -e_3' + \frac{\sigma}{1-\beta}(F_1' + G_1')G$.

A.3 Price-Dividend Ratio and Return on Equity

Conjecturing that $pd_t = \xi_0 + \xi_1'\chi_t$ and plugging the expression for r_{t+1}^d in equation 1.55 and $q_{t,t+1}$ into the Euler equation $E_t[e^{q_{t,t+1}+r_{t+1}^d}] = 1$:

$$\begin{aligned}
&E_t[\exp(-\Omega_0 - \Omega_1'\chi_{t+1} - \Omega_2'\chi_t + \kappa_0 + \kappa_1 pd_{t+1} - pd_t + \delta\Delta c_{t+1} + \sigma_d\epsilon_{t+1}^d)] = 1 \\
\implies &E_t[\exp(-\Omega_0 - \Omega_1'\chi_{t+1} - \Omega_2'\chi_t + \kappa_0 + \kappa_1(\xi_0 + \xi_1'\chi_{t+1}) - \xi_0 - \xi_1'\chi_t + \delta(e_1' + e_3')\chi_{t+1} \\
&- \delta e_3'\chi_t + \sigma_d\epsilon_{t+1}^d)] = 1 \\
\implies &[-\Omega_1' + \kappa_1\xi_1' + \delta(e_1' + e_3')]H - \Omega_2' - \xi_1' - \delta e_3' = 0 \\
&\text{and} \\
&\frac{1}{2}[-\Omega_1' + \kappa_1\xi_1' + \delta(e_1' + e_3')]GG'[-\Omega_1 + \kappa_1\xi_1 + \delta(e_1 + e_3)] + \frac{1}{2}\sigma_d^2 - \Omega_0 + \kappa_0 \\
&+ \kappa_1\xi_0 - \xi_0 = 0.
\end{aligned}$$

I solve the two equations above for ξ_0 and ξ_1: $\xi_1' = [(-\Omega_1'+\delta(e_1'+e_3'))H-\Omega_2'-\delta e_3'](I-\kappa_1 H)^{-1}$ and $\xi_0 = \frac{1}{1-\kappa_1}\{\frac{1}{2}[-\Omega_1' + \kappa_1\xi_1' + \delta(e_1' + e_3')]GG'[-\Omega_1 + \kappa_1\xi_1 + \delta(e_1 + e_3)] + [-\Omega_1' + \kappa_1\xi_1' + \delta(e_1' + e_3')]c + \frac{1}{2}\sigma_d^2 - \Omega_0 + \kappa_0\}$.

A.4 Algorithm for Optimal Monetary Policy Problem

For a given guess of the matrix $\tilde{H}$, the law of motion for $\tilde{\chi}_t$ can be used to calculate the following conditional expectations:

$$E_t\tilde{s}_{t+1} = \tilde{H}_{\tilde{s}\tilde{s}}\tilde{s}_t + \tilde{H}_{\tilde{s}\tilde{\lambda}}\tilde{\lambda}_t \tag{A.61}$$

and

$$E_t\tilde{\lambda}_{t+1} = \tilde{H}_{\tilde{\lambda}\tilde{s}}\tilde{s}_t + \tilde{H}_{\tilde{\lambda}\tilde{\lambda}}\tilde{\lambda}_t. \tag{A.62}$$

Substitute these expressions into the first-order conditions 1.37 and 1.38 to obtain

$$\Gamma_0 \begin{bmatrix} \tilde{s}_t \\ \tilde{\lambda}_t \end{bmatrix} + \Gamma_1 \begin{bmatrix} \tilde{s}_{t-1} \\ \tilde{\lambda}_{t-1} \end{bmatrix} + \Gamma_\epsilon \epsilon_t = 0,$$

where

$$\Gamma_0 \equiv \begin{bmatrix} A_0 + A_1\tilde{H}_{\tilde{s}\tilde{s}} & A_1\tilde{H}_{\tilde{s}\tilde{\lambda}} \\ 2W + \beta A'_{-1}\tilde{H}_{\tilde{\lambda}\tilde{s}} & A'_0 + \beta A'_{-1}\tilde{H}_{\tilde{\lambda}\tilde{\lambda}} \end{bmatrix},$$

$$\Gamma_1 \equiv \begin{bmatrix} A_{-1} & 0 \\ 0 & \beta^{-1}A'_1 \end{bmatrix},$$

and

$$\Gamma_\epsilon \equiv \begin{bmatrix} B \\ 0 \end{bmatrix}.$$

The law of motion is then

$$\begin{bmatrix} \tilde{s}_t \\ \tilde{\lambda}_t \end{bmatrix} = -\Gamma_0^{-1}\Gamma_1 \begin{bmatrix} \tilde{s}_{t-1} \\ \tilde{\lambda}_{t-1} \end{bmatrix} - \Gamma_0^{-1}\Gamma_\epsilon\epsilon_t.$$

The remaining step is to verify whether this law of motion corresponds to the initial guess, $H = -\Gamma_0^{-1}\Gamma_1$. If not, the guess-and-verify procedure is repeated until convergence. The algorithm can be summarized succinctly in four steps:

1. Using a guess, $\tilde{H}_{guess}$, calculate the values of Γ_0 and Γ_1.

2. Calculate $\tilde{H} = -\Gamma_0^{-1}\Gamma_1$.

3. Check whether $\|\tilde{H} - \tilde{H}_{guess}\| < \epsilon$ for some distance measure $\|.\|$ and $\epsilon > 0$. I employ the Frobenius norm, which is defined as $\|A\|^2 = trace(AA')$. If the guess and solution have converged, proceed to the next step. Otherwise, update $\tilde{H}_{guess} = \tilde{H}$, and repeat steps 1-3 until convergence.

4. Calculate $\tilde{G} = -\Gamma_0^{-1}\Gamma_\epsilon$.

Once the law of motion of $\tilde{\chi}_t$ has been solved for, variables in the expanded state vector which are redundant can be removed to form the law of motion for χ_t.

A.5 Derivation of the New Keynesian Phillips Curve

Using the definition of nominal marginal cost in equation 1.37, the profit maximization problem for the intermediate goods firm in equation 1.38 can be written as

$$\max_{P_t(j)} \left[\sum_{T=t}^{\infty} \omega^{T-t} Q_{t,T} \left(\frac{P_T(j)}{P_T} Y_T(j) - \frac{NMC_T Y_T(j)}{P_T} \right) \right].$$

Plugging in the demand function for its own good, the above expression can be rewritten as

$$\max_{P_t(j)} \left[\sum_{T=t}^{\infty} \omega^{T-t} Q_{t,T} \left(\frac{P_T(j)}{P_T} \left(\frac{P_t(j) e^{(1+\pi^*_{t+1}+\ldots+\pi^*_T)}}{P_T} \right)^{-\epsilon} - MC_T \left(\frac{P_t(j) e^{(1+\pi^*_{t+1}+\ldots+\pi^*_T)}}{P_T} \right)^{-\epsilon} \right) \right].$$

The first-order condition for the intermediate goods firm is then

$$P_t(j) = \mu \frac{E_t\left[\sum_{T=t}^{\infty} \omega^{T-t} Q_{t,T} P_T^{\epsilon} e^{(1+\pi_{t+1}^*+\ldots+\pi_T^*)(-\epsilon)} MC_T Y_T\right]}{E_t\left[\sum_{T=t}^{\infty} \omega^{T-t} Q_{t,T} P_T^{\epsilon-1} e^{(1+\pi_{t+1}^*+\ldots+\pi_T^*)(1-\epsilon)} Y_T\right]}.$$

Using the fact that $Q_{t,t+1} = \beta\frac{C_t}{C_{t+1}}M_{t,t+1}$, $M_{t,t+1} = \frac{\exp(\sigma U_{t+1})}{E_t[\exp(\sigma U_{t+1})]}$, $Q_{t,T} = Q_{t,t+1}Q_{t+1,t+2}\ldots Q_{T-1,T}$, and $M_{t,T} = M_{t,t+1}M_{t+1,t+2}\ldots M_{T-1,T}$ and the fact that the right side of the above equation has no dependence on j, the first-order condition can be written as

$$P_t^{\#} = \mu \frac{E_t\left[\sum_{T=t}^{\infty} (\omega\beta)^{T-t} M_{t,T} P_T^{\epsilon} e^{(1+\pi_{t+1}^*+\ldots+\pi_T^*)(-\epsilon)} MC_T\right]}{E_t\left[\sum_{T=t}^{\infty} (\omega\beta)^{T-t} M_{t,T} P_T^{\epsilon-1} e^{(1+\pi_{t+1}^*+\ldots+\pi_T^*)(1-\epsilon)}\right]},$$

where the superscript # denotes the optimal reset price of any firm. The numerator can be written recursively as

$$Z_{1,t} = MC_t P_t^{\epsilon} + \omega\beta E_t[M_{t,t+1} Z_{1,t+1} e^{-\epsilon\pi_{t+1}^*}]$$

and the denominator can be written recursively as

$$Z_{2,t} = P_t^{\epsilon-1} + \omega\beta E_t[M_{t,t+1} Z_{2,t+1} e^{(1-\epsilon)\pi_{t+1}^*}].$$

The previous two equations can then be written as

$$z_{1,t} = \frac{Z_{1,t}}{P_t^{\epsilon}} = MC_t + \omega\beta E_t[M_{t,t+1} z_{1,t+1} e^{-\epsilon\pi_{t+1}^*} e^{\epsilon\pi_{t+1}}]$$

and

$$z_{2,t} = \frac{Z_{2,t}}{P_t^{\epsilon-1}} = 1 + \omega\beta E_t[M_{t,t+1} z_{2,t+1} e^{(\epsilon-1)\pi_{t+1}^*} e^{\pi_{t+1}(1-\epsilon)}].$$

Now, the reset price expression can be expressed as

$$P_t^{\#} = \mu P_t \frac{z_{1,t}}{z_{2,t}}.$$

The equation describing the evolution of prices in the Calvo setting is

$$P_t = [\omega \Pi_t^{*(1-\epsilon)} P_{t-1}(j)^{1-\epsilon} + (1-\omega) P_t^{\#(1-\epsilon)}]^{\frac{1}{1-\epsilon}}. \tag{A.63}$$

Dividing both sides by P_{t-1} and raising both sides to the power $1-\epsilon$, I obtain the following expression:

$$\Pi_t^{1-\epsilon} = \omega \Pi_t^{*(1-\epsilon)} + (1-\omega)\Pi_t^{\#(1-\epsilon)}.$$

Using the fact that $\Pi_t = \Pi_t^{\#} = \Pi_t^*$ in the stochastic steady-state, log-linearizing the above equation about the stochastic steady-state, I obtain:

$$\pi_t - \pi_t^* = (1-\omega)(\pi_t^{\#} - \pi_t^*).$$

Dividing both sides of the reset price expression by $P_{t-1} e^{\pi_t^*}$ and utilizing the equation above relating the inflation rate to the reset inflation rate, the previous equation can be written as

$$e^{(\pi_t - \pi_t^*)\left(\frac{\omega}{1-\omega}\right)} = \mu \frac{z_{1,t}}{z_{2,t}}.$$

Taking logs of both sides and taking the log-linear approximation of the right hand side, I obtain

$$\begin{aligned}
\frac{\omega}{1-\omega}(\pi_t - \pi_t^*) &= \log(\mu) + \log(z_1) + \frac{1}{z_1}(z_{1,t} - z_1) - \log(z_2) - \frac{1}{z_2}(z_{2,t} - z_2) \\
&= \frac{z_{1,t} - z_1}{z_1} - \frac{z_{2,t} - z_2}{z_2},
\end{aligned}$$

where $z_1 = \frac{\mu^{-1}}{1-\omega\beta}$ and $z_2 = \frac{1}{1-\omega\beta}$. The next step is to log-linearize $z_{1,t}$ and $z_{2,t}$:

$$\frac{z_{1,t} - z_1}{z_1} = \frac{1}{z_1}(MC_t - \mu^{-1}) + \frac{\omega\beta}{z_1} E_t[z_{1,t+1} - z_1] + \omega\beta\epsilon E_t[\pi_{t+1} - \pi_t^*]$$

and

$$\frac{z_{2,t} - z_2}{z_2} = \frac{\omega\beta}{z_2} E_t[z_{2,t+1} - z_2] + \omega\beta(\epsilon - 1) E_t[\pi_{t+1} - \pi_t^*].$$

Subtracting the second equation from the first, I obtain

$$\frac{z_{1,t} - z_1}{z_1} - \frac{z_{2,t} - z_2}{z_2} = (1 - \omega\beta)\left(\frac{MC_t - \mu^{-1}}{\mu^{-1}}\right) + \omega\beta E_t\left[\frac{z_{1,t+1} - z_1}{z_1}\right] - \omega\beta E_t\left[\frac{z_{2,t+1} - z_2}{z_2}\right] + \omega\beta E_t[\pi_{t+1} - \pi_{t+1}^*].$$

Using the fact that $\frac{\omega}{1-\omega}(\pi_t - \pi_t^*) = \frac{z_{1,t} - z_1}{z_1} - \frac{z_{2,t} - z_2}{z_2}$, the above expression can be rewritten as

$$\frac{\omega}{1 - \omega}(\pi_t - \pi_t^*) = (1 - \omega\beta)\left(\frac{MC_t - \mu^{-1}}{\mu^{-1}}\right) + \omega\beta\frac{\omega}{1 - \omega} E_t[\pi_{t+1} - \pi_{t+1}^*] + \omega\beta E_t[\pi_{t+1} - \pi_{t+1}^*].$$

This expression can be simplified to obtain the New Keynesian Phillips Curve:

$$\pi_t - \pi_t^* = \frac{(1 - \omega\beta)(1 - \omega)}{\omega} \frac{MC_t - \mu^{-1}}{\mu^{-1}} + \beta E_t[\pi_{t+1} - \pi_{t+1}^*].$$

Using the definition of the output gap, the New Keynesian Phillips Curve can be expressed in terms of the output gap instead of real marginal cost:

$$\pi_t - \pi_t^* = \frac{(1 - \omega\beta)(1 - \omega)(\mu^{-1} + \varphi_0)}{\omega\varphi_0} x_t + \beta E_t[\pi_{t+1} - \pi_{t+1}^*].$$

A.6 Derivation of the Welfare-Based Quadratic Loss Function

The first summation in equation 1.29 is

$$\sum_{t=0}^{\infty} \beta^t E_{-1}[\log(C_t) + \varphi_0 \log(1 - N_t)]. \tag{A.64}$$

The second-order approximation of the first term in the summation, $\log(C_t)$, is

$$\begin{aligned}
\log(C_t) &= \log(Y_t) \\
&= \log\left(\frac{Y_t}{Y_t^f}\right) + \log\left(Y_t^f\right) \\
&= \log(X_t) + \log\left(Y_t^f\right) \\
&\approx x_t + \frac{1}{2}x_t^2 - \log(1 + \varphi_0 \mu) + a_t + ...
\end{aligned}$$

Using the fact that $Y_t = \frac{A_t N_t}{\Delta_t}$, where Δ_t is the price dispersion, the second-order approximation of the second term in the summation, $\varphi_0 \log(1 - N_t)$, is

$$\begin{aligned}
\varphi_0 \log(1 - N_t) &= \varphi_0 \log\left(1 - \frac{\Delta_t Y_t}{A_t}\right) \\
&= \varphi_0 \log\left(1 - \frac{\Delta_t X_t Y_t^f}{A_t}\right) \\
&= \varphi_0 \log(1 - N) - \frac{\varphi_0 N}{1 - N}(X_t - 1) - \frac{\varphi_0 N}{1 - N}(\Delta_t - 1) - \frac{1}{2}\frac{\varphi_0 N^2}{(1 - N)^2}(X_t - 1)^2 + ... \\
&\approx \varphi_0 \log(1 - N) - \frac{1}{\mu}\left(x_t + \frac{1}{2}x_t^2\right) - \frac{1}{\mu}\delta_t - \frac{1}{2}\frac{1}{\varphi_0 \mu^2}x_t^2 + ...
\end{aligned}$$

Price dispersion, Δ_t, is defined as

$$\begin{aligned}
\Delta_t &\equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj \\
&= (1-\omega)\left(\frac{P_t^{\#}}{P_t}\right)^{-\epsilon} + \omega\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\epsilon}\Delta_{t-1}.
\end{aligned}$$

Using equation A.63 to express $\frac{P_t^{\#}}{P_t}$ in terms of the inflation rate, the expression for price dispersion can be written as

$$\Delta_t = (1-\omega)\left[\frac{1-\omega\left(\frac{\Pi_t}{\Pi^*}\right)^{\epsilon-1}}{1-\omega}\right]^{\frac{-\epsilon}{1-\epsilon}} + \omega\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\epsilon}\Delta_{t-1}$$

Taking the second-order log-linear approximation of the above expression, the log of price dispersion, δ_t, can then be expressed recursively as

$$\delta_t = \omega\delta_{t-1} + \frac{\omega\epsilon}{2(1-\omega)}(\pi_t - \pi_t^*)^2.$$

Iterating backwards, the log price dispersion at time t can be expressed in terms of the sum of squared deviations of inflation from its target:

$$\delta_t = \omega^{t+1}\delta_{-1} + \frac{\omega\epsilon}{2(1-\omega)}\sum_{j=0}^{t}\omega^{t-j}(\pi_j - \pi_j^*)^2.$$

Therefore,

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t\delta_t &= \sum_{t=0}^{\infty}\beta^t[\omega^{t+1}\delta_{-1} + \frac{\omega\epsilon}{2(1-\omega)}\sum_{j=0}^{t}\omega^{t-j}(\pi_j - \pi_j^*)^2] \\
&= \frac{\omega\delta_{-1}}{1-\omega\beta} + \frac{\omega\epsilon}{2(1-\omega)}\sum_{t=0}^{\infty}\sum_{j=0}^{t}\beta^t\omega^{t-j}(\pi_j - \pi_j^*)^2 \\
&= \frac{\omega\delta_{-1}}{1-\omega\beta} + \frac{\omega\epsilon}{2(1-\omega)(1-\omega\beta)}\sum_{t=0}^{\infty}\beta^t(\pi_t - \pi_t^*)^2.
\end{aligned}$$

A.7 Derivation of the Per Period Utility Constraint

The lifetime utility function of the representative consumer is expressed recursively in equation 1.1 as

$$U_t = \log(C_t) + \varphi_0 \log(1 - N_t) + \frac{\beta}{\sigma} \log(E_t[\exp(\sigma U_{t+1})]).$$

The first-order approximation of the first term can be rewritten in terms of the output gap and productivity:

$$\begin{aligned}
\log(C_t) &= \log(Y_t) \\
&= \log\left(\frac{Y_t}{Y_t^f}\right) + \log\left(Y_t^f\right) \\
&= \log(X_t) + y_t^f \\
&= x_t - \log(1 + \varphi_0 \mu) + a_t + \ldots
\end{aligned}$$

The first-order approximation of the second term is

$$\begin{aligned}
\varphi_0 \log(1 - N_t) &= \varphi_0 \log\left(1 - \frac{Y_t \Delta_t}{A_t}\right) \\
&= \varphi_0 \log\left(1 - \frac{X_t Y_t^f \Delta_t}{A_t}\right) \\
&= \varphi_0 \log\left(1 - \frac{\Delta_t X_t}{1 + \varphi_0 \mu}\right) \\
&= \varphi_0 \log(1 - N) - \frac{\varphi_0 N}{1 - N}(X_t - 1) + \ldots \\
&= \varphi_0 \log\left(\frac{\varphi_0 \mu}{1 + \varphi_0 \mu}\right) - \frac{1}{\mu} x_t + \ldots
\end{aligned}$$

The first-order approximation of the third term is

$$\frac{\beta}{\sigma} \log(E_t[\exp(\sigma U_{t+1})]) = \beta U + \beta E_t[U_{t+1} - U] + \ldots$$

The log-linearized utility function can then be written as

$$U_t = -(1+\varphi_0)\log(1+\varphi_0\mu) + \varphi_0\log(\varphi_0\mu) + a_t + \left(1-\frac{1}{\mu}\right)x_t + \beta E_t[U_{t+1}].$$

By subtracting from U_t the value, U^*, I am able eliminate the constant terms from the above expression:

$$U_t - U^* = a_t + \left(1-\frac{1}{\mu}\right)x_t + \beta E_t[U_{t+1} - U^*].$$

U^* is related to the steady-state value of lifetime utility, U, as follows:

$$U^* = U - \frac{\left(1-\frac{1}{\mu}\right)x^*}{1-\beta},$$

where

$$U = \frac{\varphi_0\log(\varphi_0\mu) - (1+\varphi_0)\log(1+\varphi_0\mu) + \left(1-\frac{1}{\mu}\right)x^*}{1-\beta}.$$

A.8 Eliminating $U_t - U^*$ from the loss function

The log-linearized expression for lifetime utility can be expressed as the discounted sum of future productivity shocks and output gaps as follows:

$$\begin{aligned} U_t - U^* &= E_t\left[\sum_{k=0}^{\infty}\beta^k(a_{t+k} + \nu x_{t+k})\right] \\ &= \nu E_t\left[\sum_{k=0}^{\infty}\beta^k x_{t+k}\right] + \frac{a_t}{1-\beta} + \frac{\beta z_t}{(1-\beta)(1-\beta\rho_z)}. \end{aligned}$$

The New Keynesian Phillips Curve can similarly be expressed as the discounted sum of future output gaps:

$$\pi_t - \pi_t^* = \kappa E_t\left[\sum_{k=0}^{\infty}\beta^k x_{t+k}\right]$$

Therefore,

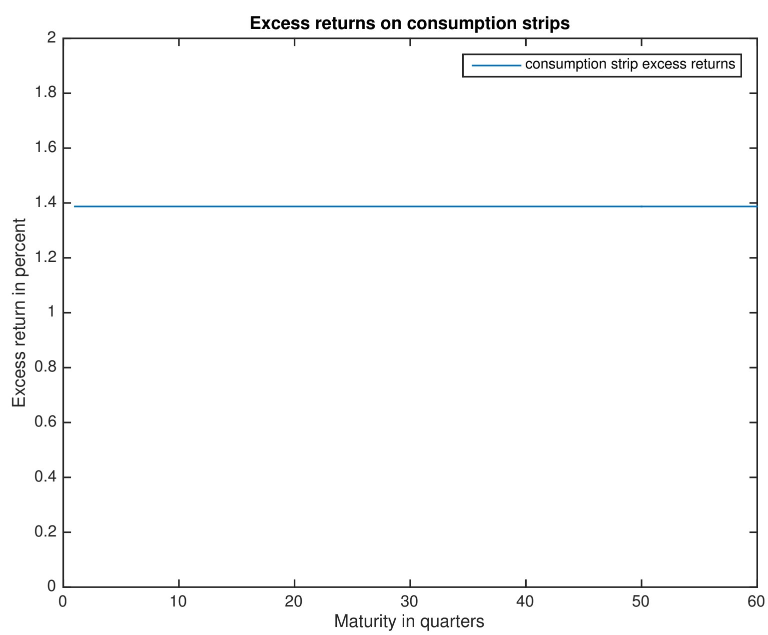
$$U_t - U^* = \frac{\nu}{\kappa}(\pi_t - \pi_t^*) + \frac{a_t}{1-\beta} + \frac{\beta z_t}{(1-\beta)(1-\beta\rho_z)}$$

and

$$\begin{aligned} U_t - U^* - E_{t-1}[U_t - U^*] = \frac{\nu}{\kappa}(\pi_t - \pi_t^*) + \frac{a_t}{1-\beta} + \frac{\beta z_t}{(1-\beta)(1-\beta\rho_z)} - \frac{\nu}{\kappa\beta}(\pi_{t-1} - \pi_{t-1}^*) \\ - \frac{a_{t-1}}{1-\beta} - \frac{\beta z_{t-1}}{(1-\beta)(1-\beta\rho_z)} + \frac{\nu x_{t-1}}{\beta}. \end{aligned}$$

A.9 Term Structure of Consumption Risk Premia

The price-consumption ratio is constant as in other long-run risks models such as Bansal and Yaron (2004) when the intertemporal elasticity of substitution is 1 as the intertemporal substitution effect is cancelled out by the wealth effect. Therefore, the term structure of consumption risk premia is flat. I plot the term structure of consumption risk premia under the baseline calibration below.



Model-implied term structure of expected excess returns on consumption strips, $E[r_n^c - r^f]$. $(\lambda_\pi = \lambda_\pi^{optimal}, \lambda_x = \lambda_x^{optimal}, \lambda_U = \lambda_U^{optimal})$

Chapter 2

Sovereign Credit Risk in a Monetary Union

2.1 Introduction

In this paper I build a structural model of sovereign credit risk in which a monetary authority of a monetary union (such as the European Central Bank) faces the problem of reducing the real burden of debt in bad times and increasing it in good times through inflation across multiple countries in a monetary union with less than perfect correlations in output. In the presence of deadweight losses due to default, sovereigns have an incentive to inflate away a portion of their nominal debt in bad states and deflate in good states. In a monetary union, the central monetary authority has one policy instrument with which to reduce the real burden of debt in these bad states. This policy instrument is the inflation rate, which must be identical across countries given the assumption of an international investor. Even without the assumption of an international investor, although there will be different inflation rates in each country, the monetary authority will have one degree of freedom in adjusting inflation across the monetary union. With less than perfect correlations in output across the countries comprising the monetary union, the central monetary authority of a monetary union will be less effective in its goal of avoiding default through strategic inflation than it would be in the case of a single country with complete control over its own monetary policy. The ability of a country with control over its own monetary policy to more effectively avoid default using strategic inflation results in lower credit spreads and Arrow-Debreu default prices for most countries. For countries with high volatility real asset values, credit spreads and Arrow-Debreu default prices will be higher when these countries conduct their own independent monetary policy since they are better able to hedge their nominal liabilities and optimally increase debt-to-GDP levels in this regime.

My objective in this paper is to quantify the fraction of sovereign credit spreads of Eurozone countries that can be attributed to loss of flexibility in monetary policy inherent in joining a monetary union. I call this the monetary union risk premium as a fraction of credit spreads. For most countries, this quantity will be positive, but for countries with high volatility real asset values, it will be negative. I also aim to quantify the costs in terms of welfare reduction posed by a loss of flexibility in monetary policy, specifically how the benefits of debt and expected losses in default change when comparing a country in a monetary union with the counterfactual of the country conducting its own monetary policy. Additionally, I show how Arrow-Debreu default prices and 10-year default rates change as a function of the monetary arrangement. More broadly speaking, the paper answers the question of how monetary policy interacts with the optimal sovereign capital structure problem and how the nature of the monetary arrangement can have significant implications for this interaction. This paper does not consider the benefits of entering a monetary union, including reduced transaction costs and exchange rate stability. Thus, it makes no claims on whether it is beneficial for a specific country to enter into a monetary union. A model in which sovereigns trade off the benefits and costs of entering a monetary union would be needed to make such claims.

This paper lies at the intersection of several strands of the macroeconomics and finance literature. It builds on an extensive literature studying default in a structural context. The majority of these papers examine the asset pricing implications of the risk of default at the level of the firm. Merton (1974) was the first to apply a contingent claims analysis to the pricing of equity and debt claims. His model allows default to occur only at maturity. Black and Cox (1976) extended Merton's model to allow default before maturity, and is the first so-called first passage model of default. Leland (1994) incorporated the tax benefits and default costs of debt to create a trade-off model in which firms choose the optimal coupon level of debt and default boundary. Leland and Toft's (1996) model improved on Leland's original model by allowing for optimal maturity choice as well as choosing the optimal amount of debt to issue.

In the international macroeconomics literature, Eaton and Gersovitz (1981) and Bulow and Rogoff (1989) laid the foundations for research into why sovereigns lend and default.

Arellano (2008), Arellano and Ramanarayanan (2012), and Yue (2010) continue their work and analyze the dynamics of credit spreads in dynamic stochastic general equilibrium models in which governments default strategically in the best interest of households. More recently, Hur, Kondo, and Perri (2017) extend Arellano's model and show that the co-movement of inflation and domestic consumption growth affects real interest rates and the likelihood of debt crises.

In the finance literature, there are a number of papers that have applied the machinery of contingent claims analysis to study sovereign default in a structural context. Gibson and Sundaresan (2001) endogenize the default trigger by building in a bargaining game between sovereigns and creditors. Credit risk premiums reflect this fact. Bodie, Gray, and Merton (2007) build a Merton-type model for the corporate, financial, household, and sovereign sectors of the economy and specify the interconnections between sectors. Andrade (2009) relates emerging market expected stock returns to sovereign yields. Jeanneret (2015) considers the optimal capital structure of the sovereign by building a Leland-type tradeoff model for sovereigns in which a sovereign trades off the returns it can earn on domestic investments by issuing debt and the costs of default. He then extracts a time series for the sovereign asset value using local stock prices and determines the model-implied credit spread. Chernov (2016) builds a sovereign contingent claims model to determine whether US CDS spreads reflect the risk of fiscal default, a state in which the budget balance cannot be restored by raising taxes or eroding the real value of debt by raising inflation.

This paper also builds upon the previous sovereign credit risk literature by considering the fact that unlike the optimal capital structure problem for firms, which takes inflation as given so firms have no control over the real value of nominal liabilities, sovereigns have the ability to affect the real values of nominal government debt through the strategic use of inflation. To my knowledge, it is the first paper to so. Papers which consider the effect of inflation on corporate default are Bhamra, Fisher, and Kuehn (2011) and Kang and

Pflueger (2015). These papers address how monetary policy and inflation can have effects on the real economy outside the sticky price paradigm of the New Keynesian literature.

Additionally, this paper adds to the literature on optimal monetary policy. New Keynesian optimal monetary policy models were summarized by Clarida, Gertler, and Galí (1999). This line of research mostly focuses on how inflation and the output gap should respond to cost-push shocks so as to minimize a quadratic loss function derived from the welfare of a representative agent. However, these papers do not consider deadweight losses in sovereign default as a possible motivation to use inflation strategically to avoid default.

In the remainder of this paper, I will introduce the model in section 2, analyze the comparative statics in section 3, go over the data sources in section 4, calibrate the model in section 5, discuss the results in section 6, and finally, conclude in section 7 with some suggestions for future work.

2.2 Model

2.2.1 Environment

I consider the case of an N country monetary union. The following presentation of the model does not presume that the parameters are symmetric across countries.

I assume that capital markets are frictionless, complete, and investors have perfect information on the state of the economy. Additionally, time is continuous and goes on forever. Superscripts and subscripts $i = 1, ..., N$ will index the country being referred to and nominal quantities will be denoted with a star. I consider N sovereign countries with infinite lifespans. Each country is constituted of a representative unlevered firm as in Jeanneret (2015), which generates an infinite stream of operating income in nominal terms of $Y^{i,*}$ and the government imposes a tax rate of τ_i on the operating income of the representative firm so that the sovereign receives an infinite stream of nominal cashflows,

$\tau_i Y^{i,*}$ at every instant. Each representative firm's nominal asset value, $V^{i,*}$, is the value of the claim on this infinite stream of nominal cashflows so the discounted value of the government's fiscal revenues is $\tau_i V^{i,*}$. The real operating cashflows of the each country's representative firm evolve according to the following geometric Brownian motion:

$$\frac{dY_t^i}{Y_t^i} = \mu_i dt + \sigma_i dW_t^i \tag{2.1}$$

$$= \mu_i dt + \sigma_i \lambda_i dX_t + \sigma_i \sqrt{1-\lambda_i^2} dZ_t^i \tag{2.2}$$

where W_t^i is a standard Brownian motion under the physical measure, and μ_i and σ_i are the expected growth rate and volatility of operating cashflows, respectively. The Brownian motion driving the cashflow process of the home representative firm, $W_t^i = \lambda_i X_t + \sqrt{1-\lambda_i^2} Z_t^i$, is comprised of a aggregate component X_t, a country-specific component Z_t^i, and is constructed so the correlation between W_t^i and X_t is $corr(W_t^i, X_t) = \lambda_i$. X_t and each Z_t^i are standard Brownian motions which are uncorrelated with all other primitive shocks in the economy. The specification of shocks follows a one factor model, so the only source of covariance between the real cashflows of each country's representative firms is the aggregate shock. The correlation between the driving Brownian motions of countries i and j is $corr(W_t^i, W_t^j) = \lambda_i \lambda_j$. Financial markets are integrated and all assets are priced by the same international investor. The model is partial equilibrium and the real pricing kernel of the international investor is specified exogenously as

$$\frac{dM_t}{M_t} = -rdt - \theta dX_t, \tag{2.3}$$

where r is the international real risk-free rate and θ is the market price of risk on the aggregate component of the real cashflow process of the representative firm. The country-specific component of the Brownian motion driving the cashflow process of each country's representative firm is not priced by the international investor since it is diversifiable risk in the sense that the international investor is risk-averse over international output which is the sum of the outputs of a large number of countries. The real risk-free rate is the same across all maturities so the real term structure is flat. Since all the countries under consideration are in Europe, international in the context of this model is European. The aggregate shock is an aggregate European shock and the price of risk will be the Sharpe ratio of a broad index of European companies spanning the entire Eurozone.

Nominal quantities are real quantities multiplied by P_t^i, the price level in country i. The price level process in country i follows the geometric Brownian motion

$$\frac{dP_t^i}{P_t^i} = a_{i,0}dt + a_{i,X}dX_t + \sum_{j=1}^{N} a_{i,j}dZ_t^j, \tag{2.4}$$

where the coefficient vector $a_i = [a_{i,0}, a_{i,X}, a_{i,1}, ..., a_{i,N}]'$ is a constant vector. The coefficients are determined endogenously as part of the optimal monetary policy problem, the details of which are given in the next section. The nominal pricing kernel in each country is $M_t^{i,*} = \frac{M_t}{P_t^i}$. By an application of Ito's lemma, the nominal pricing kernel in country i is found to follow the process

$$\frac{dM_t^{i,*}}{M_t^{i,*}} = -r_i^* dt - (\theta + a_{i,X})dX_t - \sum_{j=1}^{N} a_{i,j}dZ_t^j, \tag{2.5}$$

where $r_i^* = r + a_{i,0} - \theta a_{i,X} - a_{i,X}^2 - \sum_{j=1}^{N} a_{i,j}^2$ is the nominal risk-free rate in country i. The term $\theta a_{i,X}$ is the risk premium investors demand for inflation risk created by the aggregate shock. Note that there is no inflation risk premium for the country-specific shocks.

What is the nature of the restrictions imposed on the price level processes of each country by membership in a monetary union? In a complete markets setting, the nominal exchange rate between countries i and j is the ratio of their nominal pricing kernels, $S_t^{i,j} = \frac{M_t^{*,j}}{M_t^{*,i}} = \frac{P_t^i}{P_t^j}$. An application of Ito's rule shows that the nominal exchange rate follows the geometric Brownian motion

$$\frac{dS_t^{i,j}}{S_t^{i,j}} = \left(a_{i,0} - a_{j,0} + a_{j,X}^2 + \sum_{k=1}^{N} a_{j,k}^2 - a_{i,X}a_{j,X} - \sum_{k=1}^{N} a_{i,k}a_{j,k}\right) dt + (a_{i,X} - a_{j,X})dX_t + \sum_{k=1}^{N}(a_{i,k} - a_{j,k})dZ_t^k. \quad (2.6)$$

In a monetary union, the nominal exchange rate stays constant between any two pairs of countries i and j, imposing the restriction $dS_t^{i,j} = 0$. The implication is that the coefficient vectors of the price level processes of each country must be the same: $a_i = a_j \ \forall i, j$. There is one inflation rate for all countries in the monetary union. This result corresponds to the special case in which the real pricing kernels are identical across countries. If it were not the case that the real pricing kernels were the same, the price level processes would differ. From this point on, I drop the country-specific subscripts for the price level process and the nominal pricing kernel.

The nominal cashflow process for the representative firm of country i is $Y_t^{i,*} = Y_t P_t^{i,*}$, and by another application of Ito's lemma follows the process

$$\frac{dY_t^{i,*}}{Y_t^{i,*}} = \left(\mu_i + a_0 + \sigma_i\lambda_i a_X + \sigma_i\sqrt{1-\lambda_i^2}a_i\right) dt + (\sigma_i\lambda_i + a_X)dX_t + \left(\sigma_i\sqrt{1-\lambda_i^2} + a_i\right) dZ_t^i + \sum_{j=1(j\neq i)}^{N} a_j dZ_t^j. \quad (2.7)$$

Under the nominal risk-neutral measure, $\mathcal{Q}^*$, the nominal cashflow process can be shown to follow

$$\frac{dY_t^{i,*}}{Y_t^{i,*}} = \left[\mu_i - \theta(\sigma_i\lambda_i + a_X) + a_0 - a_X^2 - \sum_{j=1}^{N} a_j^2\right] dt + (\sigma_i\lambda_i + a_X)dX_t^{\mathcal{Q}^*} + \left(\sigma_i\sqrt{1-\lambda_i^2} + a_i\right) dZ_t^{i,\mathcal{Q}^*} + \sum_{j=1(j\neq i)}^{N} a_j dZ_t^{j,\mathcal{Q}^*}. \quad (2.8)$$

The drift and volatility of the nominal cashflow process under the nominal risk-neutral measure will be important later so I define them as $\tilde{\mu}_i^* = \mu_i - \theta(\sigma_i\lambda_i + a_X) + a_0 - a_X^2 - \sum_{j=1}^N a_j^2$ and $\sigma_i^* = \sqrt{(\sigma_i\lambda_i + a_X)^2 + (\sigma_i\sqrt{1-\lambda_i^2} + a_i)^2 + \sum_{j=1(j\neq i)}^N a_j^2}$, respectively. See Appendixes B.1 and B.2 for details on the above derivations. The volatility is minimized at $a_X = -\sigma_i\lambda_i$, $a_i = -\sigma_i\sqrt{1-\lambda_i^2}$, and $a_j = 0$ for all $j \neq i$. This illustrates a fundamental problem of the monetary authority in a monetary union. With one inflation rate for a group of countries, the monetary authority is less effective in hedging its nominal liabilities. The nominal asset value of the representative firm of country i is

$$V_t^{i,*} = E_t^{\mathcal{Q}^*}\left[\int_t^\infty e^{-r^*T} Y_T^{i,*} dT\right] \tag{2.9}$$

$$= \frac{Y_t^{i,*}}{r^* - \tilde{\mu}_i^*}. \tag{2.10}$$

The present value of fiscal revenues of country i can then be calculated as

$$V_{g,t}^{i,*} = E_t^{\mathcal{Q}^*}\left[\int_t^\infty e^{-r^*T} \tau_i Y_T^{i,*} dT\right] \tag{2.11}$$

$$= \frac{\tau_i Y_t^{i,*}}{r^* - \tilde{\mu}_i^*}. \tag{2.12}$$

See Appendix B.3 for the details of these calculations. The restriction $r^* > \tilde{\mu}_i^*$ ensures that present values are finite. Therefore, $V_{g,t}^{i,*} = \tau V_t^{i,*}$ and the present value of fiscal revenues of each country follow the same process as the nominal asset value and nominal cashflow process of the representative firm. This fact allows us to take the nominal asset value process of the representative firm as a proxy for the present value of fiscal revenues of the sovereign when performing the calibration. From this point on, I will work with the nominal asset value process of the representative firm.

2.2.2 Sovereign Debt Pricing

Country i issues an infinite maturity nominal debt contract, with level $D^{i,*}(V^{i,*})$ and continuous coupon $C^{i,*}$. Sovereigns are allowed to strategically default on their debt. The government defaults at time $T_D^i = \inf\{t \geq 0 | V_t^{i,*} \leq V_D^{i,*}\}$ when the nominal asset value of the sovereign falls to the default threshold. By Ito's lemma, the value of perpetual debt for country i satisfies the partial differential equation

$$\frac{1}{2}(\sigma_i^*)^2(V^{i,*})^2 D_{VV}^{i,*} + \tilde{\mu}_i^* V^{i,*} D_V^{i,*} + C^{i,*} = r_i^* V^{i,*}, \tag{2.13}$$

where $D_V^{i,*}$ and $D_{VV}^{i,*}$ are first and second derivatives of sovereign debt value with respect to nominal asset value. When the nominal asset value tends approaches infinity, the value of sovereign debt should approach the value of nominal risk-free debt. Additionally, when the nominal asset value approaches the default threshold, the government and its lenders restructure the debt contract and agree to a reduction $\phi_i \in [0, 1]$ in debt service. These two conditions define the boundary conditions for solving the PDE. In mathematical terms the first and second conditions are

$$\lim_{V^{i,*} \to \infty} D^{i,*}(V^{i,*}) = \frac{C^{i,*}}{r^*} \tag{2.14}$$

and

$$\lim_{V^{i,*} \to V_D^{i,*}} D^{i,*}(V^{i,*}) = \frac{(1-\phi_i)C^{i,*}}{r^*} \tag{2.15}$$

Therefore, the value of sovereign debt as defined by the boundary conditions above is

$$D^{i,*}(V^{i,*}) = E^{\mathcal{Q}^*}\left[\int_0^{T_D^i} C^{i,*} e^{-r^* t} dt\right] + E^{\mathcal{Q}^*}\left[\int_{T_D^i}^{\infty} (1-\phi_i) C^{i,*} e^{-r^* t} dt\right] \tag{2.16}$$

$$= \frac{C^{i,*}}{r^*}\left[1 - \phi_i \left(\frac{V^{i,*}}{V_D^{i,*}}\right)^{\xi^i}\right], \tag{2.17}$$

where $\xi^i = \frac{-1}{(\sigma_i^*)^2}\left(\mu_i^* - \frac{(\sigma_i^*)^2}{2} + \sqrt{\left(\mu_i^* - \frac{(\sigma_i^*)^2}{2}\right)^2 + 2(\sigma_i^*)^2 r^*}\right) < 0$ and $V_D^{i,*}$ is an exogenously given default boundary. The value $P_D^i = \phi_i \left(\frac{V^{i,*}}{V_D^{i,*}}\right)^{\xi^i}$ is the Arrow-Debreu price of default, which is defined as $P_D^i \equiv E^{\mathcal{Q}^*}[e^{-rT_D^i}]$, and has the interpretation of the price of a claim on 1 euro conditional on future default.

Sovereign debt issuance generates economic value, allowing the government to earn a return on domestic investment $r_{i,g}^*$. However, an increase in the level of debt raises the probability of default. In a situation where default is costly, a rise in the probability of default matters when determining the optimal level of debt. If default occurs, the level of firm income drops by a fraction $\gamma \in [0,1]$. Therefore, sovereigns weigh the costs and benefits of issuing debt when deciding how much debt to issue.

2.2.3 Sovereign Wealth

As in Jeanneret (2015), sovereign wealth is defined as the present value of fiscal revenues net of the present value of future debt payments taking into account the benefits and costs of debt issuance. Sovereign wealth $W^{i,*}(V^{i,*})$ is then defined as

$$W^{i,*}(V^{i,*}) = E^{\mathcal{Q}^*}\left[\int_0^\infty (\tau_i Y_t^{i,*} - C^{i,*})e^{-r^*t}dt\right] + \alpha_i D^{i,*}(V^{i,*}) + E^{\mathcal{Q}^*}\left[\int_{T_D^i}^\infty C^{i,*}\phi_i e^{-r^*t}dt\right] \tag{2.18}$$

$$- E^{\mathcal{Q}^*}\left[\int_{T_D^i}^\infty \gamma_i \tau_i Y_t^{i,*} e^{-r^*t}dt\right]$$

$$= \tau_i V^{i,*} - \gamma_i \tau_i V_D^{i,*}\left(\frac{V^{i,*}}{V_D^{i,*}}\right)^{\xi^i} + \frac{(\alpha_i - 1)C^{i,*}}{r^*}\left[1 - \phi_i\left(\frac{V^{i,*}}{V_D^{i,*}}\right)^{\xi^i}\right], \tag{2.19}$$

where $\alpha_i = E^{\mathcal{Q}^*}[\int_0^\infty r_{i,g}^* e^{-r^*t}dt] = \frac{r_{i,g}^*}{r^*}$ is the discounted benefit of issuing one unit of debt. The first term in equation 2.18 above represents the present value of fiscal revenues net of debt payments. The second term is the incentive sovereigns have to issue an additional unit of debt. The third and fourth terms are the benefits and costs for the sovereign of defaulting on its debt.

2.2.4 Optimal Default Boundary

So far I have assumed the default boundary is exogenously given and have solved for the value of debt and sovereign wealth given an arbitrary default boundary. I now endogenize the default policy of the government. Households are the eventual future beneficiaries of sovereign wealth so the government should choose a default policy to maximize sovereign wealth such that the smooth-pasting condition $\frac{\partial W^{i,*}(V^{i,*})}{\partial V^{i,*}}|_{V^{i,*}=V_D^{i,*}} = \tau_i(1-\gamma_i)$ is satisfied. The optimal default boundary is found to be

$$V_D^{i,*,opt} = \frac{C^{i,*}\phi_i(\alpha_i - 1)\xi^i}{\tau_i r^* \gamma_i (1-\xi^i)}. \tag{2.20}$$

The higher the gain from restructuring upon default, $C^{i,*}\phi_i$, the more willing the sovereign is to default and the higher the default boundary. The higher the economic costs of default, γ_i, the less willing sovereigns will be to default and the lower the default boundary.

2.2.5 Optimal Level of Debt

The next step is to endogenize the coupon rate, $C^{i,*}$, to maximize the total value, $T^{i,*}(V^{i,*}) = W^{i,*}(V^{i,*}) + D^{i,*}(V^{i,*})$, of the sovereign at $t = 0$. The total value of sovereign i is the sum of sovereign wealth (equation 2.19) and the value of debt (equation 2.17):

$$T^{i,*}(V^{i,*}) = \tau_i V^{i,*} - \gamma_i \tau_i V_D^{i,*} \left(\frac{V^{i,*}}{V_D^{i,*}}\right)^{\xi^i} + \frac{\alpha_i C^{i,*}}{r^*}\left[1 - \phi\left(\frac{V^{i,*}}{V_D^{i,*}}\right)^{\xi^i}\right]. \tag{2.21}$$

The second term on the right hand side of equation 2.21 is the expected loss in default for the sovereign and the third term is the benefits of debt. The sovereign solves the following optimization problem to determine the optimal coupon rate:

$$C^{i,*} = \arg\max_{C^{i,*}>0} T^{i,*}(V_0^{i,*}) = \arg\max_{C^{i,*}>0} W^{i,*}(V_0^{i,*}) + D^{i,*}(V_0^{i,*}) \tag{2.22}$$

$$= \frac{\tau_i V_0^{i,*} r_i^* \gamma_i (1-\xi^i)}{\phi_i(\alpha_i - 1)\xi^i} \left(\frac{\phi_i(\alpha - \xi^i)}{\alpha_i} \right)^{1/\xi^i} . \tag{2.23}$$

The optimal coupon rate is increasing in initial nominal asset value and the benefits of issuing debt. It is increasing in the economic costs of default as the higher the economic costs of default, the lower the default boundary and it is decreasing in the expected debt reduction upon default since the higher the expected debt reduction, the higher the default boundary.

2.2.6 Risk-Neutral Probabilities of Sovereign Default and Credit Spreads

The risk-neutral probability of sovereign default for country i before time T is

$$P\left(\inf_{0 \le t \le T} V_t^{i,*} \le V_D^{i,*} | V_0^{i,*} > V_D^{i,*} \right) = \Phi\left(\frac{\log\left(\frac{V_D^{i,*}}{V_0^{i,*}}\right) - (\tilde{\mu}_i^* - .5(\sigma_i^*)^2)T}{\sigma_i^* \sqrt{T}} \right)$$
$$+ \left(\frac{V_D^{i,*}}{V_0^{i,*}} \right)^{-\xi^i - 1} \Phi\left(\frac{\log\left(\frac{V_D^{i,*}}{V_0^{i,*}}\right) + (\tilde{\mu}_i^* - .5(\sigma_i^*)^2)T}{\sigma_i^* \sqrt{T}} \right), \tag{2.24}$$

where $\Phi()$ is the cumulative distribution function of a standard normal. Sovereign credit spreads are a measure of the market's perception of default risk. The above expression is the risk-neutral probability of sovereign default because the drift, $\tilde{\mu}_i^*$, is the drift under the risk-neutral measure. The physical default probability before time T would be identical to the above expression except that $\tilde{\mu}_i^*$ would be replaced by μ_i^*. The credit spread for the defaultable consol bond issued by the sovereign is the difference between the yield on this bond and the yield on a nominal risk-free consol bond. The nominal risk-free bond is still subject to inflation risk but is free from default risk. Credit spreads on the defaultable consol issued by country i can be written as

$$CS(V^{i,*}) \equiv \frac{C^{i,*}}{D^{i,*}(V^{i,*})} - r^*. \tag{2.25}$$

At the optimal coupon level and default boundary, this expression can be expressed in closed form in terms of the deep parameters of the model:

$$CS(V^{i,*}) = r^* \left[\frac{1}{1 - \frac{\alpha_i}{\alpha_i - \xi^i} \left(\frac{V^{i,*}}{V_0^*} \right)^{\xi^i}} - 1 \right]. \tag{2.26}$$

2.2.7 Optimal Monetary Policy Problem

The determination of the optimal default boundary and the coupon rate have so far taken inflation as given. The last step is for the monetary authority is to solve the optimal monetary policy problem to determine the coefficient vector $a = [a_0, a_X, a_1, ..., a_N]'$ of the price level process. The monetary authority commits to a mean inflation rate of a_0 so it takes the drift coefficient as given. Therefore, it is necessary to determine the remaining $N + 1$ coefficients. The directive of the monetary authority is to maximize an objective function consisting of the sum of the total values of the sovereigns comprising the monetary union, weighted by the relative weight that the monetary authority puts on each sovereign, where weights are proportional to the fraction of output contributed by each country to the total output of the entire monetary union. The monetary authority will be constrained in this maximization problem to keep the volatility of inflation below a certain threshold value. It solves the optimal policy problem under commitment, that is, it chooses the coefficient vector at $t = 0$ and cannot reoptimize at a later date. This is opposed to discretionary monetary policy, in which the monetary authority can reoptimize at future dates. The optimal monetary policy problem can be written succinctly as

$$a^{opt} = \underset{\sigma_P < \bar{\sigma}_P}{\arg\max} \sum_{i=1}^{N} \delta_i T^{i,*}(V_0^{i,*}), \tag{2.27}$$

where $\sigma_P = \sqrt{a_X^2 + \sum_{j=1}^N a_j^2}$ and $\bar{\sigma}_P > 0$. Without loss of generality, let the weights satisfy $\sum_{i=1}^N \delta_i = 1$. The problem above is highly nonlinear, but can be solved relatively quickly for $N < 10$. The computational time increases rapidly for higher dimensional cases.

2.3 Comparative Statics

To gain some intuition about the optimization problem solved by the monetary authority, it is instructive to see how varying the coefficient vector of the price level process changes the benefits of debt issuance, the expected loss in default, the Arrow-Debreu prices of default, and the monetary policy objective function in a simple two country model. The countries are completely symmetric in terms of parameters in the following plots. The plots show how the quantities vary with a_X, a_1, and a_2. However, I do allow the weights on the total values of each country in the monetary policy objective function to differ, where specified.

Figure 2.1 plots the Arrow-Debreu prices of default for countries 1 and 2 vs. a_X. The Arrow-Debreu prices of default for both countries 1 and 2 are increasing in a_X. Both countries share this aggregate shock, so offsetting it through countercyclical inflation will reduce the volatility of the shock to the nominal asset value of both countries, reducing the Arrow-Debreu price of default.

Figure 2.2 plots the benefits of debt for countries 1 and 2 vs. a_X. The benefits of debt are decreasing then increasing in a_X. Why is this so? For intermediate values of a_X, increasing a_X has negligible effects on the risk-neutral drift but increases volatility, leading to lower values for defaultable debt. For large positive values of a_X, the risk-neutral drift is decreasing rapidly in a_X and the volatility is increasing rapidly in a_X, so that the Arrow-Debreu price of default is increasing rapidly in a_X. The nominal-risk free rate falls, however, at large values of a_X, leading to a steep increase in the value of nominal risk-free debt. The value of defaultable debt is the value of nominal risk-free debt times one minus the Arrow-Debreu price of default. The increase in the value of the nominal risk-free debt outpaces the increase in the Arrow-Debreu price of default for values of a_X in this range.

Figure 2.3 plots the expected loss in default for countries 1 and 2 vs. a_X. The expected loss in default is the product of the default boundary and the Arrow-Debreu price of default, up to a constant multiple, so we need to understand how the default boundary changes with a_X. The curve for the optimal default boundary has the same basic shape as the curve for benefits to debt since the sovereign would like to set a low default boundary when default is likely and a high default boundary when it is not, except when nominal risk-free rates are so low that optimal coupon levels are very high despite the high levels of default risk, implying that a reduction in debt service through default would be strategically advantageous. The decreasing default boundary dominates the increasing Arrow-Debreu prices of default for intermediate values of a_X, so there is a slight negative slope to the curve for expected default loss in this range. For large positive values of a_X, both the default boundary and Arrow-Debreu prices of default are increasing so the expected loss in default increases rapidly for these a_X.

I plot only country 1's Arrow-Debreu prices of default, benefits of debt, and expected loss in default vs. the coefficients for the country-specific shocks in Figures 2.4, 2.5, and 2.6, respectively. The results for country 2 will be symmetric. For the coefficients of the country-specific shocks, a_1 and a_2, the Arrow-Debreu prices of default of each country are minimized at negative values of the coefficient of their own country-specific shock and at zero for the coefficient of the other country's country-specific shock. Why are Arrow-

Debreu prices of default minimized at less negative values for the own country-specific shock than the aggregate shock? There are two reasons for this. The first is that in this calibration the volatility of the aggregate shock is larger than the volatility of the country-specific shock: $\sigma_1\lambda_1 > \sigma_1\sqrt{1-\lambda_1^2}$. The second reason is that making a_X more negative has a first-order impact on increasing the risk-neutral drift $\tilde{\mu}_i$ through the term $-\theta a_X$, whereas this term is not present for the own country-specific shock.

The benefits of debt follow the same pattern as the Arrow-Debreu prices of default, being maximized at negative values for the coefficient of own country-specific shocks and at zero for the coefficient of the other country's country-specific shock. The debt benefit curves here are different than the one for a_X since changes in the coefficients of the country-specific shocks have only a second-order impact on the value of nominal risk-free debt through the nominal risk-free rate. The plots for expected losses in default mirrors the one for the optimal default boundary, which has a similar shape to the benefits to debt plot: high at low Arrow-Debreu prices of default and low at high Arrow-Debreu prices of default.

The monetary policy objective function for the case in which the weights are asymmetric ($\delta_1 = 2, \delta_1 = 1$) is plotted vs. a_X and the coefficients for the country-specific shocks in Figures 2.7 and 2.8, respectively. Examining the monetary policy objective function where the weights are asymmetric, it is clear that the country with the higher weight will benefit from more countercyclical inflation in its own country-specific shock than the country with the lower weight on it. In a real world context, this means that smaller countries will bear more of the costs of the loss of flexibility in monetary policy when it comes to joining a monetary union and will pay these costs through a more pronounced reduction in total sovereign value and increase in credit spreads when compared to the counterfactual of the country conducting its own independent monetary policy than will be the case for larger countries. Countries 1 and 2 both benefit equally from countercyclical inflation in the aggregate shock. The objective function decreases with increasing

a_X up to a point, then increases. This is due to a discount rate effect as discussed earlier. The nominal risk-free rate falls to a low level for high a_X, leading to a significant increase in the value of the benefits of debt. This effect outweighs the reduction in total sovereign value caused by the increased volatility in the nominal asset value process of the representative firm for a_X in this range. For any reasonable calibration, however, the imperative for the monetary authority to keep the volatility of inflation low will dictate that the optimal value of a_X is negative.

Asymmetries in the parameter values can also affect the degree to which monetary policy is skewed to benefit one country at the expense of another even in the case of the weights on both countries in the objective function being equal ($\delta_1 = \delta_2 = 1$). Asymmetries in the volatility of the real cashflow processes of the representative firms of each country, σ_i, have the greatest impact, all other parameters equal. The country with the lower volatility will benefit more from membership in a monetary union as the monetary authority makes inflation more countercyclical with respect to the country-specific shock of the lower volatility country. This seems counterintuitive, but can be understood by the fact that total sovereign value is higher for the country with the lower volatility so it takes precedence in a monetary policy objective function which is the sum of the total values of each sovereign. Asymmetries in drift, μ_i, have a much less noticeable impact.

2.4 Data

I will focus on the time period from 2005-2016 for a sample of eight Eurozone countries including Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, and Spain. These countries collectively account for 92% of total Eurozone output, and represent the eight largest Eurozone countries in terms of output. Quarterly data on 10 year sovereign yields comes from the St. Louis Fred database. To calculate credit spreads, it is necessary to identify a risk-free instrument denominated in Euros. The standard practice is to assume German bonds are free of default risk. Therefore, credit spreads are calculated by subtracting the yield on the 10 year German bond from the yield on the 10 year bonds of each country. Therefore, German bonds by definition have credit spreads of zero. Summary statistics for credit spreads are shown in Table 2.1 for the period from 2005-2016. Quarterly data on real GDP for the eight Eurozone countries and Harmonized Index of Consumer Prices data also come from the St. Louis Fred database. Average annual Eurozone GDP growth is measured as the annualized ($\times 4$) log difference in quarterly GDP. The annual volatility of European GDP growth is measured as the annualized ($\times\sqrt{4}$) volatility of the log difference in quarterly GDP. Annual inflation volatility is measured as the annualized volatility of the log difference in the quarterly Harmonized Index of Consumer Prices.

2.5 Calibration

In the following section, I lay out the baseline calibration, which will be used to determine the fraction of credit spreads that can be attributed to membership in a monetary union. The model is calibrated at an annual frequency. First, I will calibrate the parameters which will be constant across all countries. I normalize $V_0^* = 100$ without loss of generality. The relative size and importance of each country in the monetary policy objective function is captured by the δ_i parameters. The real risk-free interest rate, r, is taken to be 2% annually, consistent with the asset pricing literature. The return on government investment, r_g^*, is estimated by Jeanneret (2015) to average out to 0.26% for European countries using a nominal risk-free rate of 3.52%, implying an $\alpha = 0.074$. I use α instead of r_g^* directly since the nominal risk-free rate is endogenous in my model and decreasing the nominal risk-free rate through inflation would artificially increase α. The parameter, λ, specifying the correlation of the driving Brownian motions of the real cashflow processes of the representative firms with the aggregate shock is determined by the fact that the average pairwise correlation of quarterly real GDP growth in my sample of Eurozone countries over the period from 2005-2016 is 0.72. Then since all countries are symmetric in this parameter value, it must be the case that $\lambda = \sqrt{0.72} = 0.85$. The matrix showing the pairwise correlation in GDP growth for every country in the sample is shown in Table 2.3. The expected loss, ϕ, to debtholders upon default is set at 0.60 according to the market convention for the quotation of sovereign credit default swap contracts. The corporate tax rate, $\tau = 0.30$, is taken from Jeanneret (2015). The value of the expected economic contraction upon default, $\gamma = 0.05$, is taken from Jeanneret (2015) and Mendoza and Yue (2012). This parameter has minor effects on credit spreads and other quantities of interest. The market price of aggregate risk, $\theta = 0.15$, is set to the Sharpe ratio of the MSCI European Index, a broad index comprised of large and medium cap stocks across 15 developed markets in Europe. The volatility of inflation constraint, $\bar{\sigma}_P$, is set to 0.0167, matching the annual volatility of the growth in the Harmonized Index of Consumer Prices over the period from 2005-2016. I assume the monetary authority commits to a mean inflation rate of 0 so $a_0 = 0$. This parameter does not play an important role in the analysis.

Finally, it is necessary to calibrate country-specific parameters. Chen (2013) argues that government revenue is perfectly correlated with GDP but is more volatile. In my setup, with government revenue coming from the taxation of a representative firm, this means that the real cashflow process of the representative firm will be perfectly correlated with GDP but more volatile than it. I also impose the additional restriction that the drift of the cashflow process of the representative firm is the same as the mean of real GDP growth, $\mu_i = \mu_{i,GDP}$. The drift of the real cashflow process of the representative firm of each country is the average of annual real GDP growth over the sample period. The volatility of the real cashflow process of the representative firm is a multiple, m_i, of the volatility of real GDP growth in the sample, $\sigma_i = m_i \sigma_{i,GDP}$. I choose m_i for each country in order to match the level of credit spreads in that particular country. I match these spreads assuming that the country conducts its own monetary policy since to match these spreads in a monetary union, I would need knowledge of the other countries' m_i's. As a result, the level of credit spreads of the countries assuming a monetary union arrangement will not match the levels in the data, but will be close. Since my objective is to estimate the fraction of credit spreads attributable to loss of flexibility in monetary policy in a monetary union, matching the exact level of credit spreads is not essential. The final set of parameters to calibrate are the δ_i, the weights on the total value of each sovereign in the monetary policy objective function. The weights are determined by finding the relative contribution to total output, as defined by the total output of the eight countries in the sample, that each individual country makes averaged over the time period under consideration. The mean and volatility of GDP growth of each country as well as the m_i, σ_i, and δ_i are shown in Table 2.4.

2.6 Results

In the following tables, I compare the benefits of debt, expected losses in default, credit spreads, Arrow-Debreu prices of default, and 10-year default probabilities for the case in which the countries are assumed to be in a monetary union and the counterfactual case in which they are assumed to conduct their own independent monetary policies. I also calculate the monetary union risk premium as a fraction of credit spreads. First, I examine the baseline calibration, then provide a discussion of the quantitative results given in Tables 2.5 through 2.8.

In Table 2.5, we can see that there is a reduction in welfare across the board from being a member of a monetary union compared to the counterfactual of each country conducting its own monetary policy. The country that would benefit the least, in percentage terms, in a transition from a monetary union to an independent monetary policy (Table 2.6) is Belgium. Italy and Spain are the two countries that would benefit the most, in percentage terms, in transitioning. This is due to the high volatility of the nominal asset value processes for the representative firms of Italy and Spain and the volatility reducing benefits of an independent monetary policy regime.

The results for credit spreads and Arrow-Debreu prices of default support the previous findings and are shown in Tables 2.7 and 2.8. Both Italy's and Spain's credit spreads and Arrow-Debreu prices of default would increase the most, in percentage terms, in transitioning from a monetary union arrangement to an independent monetary policy arrangement. At first blush, this seems to contradict the welfare results given above. However, in a transition to an independent monetary policy, Italy and Spain would optimally take on much more debt, as they would benefit the most from the reduction in volatility that comes with an independent monetary policy regime. Upon optimally issuing additional debt, credit spreads and Arrow-Debreu prices of default would increase. In fact, these are the only two countries that would experience an increase in credit spreads and Arrow-Debreu prices of default in transitioning to an independent monetary policy. For the rest of the countries in the sample, the volatility reduction afforded by transitioning to an independent monetary policy is not great enough to encourage the sovereign to issue significantly more debt. Instead, the reduction in volatility will lead to lower credit spreads and Arrow-Debreu prices of default. Italy and Spain would also experience the biggest increase in 10-year default probabilities as a result of transitioning.

Finland's and the Netherland's credit spreads and Arrow-Debreu prices of default would decrease the most, in percentage terms, in a transition. However, their 10-year default probabilities would increase. The marked difference in results between the Arrow-Debreu prices of default and the 10-year default probabilities for Finland and the Netherlands can be chalked up to the fact that the low volatility of the real asset value processes of Finland and the Netherlands make volatility reduction more apparent in the long run, whereas for countries with higher volatilities, this reduction will be more apparent over shorter horizons. This is due to the fact that the expected survival time for a low volatility countries is longer than than the expected survival time for a high volatility country. Belgium and France would see the biggest decreases, in percentage terms, in 10-year default probabilities.

The monetary union risk premium as a fraction of credit spreads is defined as the credit spread assuming the country is in a monetary union minus the credit spread assuming it conducts its own monetary policy divided by the credit spread assuming the country is in a monetary union. The monetary union risk premium is the risk premium investors demand on sovereign debt due to loss of flexibility in monetary policy associated with entering a monetary union. The results are shown in Figure 2.8. The credit spreads of Finland and the Netherlands have the highest monetary union risk premium as a fraction of spreads and Italy and Spain have the lowest. In fact, the risk premium is negative for Italy and Spain. From these results it is clear that the countries which are most hurt by membership in the monetary union are also the countries with the lowest monetary union risk premium as a fraction of credit spreads: Italy and Spain. These two countries are also the least credit-worthy countries in the sample. Conversely, the countries that are hurt the least by monetary union membership are Belgium and France. However, the countries that have the highest monetary union risk premium as a fraction of credit spreads are the two most credit-worthy countries in the sample: Finland and the Netherlands.

Belgium and France, due to their higher volatilities, take on more debt than Finland and the Netherlands in a transition from a monetary union to an independent monetary policy leading to a less pronounced reduction in credit spreads and a lower monetary union risk premium as a fraction of credit spreads. From the discussion above, a consistent pattern emerges: a country with a higher volatility issues more debt in a transition from a monetary union arrangement to an independent monetary policy arrangement, causing them to benefit more from an independent monetary policy than an otherwise similar country with a lower volatility.

2.7 Conclusion

This paper studies a structural model of sovereign credit risk in a monetary union in which the inflation rate is endogenously chosen to maximize an objective function of the weighted sum of the total values of each sovereign. Inflation reduces the real value of nominal debt in bad times and increases it in good times. Therefore, when default is costly, inflation serves as an important part of the capital structure problem of the sovereign. In a monetary union, the effectiveness of this hedging mechanism is reduced since the nominal asset values of the representative firms in each country have imperfectly correlated shocks. The weights correspond to the relative contribution of each country to the output of the monetary union. The model provides estimates on how transitioning from a monetary union to conducting independent monetary policy affects the benefits of debt, expected losses in default, credit spreads, Arrow-Debreu prices of default, and 10-year probabilities of default. The model also quantifies the monetary union risk premium as a fraction of credit spreads. These estimates range from -4.6% to 11.8% of credit spreads, depending on the country under consideration. The countries with the highest monetary union risk premium as a fraction of spreads are Finland and the Netherlands and those with the lowest are Italy and Spain. Keep in mind, however, that these estimates do not consider the benefits that joining a monetary union has on output, including lower transaction costs and exchange rate stability.

There are many possibilities to extend the model in this paper to explore the trade-off between entering a monetary union and retaining control of monetary policy. A benefit of entering a monetary union might be decreased transaction costs for trading and lower exchange rate volatility between member countries and a cost would be the decreased flexibility in setting inflation rates explored in this paper. Quantifying the benefits of decreased transaction costs and lower exchange rate volatility is a challenging direction for future work in this area.

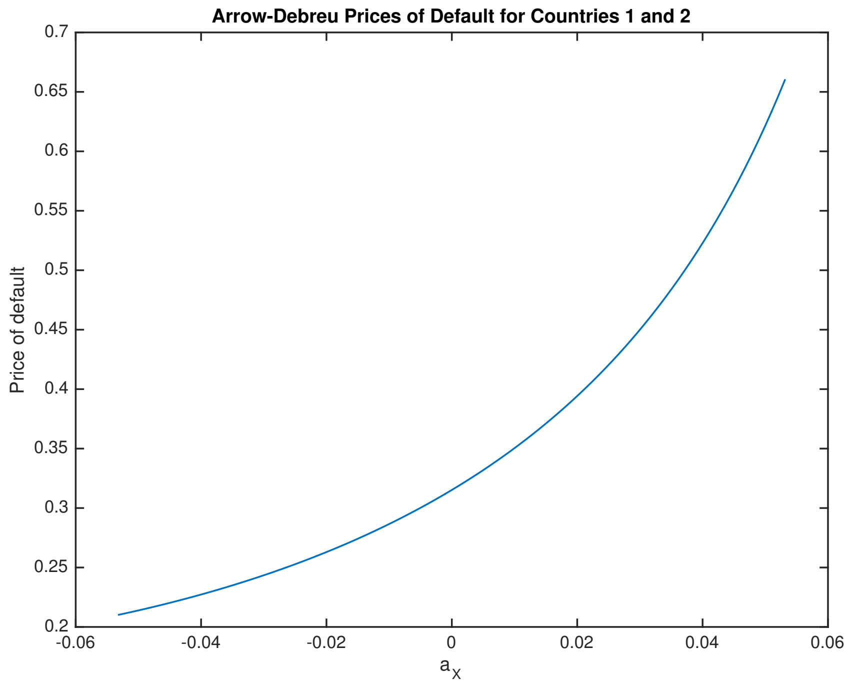


Figure 2.1: $r = 0.02$, $\alpha_1 = \alpha_2 = 0.074$, $\lambda_1 = \lambda_2 = 0.85$, $\phi_1 = \phi_2 = 0.60$, $\tau_1 = \tau_2 = 0.30$, $\gamma_1 = \gamma_2 = 0.05$, $\theta = 0.15$, $\mu_1 = \mu_2 = 0.01$, $\sigma_1 = \sigma_2 = 0.25$

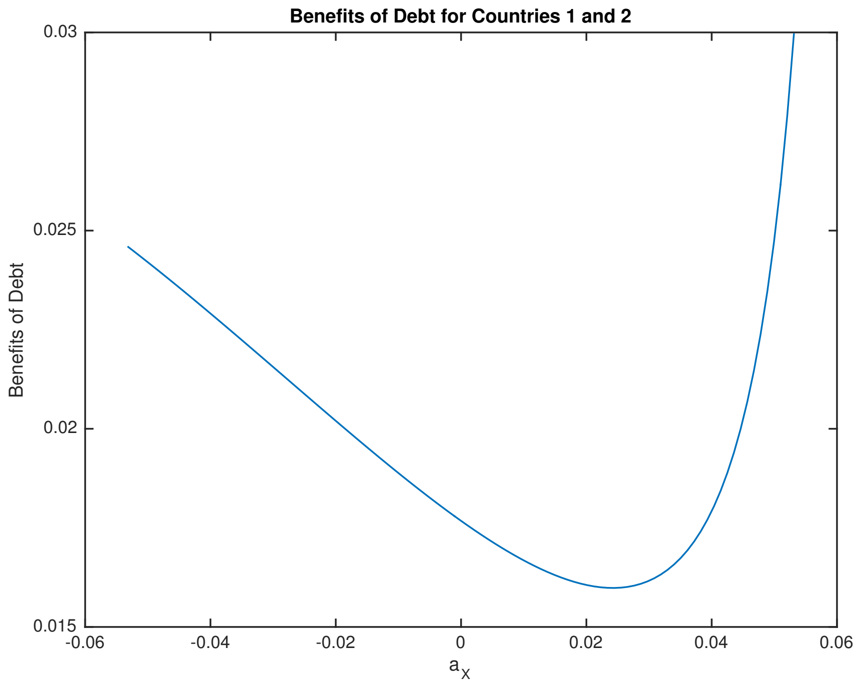


Figure 2.2: $r = 0.02$, $\alpha_1 = \alpha_2 = 0.074$, $\lambda_1 = \lambda_2 = 0.85$, $\phi_1 = \phi_2 = 0.60$, $\tau_1 = \tau_2 = 0.30$, $\gamma_1 = \gamma_2 = 0.05$, $\theta = 0.15$, $\mu_1 = \mu_2 = 0.01$, $\sigma_1 = \sigma_2 = 0.25$

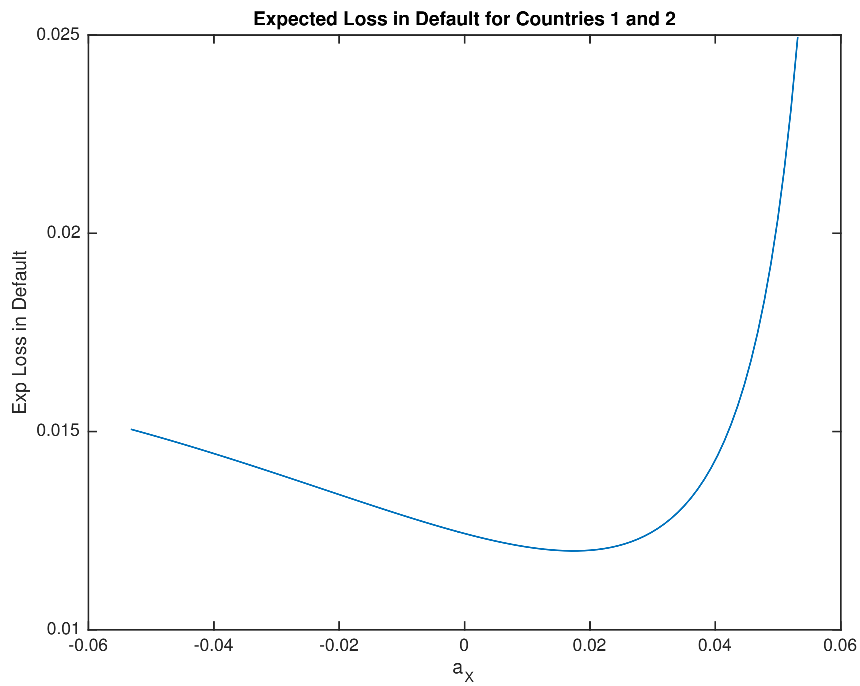


Figure 2.3: $r = 0.02$, $\alpha_1 = \alpha_2 = 0.074$, $\lambda_1 = \lambda_2 = 0.85$, $\phi_1 = \phi_2 = 0.60$, $\tau_1 = \tau_2 = 0.30$, $\gamma_1 = \gamma_2 = 0.05$, $\theta = 0.15$, $\mu_1 = \mu_2 = 0.01$, $\sigma_1 = \sigma_2 = 0.25$

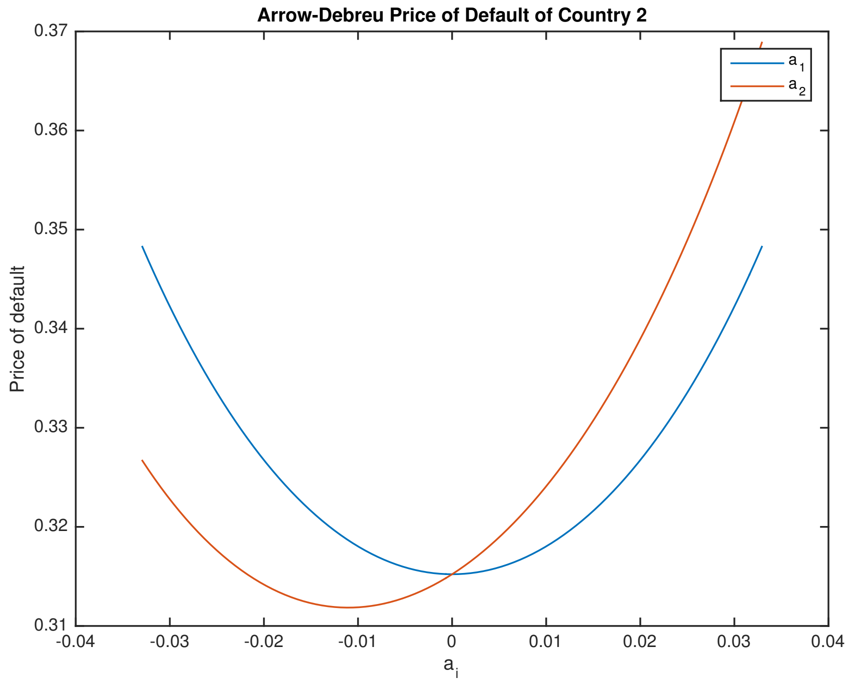


Figure 2.4: $r = 0.02$, $\alpha_1 = 0.074$, $\lambda_1 = 0.85$, $\phi_1 = 0.60$, $\tau_1 = 0.30$, $\gamma_1 = 0.05$, $\theta = 0.15$, $\mu_1 = 0.01$, $\sigma_1 = 0.25$

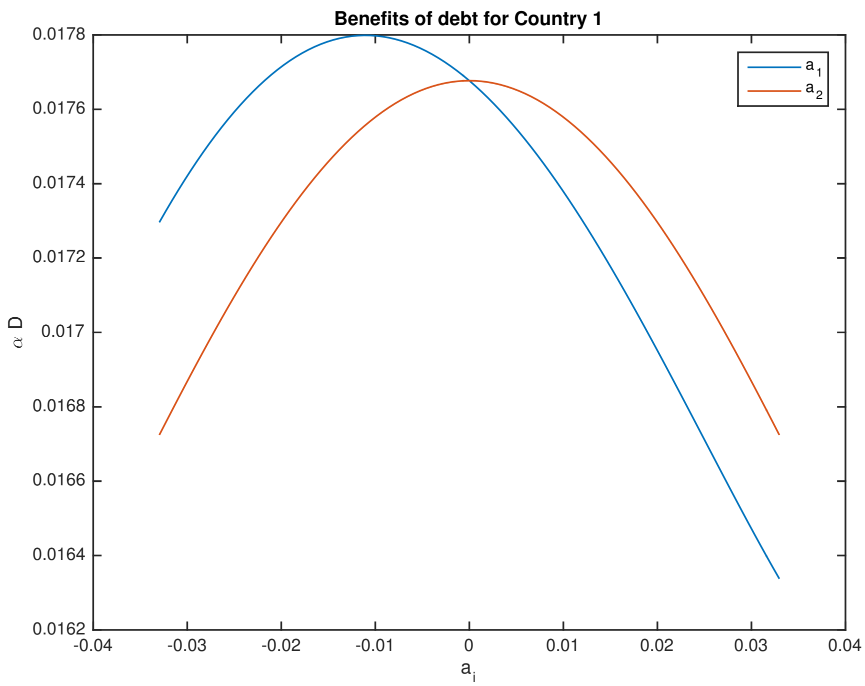


Figure 2.5: $r = 0.02$, $\alpha_1 = 0.074$, $\lambda_1 = 0.85$, $\phi_1 = 0.60$, $\tau_1 = 0.30$, $\gamma_1 = 0.05$, $\theta = 0.15$, $\mu_1 = 0.01$, $\sigma_1 = 0.25$

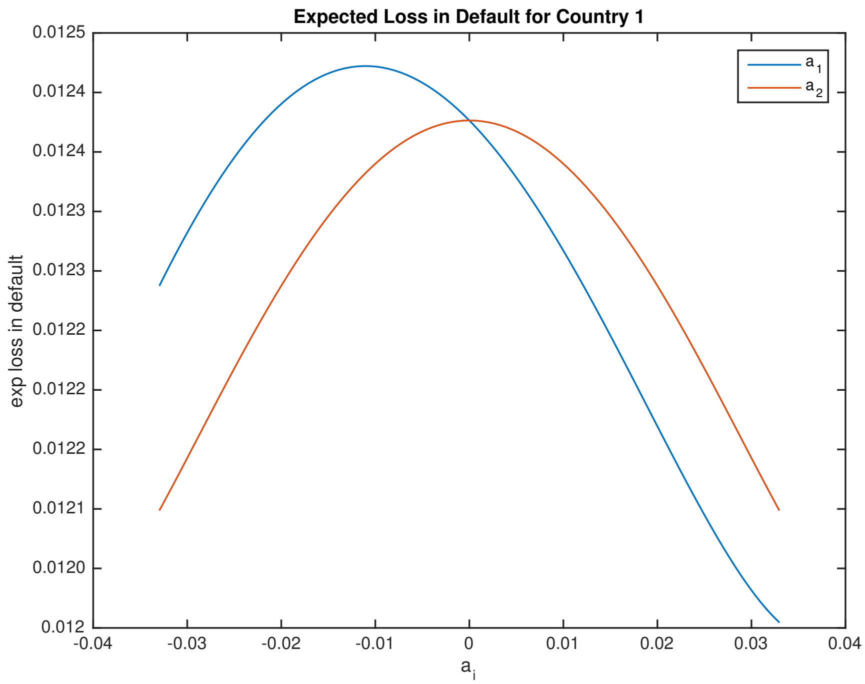


Figure 2.6: $r = 0.02$, $\alpha_1 = 0.074$, $\lambda_1 = 0.85$, $\phi_1 = 0.60$, $\tau_1 = 0.30$, $\gamma_1 = 0.05$, $\theta = 0.15$, $\mu_1 = 0.01$, $\sigma_1 = 0.25$

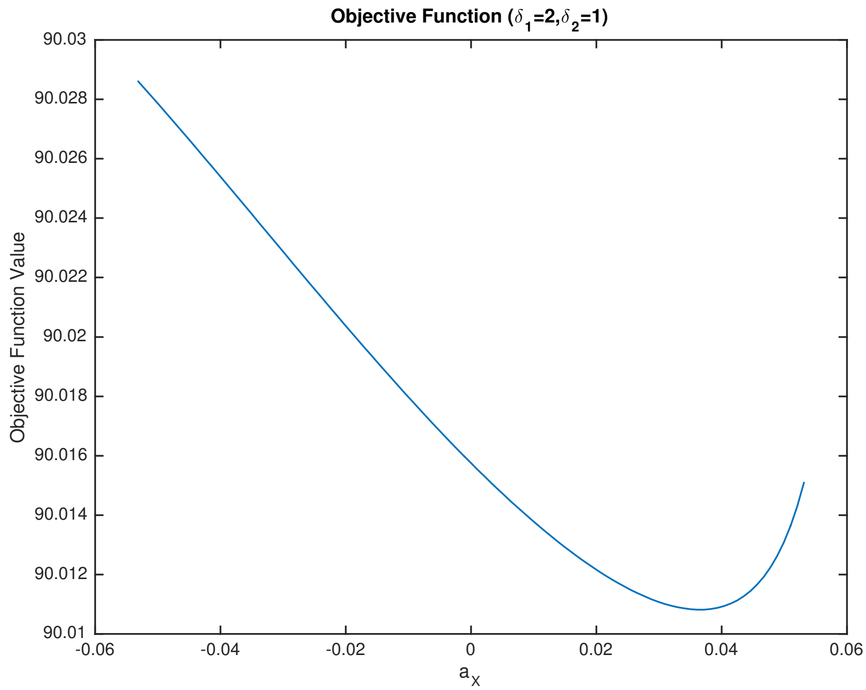


Figure 2.7: $r = 0.02$, $\alpha_1 = \alpha_2 = 0.074$, $\lambda_1 = \lambda_2 = 0.85$, $\phi_1 = \phi_2 = 0.60$, $\tau_1 = \tau_2 = 0.30$, $\gamma_1 = \gamma_2 = 0.05$, $\theta = 0.15$, $\mu_1 = \mu_2 = 0.01$, $\sigma_1 = \sigma_2 = 0.25$

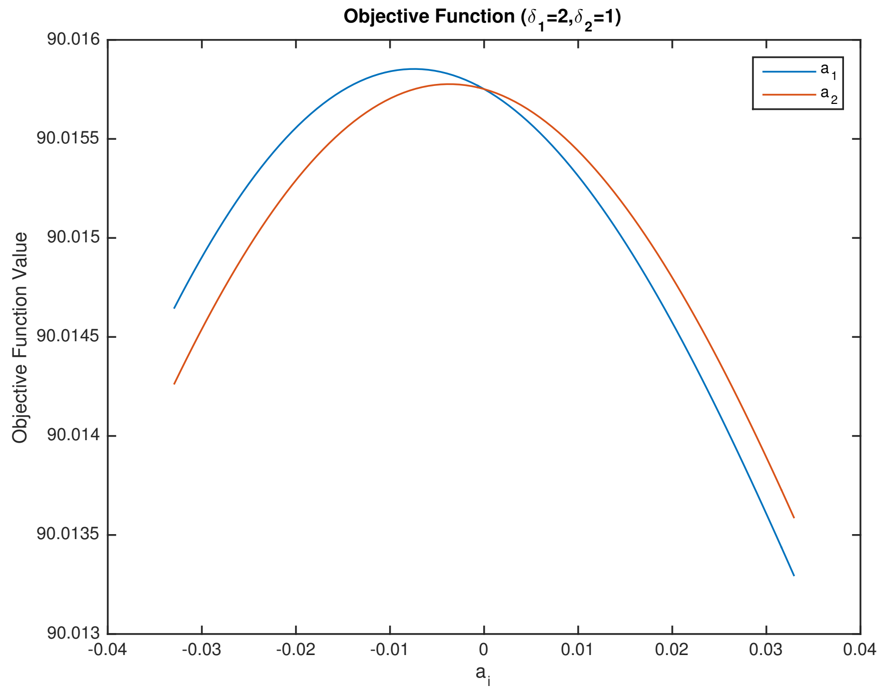


Figure 2.8: $r = 0.02$, $\alpha_1 = \alpha_2 = 0.074$, $\lambda_1 = \lambda_2 = 0.85$, $\phi_1 = \phi_2 = 0.60$, $\tau_1 = \tau_2 = 0.30$, $\gamma_1 = \gamma_2 = 0.05$, $\theta = 0.15$, $\mu_1 = \mu_2 = 0.01$, $\sigma_1 = \sigma_2 = 0.25$

Table 2.1: Credit Spread Summary Statistics (2005-2016)

	N	Mean (bp)	St. Dev. (bp)	Min	Max
Austria	48	38	31	0	122
Belgium	48	61	55	4	253
Finland	48	25	18	-4	80
France	48	40	32	2	135
Germany	48	0	0.00	0	0
Italy	48	148	121	14	468
Netherlands	48	25	17	0	67
Spain	48	142	136	1	507

Table 2.2: Calibration of Parameters Constant Across Countries

V_0^*	α	λ	ϕ	τ	γ	θ	$\bar{\sigma}_P$
100	0.074	0.85	0.60	0.30	0.05	0.15	0.0167

Table 2.3: Quarterly Correlation in GDP growth 2005-2016

	Austria	Belgium	Finland	France	Germany	Italy	Netherlands	Spain
Austria	1	0.75	0.61	0.77	0.76	0.76	0.59	0.53
Belgium		1	0.66	0.78	0.75	0.84	0.63	0.62
Finland			1	0.74	0.83	0.72	0.78	0.57
France				1	0.85	0.77	0.53	0.93
Germany					1	0.75	0.72	0.94
Italy						1	0.73	0.86
Netherlands							1	0.75
Spain								1

Table 2.4: Calibration of Country-Specific Parameters

	$\mu_{i,GDP} = \mu_i$	$\sigma_{i,GDP}$	m_i	$\sigma_i = m_i \sigma_{i,GDP}$	δ_i
Austria	0.0133	0.0151	16.03	0.2420	0.034
Belgium	0.0122	0.0115	27.33	0.3143	0.042
Finland	0.0060	0.0279	6.20	0.1730	0.022
France	0.0092	0.0100	23.97	0.2397	0.23
Germany	0.0147	0.0192	5.2E-5	1E-6	0.30
Italy	-0.0020	0.0153	31.51	0.4820	0.19
Netherlands	0.0129	0.0146	12.82	0.1871	0.072
Spain	0.0090	0.0139	34.90	0.4851	0.12

Table 2.5: Benefits of Debt and Expected Losses in Default ($\times 10^4$)

	Ben. of Debt -Exp. Loss in Def. (m.u.)	Ben. of Debt -Exp. Loss in Def. (ind.)	Ben. of Debt (m.u.)	Ben. of Debt (ind.)	Exp. Loss in Def. (m.u.)	Exp. Loss in Def. (ind.)
Austria	58.803	74.943	187.001	213.496	128.198	138.553
Belgium	39.164	44.642	159.886	165.031	120.722	120.389
Finland	96.432	130.353	247.543	297.819	151.111	167.466
France	55.955	70.635	182.338	206.470	126.383	135.835
Italy	58.211	96.972	360.366	658.307	302.155	561.335
Netherlands	99.905	135.623	252.879	305.291	152.974	169.668
Spain	53.345	85.914	323.027	565.103	269.681	480.189

Table 2.6: % Change in Benefits of Debt and Expected Losses in Default in Transitioning From a Monetary Union to an Independent Monetary Policy

	% Change Ben. of Debt -Exp. Loss in Def.	% Change Ben. of Debt	% Change Exp. Loss in Def.
Austria	27.5	14.2	8.1
Belgium	14.0	3.2	-0.3
Finland	35.2	20.3	10.8
France	26.2	13.2	7.5
Italy	66.6	82.7	85.8
Netherlands	35.8	20.7	10.9
Spain	61.1	74.9	78.1

Table 2.7: Credit Spreads, Arrow-Debreu Prices of Default, 10-Year Default Probabilities

	Credit Spreads in bp (m.u.)	Credit Spreads in bp (ind.)	A-D Price of Def. (m.u.)	A-D Price of Def. (ind.)	10-year Def. Prob. in % (m.u.)	10-year Def. Prob. in % (ind.)
Austria	42	38	0.290	0.246	0.005717	0.005320
Belgium	65	61	0.411	0.359	0.1252	0.08905
Finland	29	25	0.209	0.171	5.905E-5	6.409E-5
France	44	40	0.301	0.256	0.004334	0.003757
Italy	142	148	0.691	0.771	18.23	32.73
Netherlands	28	25	0.204	0.167	4.175E-4	5.365E-4
Spain	136	142	0.673	0.753	15.46	27.80

Table 2.8: Monetary Union Risk Premium as a Fraction of Credit Spreads and % Change in Credit Spreads, Arrow-Debreu Prices of Default, and 10-Year Probabilities of Default in Transitioning From a Monetary Union to an Independent Monetary Policy

	Monetary Union Risk Premium as Fraction of Credit Spread in %	% Change Credit Spreads	% Change A-D Price of Def.	% Change 10-year Def. Prob.
Austria	9.0	-9.0	-15.1	-6.9
Belgium	6.9	-6.9	-12.7	-28.9
Finland	11.5	-11.5	-18.2	8.5
France	8.7	-8.7	-15.0	-13.3
Italy	-4.6	4.6	11.6	79.5
Netherlands	11.8	-11.8	-18.1	28.5
Spain	-4.7	4.7	11.9	79.8

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Appendix

B.1 Nominal Exchange Rate, Nominal Pricing Kernel, and Nominal Cashflow Process

The nominal exchange rate between countries i and j is expressed in differential form by applying Ito's lemma to $S_t^{i,j} = \frac{M_t^{i,*}}{M_t^{j,*}} = \frac{P_t^i}{P_t^j}$.

$$
\begin{aligned}
\frac{dS_t^{i,j}}{S_t^{i,j}} &= \frac{dP_t^i}{P_t^i} - \frac{dP_t^j}{P_t^j} + \frac{d[P^j, P^j]_t}{(P_t^j)^2} - \frac{d[P^i, P^j]_t}{P_t^i P_t^j} \\
&= a_{i,0}dt + a_{i,X}dX_t + \sum_{k=1}^{N} a_{i,k}dZ_t^k - a_{j,0}dt - a_{j,X}dX_t - \sum_{k=1}^{N} a_{j,k}dZ_t^k + \frac{(a_{j,X}^2 + \sum_{k=1}^{N} a_{j,k}^2)(P_t^j)^2 dt}{(P_t^j)^2} \\
&- \frac{(a_{i,X}a_{j,X} + \sum_{k=1}^{N} a_{i,k}a_{j,k})P_t^i P_t^j}{P_t^i P_t^j} \\
&= \left(a_{i,0} - a_{j,0} + a_{j,X}^2 + \sum_{k=1}^{N} a_{j,k}^2 - a_{i,X}a_{j,X} - \sum_{k=1}^{N} a_{i,k}a_{j,k} \right) dt + (a_{i,X} - a_{j,X})dX_t \\
&+ \sum_{k=1}^{N} (a_{i,k} - a_{j,k})dZ_t^k.
\end{aligned}
$$

Recognizing that in a monetary union the exchange rate is constant through time for each pair of countries i and j, the country-specific subscripts can be dropped from the price level process. The nominal pricing kernel can then be written in differential form by applying Ito's lemma to the expression $M_t^* = \frac{M_t}{P_t}$, the real pricing kernel divided by the price level.

$$\begin{aligned}\frac{dM_t^*}{M_t^*} &= \frac{dM_t}{M_t} - \frac{dP_t}{P_t} + \frac{d[P,P]_t}{P_t^2} - \frac{d[M,P]_t}{M_tP_t} \\ &= -rdt - \theta dX_t - a_0dt - a_XdX_t - \sum_{j=1}^{N} a_jdZ_t^j + \frac{(a_X^2 + \sum_{j=1}^{N} a_j^2)P_t^2dt}{P_t^2} + \frac{\theta a_X M_tP_tdt}{M_tP_t} \\ &= -\left(r + a_0 - \theta a_X - a_X^2 - \sum_{j=1}^{N} a_j^2\right)dt - (\theta + a_X)dX_t - \sum_{j=1}^{N} a_jdZ_t^j.\end{aligned}$$

The nominal cashflow process for the representative firm of country i can similarly be expressed in differential form by applying Ito's lemma to $Y_t^{i,*} = P_tY_t^i$.

$$\begin{aligned}\frac{dY_t^{i,*}}{Y_t^{i,*}} &= \frac{dP_t}{P_t} + \frac{dY_t^i}{Y_t^i} + \frac{dP_tdY_t^i}{P_tY_t^i} \\ &= (a_0dt + a_XdX_t + \sum_{j=1}^{N} a_jdZ_t^j) + (\mu_idt + \sigma_idW_t^i) + \frac{(\sigma_i\lambda_ia_X + \sigma_i\sqrt{1-\lambda_i^2}a_i)P_tY_t^idt}{P_tY_t^i} \\ &= \left(\mu_i + a_0 + \sigma_i\lambda_ia_X + \sigma_i\sqrt{1-\lambda_i^2}a_i\right)dt + \left(\sigma_i\sqrt{1-\lambda_i^2} + a_i\right)dZ_t^i + (\sigma_i\lambda_i + a_X)dX_t \\ &+ \sum_{j=1 (j\neq i)}^{N} a_jdZ_t^j\end{aligned}$$

B.2 Risk-Neutral Measure

Given the exogenously specified process for the nominal pricing kernel, the risk-neutral dynamics of the nominal cashflow process of the representative firm in each country can be derived. The nominal pricing kernel, M_t^*, evolves according to

$$\frac{dM_t^*}{M_t^*} = -r^*dt - (\theta + a_X)dX_t - \sum_{j=1}^{N} a_jdZ_t^j.$$

The density process for the risk-neutral measure is defined as

$$\frac{d\xi_t^*}{\xi_t^*} = E_t\left[\frac{dQ^*}{dP}\right].$$

The density process and the pricing kernel are related by

$$\xi_t^* = B_t^* M_t^*$$

where

$$B_t^* = e^{r^* t}$$

is the time t price of the nominal risk-free bond. B_0^* has been normalized to 1. Applying Ito's lemma to the expression for the density process, I obtain

$$\begin{aligned}
d\xi_t^* &= B_t^* dM_t^* + dB_t^* M_t^* \\
&= B_t^*(-r^* M_t^* dt - (\theta + a_X) M_t^* dX_t - \sum_{j=1}^{N} a_j M_t^* dZ_t^j) + r^* B_t^* dt M_t^* \\
&= -(\theta + a_X) B_t^* M_t^* dX_t - \sum_{j=1}^{N} a_j B_t^* M_t^* dZ_t^j.
\end{aligned}$$

I now replace M_t^* with $\frac{\xi_t^*}{B_t^*}$ and divide through by ξ_t^* to obtain the expression

$$\frac{d\xi_t^*}{\xi_t^*} = -(\theta + a_X) dX_t - \sum_{j=1}^{N} a_j dZ_t^j.$$

By the Girsanov Theorem, the new Brownian motions under the risk-neutral measure are given by

$$dX_t^{\mathcal{Q}^*} = dX_t + (\theta + a_X) dt$$

and

$$dZ_t^{j,\mathcal{Q}^*} = dZ_t^j + a_j dt.$$

for all j between 1 and N. The risk-neutral dynamics of the nominal cashflow process of the representative firm of country i can be derived now as

$$\begin{aligned}
\frac{dY_t^{i,*}}{Y_t^{i,*}} &= \left(\mu_i + a_0 + \sigma_i \lambda_i a_X + \sigma_i \sqrt{1-\lambda_i^2} a_i\right) dt + (\sigma_i \lambda_i + a_X) dX_t + \left(\sigma_i \sqrt{1-\lambda_i^2} + a_i\right) dZ_t^i \\
&+ \sum_{j=1 (j\neq i)}^{N} a_j dZ_t^j \\
&= \left(\mu_i + a_0 + \sigma_i \lambda_i a_X + \sigma_i \sqrt{1-\lambda_i^2} a_i\right) dt + (\sigma_i \lambda_i + a_X)[dX_t^{\mathcal{Q}^*} - (\theta + a_X) dt] \\
&+ \left(\sigma_i \sqrt{1-\lambda_i^2} + a_i\right) (dZ_t^{i,\mathcal{Q}^*} - a_i dt) + \sum_{j=1 (j\neq i)}^{N} a_j (dZ_t^{j,\mathcal{Q}^*} - a_j dt) \\
&= \left[\mu_i - \theta(\sigma_i \lambda_i + a_X) + a_0 - a_X^2 - \sum_{j=1}^{N} a_j^2\right] dt + (\sigma_i \lambda_i + a_X) dX_t^{\mathcal{Q}^*} + \left(\sigma_i \sqrt{1-\lambda_i^2} + a_i\right) dZ_t^{i,\mathcal{Q}^*} \\
&+ \sum_{j=1 (j\neq i)}^{N} a_j dZ_t^{j,\mathcal{Q}^*}.
\end{aligned}$$

B.3 Asset Values of the Representative Firm and Government

The nominal asset value of the representative firm of country i, which is the present value of nominal cashflows can be calculated as

$$
\begin{aligned}
V_t^{i,*} &= E_t\left[\int_t^\infty M_T^* Y_T^{i,*} dT\right] \\
&= \int_t^\infty E_t[M_T^* Y_T^{i,*}]dT \\
&= \int_t^\infty E_t^{\mathcal{Q}^*}[e^{-r^* t} Y_T^{i,*}]dT \\
&= \int_t^\infty Y_t^{i,*} e^{-(r^* - \tilde{\mu}_i^*)T} dT \\
&= \frac{Y_t^{i,*}}{r^* - \tilde{\mu}_i^*}.
\end{aligned}
$$

Similarly, the nominal asset value of the government of country i is the present value of the tax revenues of the representative firm:

$$
\begin{aligned}
V_{g,t}^{i,*} &= E_t\left[\int_t^\infty M_T^* \tau_i Y_T^{i,*} dT\right] \\
&= \int_t^\infty E_t[M_T^* \tau_i Y_T^{i,*}]dT \\
&= \int_t^\infty E_t^{\mathcal{Q}^*}[e^{-r^* t} \tau_i Y_T^{i,*}]dT \\
&= \int_t^\infty \tau_i Y_t^{i,*} e^{-(r^* - \tilde{\mu}_i^*)T} dT \\
&= \frac{\tau_i Y_t^{i,*}}{r^* - \tilde{\mu}_i^*}.
\end{aligned}
$$

According to these equations, it is clear that $V_{g,t}^{i,*} = \tau_i V_t^{i,*} = \frac{\tau_i Y_t^{i,*}}{r^* - \tilde{\mu}_i^*}$ and the nominal asset value of the government, the nominal asset value of the representative firm, and the nominal cashflows of the representative firm all follow the same GBM.

Chapter 3

A Heterogeneous-Agent Incomplete Markets Model of the Term Structure of Interest Rates

3.1 Introduction

Recently, Constantinides and Ghosh (2013), hereafter referred to as CG, demonstrated that a heterogeneous-agent incomplete markets asset pricing model, in which agents have recursive preferences, and a single state variable, termed household risk, that drives conditional cross-sectional moments of relative household consumption growth can account for many key features of equity markets. In the model, consumption and dividend growth are deliberately modeled as i.i.d. processes in order to demonstrate that their results do not depend on predictability of consumption and dividend growth processes, as long run risks models such as Bansal and Yaron (2004) do. Beeler and Campbell (2012) provide criticism of long run risk models, showing that predictability of consumption and dividend growth is limited in U.S. data. Although both the volatility and third central moment of the cross-sectional distribution are countercyclical, it is primarily the countercyclical behavior of the third central moment that accounts for the low risk-free rate and price-dividend ratio and high equity premium in recessions. This paper applies the asset pricing model of CG to the term structure of interest rates, and investigates whether or not it can match the means, volatilities, and cyclical properties of interest rates in the data, in particular of nominal interest rates. We also test whether the model generates time-varying expected excess returns on nominal bonds, accounting for the failure of the expectations hypothesis.

The empirical underpinnings of the CG model rely on results by Brav, Constantinides, and Geczy (2002), Cogley (2002), and Guvenen, Ozkan, and Song (2012). Brav, Constantinides, and Geczy show that a stochastic discount factor calculated as the weighted average of individual households' marginal rates of substitution can rationalize the equity premium with low levels of relative risk aversion. Since the equity premium is not explained with an SDF calculated as the per capita marginal rate of substitution, there is evidence of incomplete consumption insurance. Cogley calibrated a model with incomplete consumption insurance that recognizes the cross-sectional variance and skewness of household consumption growth and finds that both the cross-sectional variance and skewness are weakly correlated with stock returns and generate equity premia of 2 percent or less at low degrees of risk aversion. Guvenen, Ozkan, and Song find that the skewness, not the volatility of idiosyncratic earnings shocks is strongly countercyclical by using a confidential dataset from the U.S. Social Security Administration which contains a nationally representative panel of millions of earnings histories for 10% of all U.S. males from 1978-2010.

The model is able to account for several of the empirical regularities observed in the term structure literature, such as deviations from the expectations hypothesis. The failure of the expectations hypothesis, which requires risk premia on bonds to be constant (zero), is documented by Campbell and Shiller (1991) and Fama and Bliss (1987). Their findings indicate that risk premia on bonds are time-varying. The model of CG generates time-varying risk premia through its household risk variable, accounting for the failure of the expectations hypothesis. The expectations hypothesis is tested by running long-rate regressions

$$y^{\$}_{n-1,t+1} - y^{\$}_{n,t} = \alpha_n + \beta_n \frac{1}{n-1}\left(y^{\$}_{n,t} - y^{\$}_{1,t}\right) + \text{error}$$

where $y^{\$}_{n,t} = -\frac{1}{n}\log(P^{\$}_{n,t})$ and $P^{\$}_{n,t}$ is the price of a nominal bond with maturity n. If the expectations hypothesis were to hold, $\beta_n = 1$ for all n, but Campbell and Shiller show that $\beta_n < 1$ and in fact is *negative* and decreasing in n. The model in this paper replicates these findings. Additionally, the model generates a downward sloping real yield curve on average, which is supported by empirical evidence on U.K. index-linked bonds as documented by Evans (1998) and Piazzesi and Schneider (2006). Data for U.S. index-linked bonds indicate a positively sloped term structure but the data dates back to only 1997 and the market was very illiquid during the early years. The model's nominal yield curve is upward sloping on average, an observation that holds robustly in the data.

Real yields in the model are procyclical, consistent with empirical findings of Chapman (1997). The countercyclical nature of nominal yields is inconsistent with Fama (1990) but consistent with de Lint and Stolin (2003) and Ang et al. (2007).

This work is very similar in spirit to Wachter (2006) and Bansal and Shaliastovich (2012) in that it takes an asset pricing model linked to macroeconomic fundamentals and examines its implications for the term structure of interest rates. Wachter employs Campbell and Cochrane's (1999) external habit formation model to generate a time-varying price of risk. In her model, nominal bonds depend on past consumption growth through habit and on expected inflation. Bansal and Shaliastovich employ Bansal and Yaron's (2004) long run risk model and add in uncertainty in the long-run expectation of inflation to generate time-varying expected excess returns on nominal bonds. They show that their model quantitatively captures the documented bond predictability features in the data. The strength of this paper relative to these other two papers is twofold. The first strength is that it uses a calibration for the real side of the economy that is consistent with equity return data. Wachter and Bansal and Shaliastovich must use a different calibration for their term structure papers than the original models of Campbell (1999) and Bansal (2004) that were fit to equity return data. The second, and perhaps more important strength, is that the time series of the state variable driving the conditional moments of relative household consumption growth is directly observable from the data in this model. This allows a simple way to verify whether the results of an estimation of the model is consistent with the properties of the underlying macro variables.

Despite the fact that yields implied by the model are driven only by a single state variable which drives conditional cross-sectional moments of relative household growth and inflation, the implied nominal risk-free rate and yield spread capture many of the short and long-run fluctuations of their data counterparts. The purpose of this paper is to determine the implications for the term structure of interest rates of a model which captures features of equity returns, and in that context, it does a good job as it retains all of the attractive features of the original Constantinides and Ghosh (2013) paper including a low risk-free rate, high equity premium, procyclical price-dividend ratio and risk-free rate, and countercyclical expected market return; and further, it produces realistic means and volatilities of bond yields and accounts for the failure of the expectations hypothesis.

The second section introduces the model, the third section presents the solution of the model, the fourth section discusses the calibration of the model, the fifth section describes the results and some of implications of the model for the pricing of real and nominal bonds, and the fifth section provides some concluding comments.

3.2 Model

3.2.1 Specification of the Economy

The specification of the economy follows CG. The economy is an exchange economy with a single nondurable consumption good. There are an arbitrary number of securities in positive or zero net supply. However, there are no markets for trading households' wealth portfolios, so the market is incomplete in this setting. The traded securities in positive net supply is defined as the "market" and the market pays a net dividend D_t at time t and has ex-dividend price P_t, and a normalized supply of one unit. Households are endowed with an equal number of market shares initially but can trade in these shares afterwards.

Aggregate consumption is denoted by C_t, log consumption will be $c_t \equiv \log(C_t)$, and consumption growth is $\Delta c_{t+1} \equiv c_{t+1} - c_t$. Aggregate consumption growth is IID normal: $\Delta c_{t+1} = \mu + \sigma_a \epsilon_{t+1}$, where $\epsilon_t \sim N(0,1)$. Note that by construction, aggregate consumption growth is uncorrelated with business cycles.

Aggregate labor income is defined as $I_t = C_t - D_t$. There are an infinite number of households normalized to one. Household i is endowed with labor income $I_{i,t} = \delta_{i,t} C_t - D_t$, where

$$\delta_{i,t} = \exp\left[\sum_{s=1}^{t}\left\{\left(j_{i,s}^{1/2}\sigma\theta_{i,s} - j_{i,s}\sigma^2/2\right) + \left(\hat{j}_{i,s}^{1/2}\hat{\sigma}\hat{\theta}_{i,s} - \hat{j}_{i,s}\hat{\sigma}^2/2\right)\right\}\right]. \tag{3.1}$$

The exponent contains two terms: the first captures shocks related to the business cycle. The single state variable in our model ω_t drives the business cycle through $j_{i,s}$ which is exponentially distributed with $P\left[j_{i,s} = n\right] = e^{-\omega_s}\omega_s^n/n!$, $n = 0, 1, ...\infty$, $E\left[j_{i,s}\right] = \omega_s$, and independent of all other primitive random variables. $\theta_{i,s} \sim N(0,1)$ is an IID random variable independent of all other primitive random variables. Thus, $j_{i,s}^{1/2}\sigma\theta_{i,s} - j_{i,s}\sigma^2/2$ are normal, conditional on the realization of $j_{i,s}$ with volatility $j_{i,s}^{1/2}\sigma$. The second captures idiosyncratic shocks and is defined identically to the first term with the difference being that $\hat{\omega}$ is a constant instead of a state variable.

In this setting, household income shocks are permanent. Since the number of households is infinite and income shocks are symmetric across households, we can apply a law of large numbers and show that $I_t = C_t - D_t$. Also, with this specification of household income and symmetric and homogeneous household preferences, households choose not to trade so that household consumption is $C_{i,t} = I_{i,t} + D_t = \delta_{i,t}C_t$. More details will be provided later in this subsection about how it is proven that no-trade is an equilibrium in this economy.

For convenience, define the variable x_t in terms of ω_t as $x_t \equiv \left(e^{\gamma(\gamma-1)\sigma^2/2} - 1\right)\omega_t$. The calibration of CG limits $\gamma > 1$, implying $x_t > 0$. There is a one-to-one mapping from x_t to ω_t, so the dynamics of x_t will be specified in place of ω_t. x_t follows a square root process:

$$x_{t+1} = x_t + \kappa\left(\bar{x} - x_t\right) + \sigma_x\sqrt{x_t}\epsilon_{x,t+1} \tag{3.2}$$

where $\epsilon_{x,t+1} \sim N(0,1)$, IID and independent of all primitive random variables. $\bar{x} > 0$ and $2\kappa\bar{x} > \sigma_x^2$ so the Feller condition is satisfied. Note that in discrete time, the square root process can still become negative even if the Feller condition holds. The autocorrelation of x_t is $1 - \kappa$ so the autocorrelation of real and nominal bond yields will be $1 - \kappa$ since bond yields are affine in x_t. The volatility of household risk is countercyclical and this accounts for the fact that the variance of bond yields are higher in recessions.

The state variable ω_t and therefore x_t can be viewed as a household risk variable in the context of our model. The logarithm of relative household consumption growth is

$$\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right) = \log(\delta_{i,t+1}) - \log(\delta_{i,t}) = j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2 \tag{3.3}$$

with conditional first moment (μ_1) and conditional second and third *central* moments ($\mu_{c,2}$ and $\mu_{c,3}$):

$$\mu_1\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right) = -\sigma^2\omega_{t+1}/2 - \hat{\sigma}^2\hat{\omega}/2 \tag{3.4}$$

$$\mu_{c,2}\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right) = \left(\sigma^2 + \sigma^4/4\right)\omega_{t+1} + \left(\hat{\sigma}^2 + \hat{\sigma}^4/4\right)\hat{\omega} \tag{3.5}$$

$$\mu_{c,3}\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right) = -\left(3\sigma^4/2 + \sigma^6/8\right)\omega_{t+1} - \left(3\hat{\sigma}^4/2 + \hat{\sigma}^6/8\right)\hat{\omega} \tag{3.6}$$

Note that an increase in household risk results in an increase in the variance and a more negative skewness of the cross-sectional distribution of household consumption growth. A high level of household risk is therefore associated with recessions.

Households' preferences have a recursive structure:

$$U_{i,t} = \left\{(1-\delta)C_{i,t}^{1-1/\psi} + \delta\left(E_t\left[U_{i,t+1}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}} \tag{3.7}$$

where δ is the subjective discount factor, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$. Epstein and Zin (1989) show that the SDF of household i is

$$M_{i,t+1} = \exp\left(\theta\log\delta - \frac{\theta}{\psi}\Delta c_{i,t+1} + (\theta-1)r_{i,c,t+1}\right) \tag{3.8}$$

where $\Delta c_{i,t+1} \equiv c_{i,t+1} - c_{i,t}$ and $r_{i,c,t+1} \equiv \log\left(\frac{P_{i,c,t+1}+C_{i,t+1}}{P_{i,c,t}}\right)$ is the log return on the i^{th} household's private valuation of its wealth portfolio.

Let $P_{i,c,t}$ be the price of the i^{th} household's private valuation of its wealth portfolio and $Z_{i,c,t} \equiv P_{i,c,t}/C_{i,t}$, and $z_{i,c,t} \equiv \log(Z_{i,c,t})$. CG prove by induction in their Appendix B that the wealth-to-consumption ratio is a function of only the state variable ω_t so that $z_{i,c,t} = z_{c,t}$ and each household's private valuation of any security other than the households' wealth portfolios is common so that no trade is an equilibrium. They then conjecture that $z_{c,t}$ is an affine function of x_t, $z_{c,t} = A_0 + A_1 x_t$, and log-linearize the SDF, obtaining two equations determining A_0 and A_1. See Appendix D of CG for a derivation. Households' common SDF is given by

$$M_{t+1} = \exp\left\{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma \Delta c_{t+1} + (\theta - 1)[h_0 + h_1 A_0 - (A_0 + A_1 x_t)] + \lambda x_{t+1}\right\} \tag{3.9}$$

where h_0, h_1, A_0, A_1, and λ can be found in Appendix C.1.

Letting R_{t+1} be the real return on a real asset, R_{t+1} satisfies the standard Euler equation

$$\mathrm{E}_t[M_{t+1} R_{t+1}] = 1 \tag{3.10}$$

where M_{t+1} is the real pricing kernel. It follows from the above equation that the price of risk can be written as

$$\frac{\mathrm{E}_t[R_{t+1} - R_{f,t}]}{\sigma_t[R_{t+1}]} = -\rho_t(M_{t+1}, R_{t+1}) \frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]}. \tag{3.11}$$

Using the fact that M_{t+1} is lognormal based on time-t information in our model,

$$\frac{\mathrm{E}_t[R_{t+1} - R_{f,t}]}{\sigma_t[R_{t+1}]} = -\rho_t(M_{t+1}, R_{t+1}) \sqrt{e^{\gamma^2 \sigma_a^2 + \lambda^2 \sigma_x^2 x_t} - 1}. \tag{3.12}$$

Since high household risk is associated with recessionary periods, the maximum Sharpe ratio, the ratio of volatility of the stochastic discount factor to its mean, varies counter-cyclically. Since real bonds will be affine functions of x_t, $\rho_t(M_{t+1}, R_{t+1})$ will be nonzero so that risk premia on real bonds will vary through time. This will provide the mechanism through which the failure of the expectations hypothesis will be achieved, since a similar argument will apply when the nominal pricing kernel is substituted for the real pricing kernel on the right hand side of equation 3.11 and the returns on the left side are nominal returns on nominal bonds.

Note that the interpretation of the variable x_t as a variable associated with recessions when high and good times when low depends crucially on the parameter λ being greater than 0. If $\lambda > 0$, assets that have high payoffs when x_{t+1} is low will command a positive risk premium and assets that have high payoffs when x_{t+1} is high will have a negative risk premium. The opposite will be the case if $\lambda < 0$: assets that have high payoffs when x_{t+1} is low will have a negative risk premium and assets that have high payoffs when x_{t+1} is high will command a positive risk premium.

3.2.2 Exogenous Inflation Process

In order to model nominal bonds, it is necessary to specify a process for inflation. Inflation is modeled as an exogenous process. Letting P_t denote the price level, $p_t \equiv \log(P_t)$, log inflation will be $\pi_{t+1} \equiv p_{t+1} - p_t$. It is assumed that log inflation follows the process

$$\pi_{t+1} = \eta_0 + \eta_1 x_{t+1} + \sigma_\pi \epsilon_{\pi,t+1} \tag{3.13}$$

where $\epsilon_{\pi,t+1}$ is independent of all other primitive random variables in the economy. The process for inflation is an affine function of household risk and a zero-mean normal random variable. The process for inflation is exposed to all underlying sources of uncertainty in the economy so there will be an inflation risk premium associated with household risk. If $\eta_1 > 0$, there will be a positive inflation risk premium and the opposite is true for

$\eta_1 < 0$. It should be noted that the nth order autocorrelation of π_{t+1} is equal to the nth order autocorrelation of x_{t+1} in this specification. I choose not to introduce another state variable in order to maintain the parsimony of the model.

3.3 Model Solution

In this section, prices and yields are calculated for nominal and real bonds. Prices and yields will be obtained in closed form, with prices exponential affine in the state variable and yields affine in the state variable. The model fits into the Duffie and Kan (1996) class of affine term-structure models, translated into discrete time by Backus, Foresi, and Telmer (2001). Let $P^r_{n,t}$ denote the real price of a real bond maturing in n periods and $P^\$_{n,t}$ denote the nominal price of a nominal bond. The real return on an n-period real bond is $R_{n,t} = \frac{P^r_{n-1,t+1}}{P^r_{n,t}}$ with $r_{n,t} \equiv \log(R_{n,t})$ and the nominal return on an n-period nominal bond is $R^\$_{n,t} = \frac{P^\$_{n-1,t+1}}{P^\$_{n,t}}$ with $r^\$_{n,t} \equiv \log(R^\$_{n,t})$. Additionally, the real yield on a real bond is $y_{n,t} = -\frac{1}{n}\log(P^r_{n,t})$ and the nominal yield on a nominal bond is $y^\$_{n,t} = -\frac{1}{n}\log(P^\$_{n,t})$.

3.3.1 Real bonds

Bond prices are determined recursively using the Euler equation 3.10, for real bonds this will be

$$P^r_{n,t} = E_t[M_{t+1}P^r_{n-1,t+1}] \tag{3.14}$$

with M_{t+1} being defined in equation 3.9. When $n = 0$, the real bond is worth one unit of the consumption good, so $P^r_{0,t} = 1$. Closed form expressions for real bond prices are obtained by conjecturing and verifying that $P^r_{n,t} = e^{W_n+V_n x_t}$. In this way, a recursive relationship between the coefficients is obtained so that bond prices can be calculated for

$n = 1, 2, \ldots$ The results of the calculation are

$$W_n = \theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma\mu + \frac{1}{2}\gamma^2\sigma_a^2 + (\theta-1)(h_0 + h_1 A_0 - A_0) + W_{n-1} + \kappa\bar{x}(\lambda + V_{n-1}) \tag{3.15}$$

$$V_n = -(\theta - 1)A_1 + (1-\kappa)(\lambda + V_{n-1}) + \frac{1}{2}(\lambda + V_{n-1})^2\sigma_x^2 \tag{3.16}$$

with $W_0 = 0$ and $V_0 = 0$. Details can be found in Appendix C.2. Real yields will be $y_{n,t} = -\frac{W_n + V_n x_t}{n}$. Accordingly, the real risk-free in this economy can be calculated by noting that $r_{f,t} = r_{1,t} = y_{1,t}$. Therefore, this will be

$$\begin{aligned} r_{f,t} = -\theta \log \delta - \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) + \gamma\mu - \frac{1}{2}\gamma^2\sigma_a^2 - (\theta - 1)(h_0 + h_1 A_0 - A_0) - \kappa\bar{x}\lambda \\ - \left[-(\theta-1)A_1 + (1-\kappa)\lambda + \frac{1}{2}\lambda^2\sigma_x^2\right] x_t. \end{aligned} \tag{3.17}$$

Since there is positive autocorrelation in the log of the real stochastic discount factor due to the positive autocorrelation of x_t, the real term structure is downward sloping.

3.3.2 Nominal bonds

The equation for nominal bonds can be formulated by noting that the pricing relationship for real bond prices, equation 3.14, must hold for real prices of nominal bonds. Therefore, the following equation is obtained: $\frac{P^\$_{n,t}}{P_t} = \mathrm{E}_t\left[M_{t+1}\frac{P^\$_{n-1,t+1}}{P_{t+1}}\right]$. When $n = 0$, $\frac{P^\$_{0,t}}{P_t} = \frac{1}{P_t}$. This can be rewritten as

$$P^\$_{n,t} = \mathrm{E}_t\left[M_{t+1}\frac{P_t}{P_{t+1}}P^\$_{n-1,t+1}\right] \tag{3.18}$$

with $P^\$_{0,t} = 1$. The nominal pricing kernel in this economy is $M^\$_{t+1} = M_{t+1}\frac{P_t}{P_{t+1}} = M_{t+1}e^{-\pi_{t+1}}$. Closed form expressions for nominal bond prices can be obtained by conjecturing and verifying that $P^\$_{n,t} = e^{W^\$_n + V^\$_n x_t}$. Again, a recursive relationship between the coefficients is obtained. The results are

$$W_n^\$ = \theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma\mu + \frac{1}{2}\gamma^2\sigma_a^2 + (\theta - 1)(h_0 + h_1 A_0 - A_0) + W_{n-1}^\$ \\ + \kappa\bar{x}(\lambda + V_{n-1}^\$ - \eta_1) - \eta_0 + \frac{1}{2}\sigma_\pi^2 \quad (3.19)$$

$$V_n^\$ = -(\theta - 1)A_1 + (1 - \kappa)(\lambda + V_{n-1}^\$ - \eta_1) + \frac{1}{2}(\lambda + V_{n-1}^\$ - \eta_1)^2\sigma_x^2 \quad (3.20)$$

with $W_0^\$ = 0$ and $V_0^\$ = 0$. Again, the details can be found in Appendix C.2. Nominal yields will be $y_{n,t}^\$ = -\frac{W_n^\$ + V_n^\$ x_t}{n}$.The nominal risk-free rate can be calculated by noting that $r_{f,t}^\$ = r_{1,t}^\$ = y_{1,t}^\$$. The result of the calculation is

$$r_{f,t}^\$ = -\theta \log \delta - \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) + \gamma\mu - \frac{1}{2}\gamma^2\sigma_a^2 - (\theta-1)(h_0 + h_1 A_0 - A_0) - \kappa\bar{x}(\lambda - \eta_1) + \eta_0 - \frac{1}{2}\sigma_\pi^2 \\ - \left[-(\theta - 1)A_1 + (1 - \kappa)(\lambda - \eta_1) + \frac{1}{2}(\lambda - \eta_1)^2\sigma_x^2\right] x_t. \quad (3.21)$$

This expression can be rewritten as

$$\begin{aligned} r_{f,t}^\$ &= r_{f,t} + \kappa\bar{x}\eta_1 + \eta_0 - \frac{1}{2}\sigma_\pi^2 - \left[-(1 - \kappa)\eta_1 + \left(\frac{1}{2}\eta_1^2 - \lambda\eta_1\right)\sigma_x^2\right] x_t \\ &= r_{f,t} + \mathrm{E}_t[\pi_{t+1}] + \lambda\eta_1\sigma_x^2 x_t - \frac{1}{2}\sigma_\pi^2 - \frac{1}{2}\eta_1^2\sigma_x^2 x_t. \end{aligned} \quad (3.22)$$

The last two terms on the second line of the right hand side of the above equation are adjustments for Jensen's inequality. In our calibration, it will turn out that $\lambda > 0$ and $\eta_1 > 0$ so the third term is the inflation risk premium due to household risk that the nominal risk-free rate carries relative to the real risk-free rate because inflation is high when the economy is in a recessionary period and low when the economy is doing well, thereby implying that nominal bonds have low payoffs in bad times and high payoffs in good times. The slope of the real pricing kernel will depend on whether η_1 is larger than, smaller than, or equal to λ. If $\eta_1 > \lambda$, the nominal pricing kernel will have negative autocorrelation and the nominal term structure will slope upwards, however, if $\eta_1 < \lambda$, the nominal pricing kernel will have positive autocorrelation and the nominal term structure will slope downwards. In the case that $\eta_1 = \lambda$, the nominal term structure will be flat across all maturities.

3.4 Calibration

The parameters of aggregate consumption growth (μ and σ_a), household income shocks (σ, $\hat{\sigma}$, and $\hat{\omega}$), the state variable (κ, $\bar{x}$, and σ_x), and the preference parameters (δ, γ, and ψ) are the same as those used in CG for their quarterly estimation, 1947:Q1-2009:Q4 detailed in their Table 4. By using the same parameters, the model retains all of the attractive properties of the original model laid out in CG and can account for many features of the nominal term structure. The goal of this paper, after all, is to examine the implications of a model, which is calibrated primarily to match equity returns, when applied to the term structure of interest rates. CG estimate the parameters of their model using GMM to match the following twelve moments: the mean and variance of aggregate consumption growth, dividend growth, and market return; and the mean, variance, and autocorrelation of the real risk free rate and market-wide price-dividend ratio.

The parameters of the inflation process η_0, η_1, and σ_π are chosen to match the level of the nominal term structure, the nominal term premium (the difference between the yield of the 5 year nominal bond and the 3 month nominal bond), and the volatility of inflation in the data. I then compare these results with the results of a linear regression of inflation on the time series of x_t extracted from the data. The time series for x_t can be derived from the time series of conditional moments of the distribution of relative household consumption growth. The data for relative household consumption growth comes from the CEX database and is available at a quarterly frequency over the time period from 1982-2009. Inflation is the log first difference of quarterly observations of the consumer price index over the same time period. Table 3.1 displays the results of the regression of inflation on household risk and table 3.2 displays all of the parameter values for the calibration used in our model solution. In the regression, the coefficient on x_t is 0.315 and the residual has a standard deviation of 0.006 and in the calibration the coefficient on x_t is 0.455 and the standard deviation of the residual is 0.004.

In comparison to Wachter (2006) and Bansal and Shaliastovich (2012), I am able to demonstrate empirically and directly the covariance of inflation with the state variable responsible for generating an upward sloping term structure. There is a positive and significant relationship between inflation and household risk (x_t) as shown in the regression results. The standard errors are corrected for heteroskedasticity and autocorrelation as in Newey and West (1987). The residuals from the regression show no sign of autocorrelation or heteroskedasticity. The results from the regression are close to the parameters needed to match term structure moments, giving strong evidence for the validity of the model.

In the next version of this paper, a full GMM estimation will be done whereby the parameters of the inflation process will be estimated jointly with all the other parameters and moments from the term structure.

3.5 Results

3.5.1 Implications for the Term Structure

Real yields for the 3 month and 5 year zero-coupon bond are plotted as a function of the state variable, x_t, in Figure 3.1. Real yields are procyclical. During recessions, households have a precautionary savings motive due to incomplete insurance markets for idiosyncratic labor shocks. This precautionary savings effect drives down real interest rates. The opposite is true in good times: households will borrow against future income. Real yield spreads (difference between the 5 year and 3 month yield) are procyclical, becoming more negative during recessions and less negative during good times.

The same plot is made for nominal yields in Figure 3.2. Nominal yields are countercyclical. The precautionary savings effect also drives down nominal interest rates during recessions just as it does for real bonds, but simultaneously there is another opposing effect at work for nominal bonds: inflation is high during recessions so investors demand compensation for expected inflation plus a premium for bearing inflation risk. This compensation for expected inflation plus the premium for bearing inflation risk overcomes the precautionary savings effect to make nominal yields higher in recessions. The opposite is again true in good times. Nominal yield spreads are countercyclical, becoming more positive during recessions and less positive during good times.

The real yield curve is downward sloping because long bonds provide a hedge against uncertainty in the future state of the economy. If the persistence of x_t were increased, the real yield curve would have a more negative slope because long bonds would be even more valuable as a hedge due to increased uncertainty over the future state of the economy from a shock to household risk today since the effect of these shocks would not die out as quickly. Accordingly, the real yield curve would be less negatively sloped if the persistence of x_t were decreased. The real yield curve is plotted for the unconditional mean of x_t ($\bar{x}$) and the 25% quantile, median, and 75% quantile of the stationary distribution of x_t. The quantiles of the stationary distribution are found via simulation.

The nominal yield curve is upward sloping because the effect of the inflation risk premium increases with the maturity of the bond. This occurs because inflation is a persistent process, implying that near term inflation will be known with a high degree of certainty whereas inflation far into the future will be more uncertain. Therefore, inflation risk premiums are higher at longer maturities. The nominal yield curve is also plotted for the unconditional mean of x_t ($\bar{x}$) and the 25% quantile, median, and 75% quantile of the stationary distribution of x_t. Table 3.3 details the data and model-generated means and volatilities for the 3 month, 1 year, 2 year, 3 year, 4 year, and 5 year zero-coupon nominal and real bond yields.

The model slightly underpredicts mean nominal yields over the intermediate maturities of the term structure, having a convex shape rather than the concave shape observed in the data. The volatilities of the model generated nominal yields are less than in the data at the short end but get closer at the long end as the volatilities of the model generated yields increase in bonds' time to maturity whereas volatilities of yields in the data actually decrease in bonds' time to maturity. The model generated yields on nominal bonds are higher than those for real bonds at all maturities. Expected inflation of 2.10% annually is mostly responsible for this fact, but the inflation risk premium due to household risk is equal to $\lambda\eta_1\sigma_x^2 x_t$. When evaluated at the unconditional mean of x_t, $\bar{x}$, this premium is equal to 0.036% quarterly or 0.14% in annual terms.

3.5.2 Long Rate Regressions

Long rate regressions of the form

$$y^{\$}_{n-1,t+1} - y^{\$}_{n,t} = \text{constant} + \beta_n \frac{1}{n-1}\left(y^{\$}_{n,t} - y^{\$}_{1,t}\right) + \text{error} \tag{3.23}$$

were carried out as in Campbell and Shiller (1991) to test the hypothesis of constant risk premia on bonds, also known as the generalized expectations hypothesis. If risk premia are constant, the coefficient $\beta_n = 1$ and $\alpha_n = 0$ for all n. To see why this is so, using the definitions of returns and yields,

$$r^{\$}_{n,t} = y^{\$}_{n,t} - (n-1)\left(y^{\$}_{n-1,t+1} - y^{\$}_{n,t}\right). \tag{3.24}$$

After rearranging terms and taking expectations on both sides, the following equation is obtained:

$$\mathrm{E}_t\left[y^{\$}_{n-1,t+1} - y^{\$}_{n,t}\right] = \frac{1}{n-1}\left(y^{\$}_{n,t} - y^{\$}_{1,t}\right) - \frac{1}{n-1}\mathrm{E}_t\left[r^{\$}_{n,t} - y^{\$}_{1,t}\right] \tag{3.25}$$

The second term on the right side of the above equation is the scaled risk premium on the n-period bond. A regression of changes in yield spreads on the scaled yield spread will produce a coefficient of one, only if the third term is constant. In other words, only if the risk premium on the n-period bond is constant. Based on equation 3.12, this asset pricing model should generate time-varying risk premia on bonds, resulting in a failure of the expectations hypothesis. Of course, matching the coefficients obtained from running regressions on actual yields will be the real test of the model.

Campbell and Shiller (1991) find a coefficient that is negative at all maturities and significantly different from one. The same regression run on nominal bond yields in the data for the period beginning in the second quarter of 1952 and ending in the fourth quarter of 2009, and the results are very similar to the findings of Campbell and Shiller (1991). In particular, there are decreasing coefficients across maturities with the regression coefficients being negative at all maturities. The coefficient β_n can be determined analytically since yields are available in closed form and are affine in the state variable x_t. The calculations are given in Appendix C.3.

$$\beta_n = \frac{-nV^{\$}_{n-1}(1-\kappa) + (n-1)V^{\$}_n}{n\left(V^{\$}_1 - \frac{V^{\$}_n}{n}\right)} \tag{3.26}$$

The slope of the regression coefficients across maturities is decreasing in the data. The model is able to replicate this stylized fact. Taking the partial derivative of the above expression with respect to n:

$$\frac{\partial \beta_n}{\partial n} = \frac{V_n^{\$}[(1-\kappa)V_{n-1}^{\$} - V_n^{\$}]}{n^2 \left(V_1^{\$} - \frac{V_n^{\$}}{n}\right)^2}, \tag{3.27}$$

and using the fact that $V_n^{\$} < 0$ for all n and $V_n^{\$} < V_{n-1}^{\$}$, the expression on the right hand side of the above equation is negative for all values of n. This implies that the coefficients are negative across all maturities and decreasing as observed in the data. The results in Figure 3.5 provide support for time-varying risk premia on nominal bonds generated by our model. The coefficients from the regression are close to -3 at the short end of the term structure and are decreasing in time to maturity and decrease to about -3.5 at the five year maturity. The simulated nominal bond yields from the model show a larger departure from the expectations hypothesis than the data does. The same long-rate regression is run on model-generated data from real bond yields to determine whether the failure of the expectations hypothesis holds for real bonds as well, supporting the fact that real risk premia are time-varying in the model as well. The coefficients for the long rate regressions on real bonds are of the exact same form as in equation 3.26 with nominal coefficients being replaced by their real counterparts. Using the fact that $V_n > 0$ for all n and $V_n > V_{n-1}$, the coefficients are negative and decrease with maturity. The results from these regressions are plotted in Figure 3.6. Interestingly, the coefficients are obtained in the real case are very similar to those obtained in the nominal case.

3.5.3 Implications for the Time Series of x_t

In the section introducing the model, we note that an increase in x_t results in an increase in the variance and a more negative skewness of the cross-sectional distribution of relative household consumption growth. As mentioned in CG, data on relative household consumption growth is available only for the period from quarter one of 1982 to quarter four of 2009. They plot the skewness and volatility of the distribution of relative household consumption growth over this time period in Figure 3.8, showing that the volatility increases slightly during the recessions in the early 1980s and 1990s, but shows no appreciable increase during the recessions of the 2000s. The skewness of the distribution, however, becomes substantially more negative during recessions. There is also a strong overall trend of skewness becoming less negative over the entire period from quarter one of 1982 to quarter four of 2009.

In Figure 3.7, the time series data for log inflation and the time series for x_t are plotted for the period from 1982:Q1-2009:Q4. From this plot, it is easy to see the source of the inflation risk premium due to household risk in the model. Inflation is highest when household risk is high. The positive correlation between inflation and household risk is especially pronounced during the late 1980s and early 1990s, implying large inflation risk premiums during these periods and high term premiums in correspondence with term structure data from that period. It is less pronounced in the 2000s, which corresponds with the low inflation risk premiums and low term premiums observed in the data at that time.

3.5.4 Higher Order Moments of Nominal Bond Yield Changes

Piazzesi (2010) finds that changes in nominal bond yields have negative skewness and excess kurtosis across all maturities from one month to five years for data from 1964-2001. In a shorter subsample from 1990-2001, the skewness and kurtosis effects disappear and nominal bond yield changes are more Gaussian. In my model, nominal bond yield changes have a negative skewness and excess kurtosis driven by the fact that the household risk state variable follows a discrete time square root process which has a skewed and leptokurtic stationary distribution. Had the state variable followed a process with a stationary distribution that had 0 skewness and a kurtosis of 3, such as a simple AR(1) process, yield changes would also have had 0 skewness and a kurtosis of 3. In Table 3.4, I list the skewness and kurtosis of nominal bond yield changes for bonds of different maturities up to 5 years. These moments were determined via simulation.

3.6 Conclusion

This paper presents a heterogeneous-agent incomplete markets asset pricing model and applies it to calculate the term structure of nominal and real interest rates. The model is calibrated to capture many of the features of equity returns, and as such, all parameters not related to the inflation process are fit to equity return data. Despite this, the model is able to account for a number of empirical regularities concerning the term structure including the downward sloping real term structure, the upward sloping nominal term structure, procyclical real yields, and countercyclical nominal yields. The model also closely matches the means and volatilities of 3 month, 1 year, 2 year, 3 year, 4 year, and 5 year nominal zero-coupon bond yields. It can also quantitatively match the negative skewness and excess kurtosis of nominal bond yield changes and the failure of the expectations hypothesis in the data by generating a countercyclical time-varying risk premium on nominal and real bonds.

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	Dependent variable:
	π_t
x_t	0.315***
	(0.043)
Constant	0.004***
	(0.001)
Observations	111
R^2	0.108
Adjusted R^2	0.099
Residual Std. Error	0.006 (df = 109)
F Statistic	13.12*** (df = 1; 109)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 3.1: Regression of inflation (π_t) on household risk (x_t) and a constant term for quarterly data over the period from 1982 to 2009. Standard errors are heteroskedasticity and autocorrelation consistent as in Newey and West (1987).

	Calibration
γ	12.59
ψ	1.11
δ	0.990
μ	0.007
σ_a	0.006
κ	0.039
$\bar{x}$	0.010
σ_x	0.028
σ	0.030
$\hat{\sigma}$	0.030
$\hat{\omega}$	0.986
η_0	0.005245
η_1	0.455
σ_π	0.004259

Table 3.2: Parameter values used in our solution of the model. All but the last three parameters for the inflation process are taken directly from Table 4 of CG. CG estimate the parameters of their model using GMM to match the following twelve moments: the mean and variance of aggregate consumption growth, dividend growth, and market return; and the mean, variance, and autocorrelation of the real risk free rate and market-wide price-dividend ratio. The last three parameters are parameters of the inflation process and are chosen to match the level of the term structure, the term premium, and the volatility of inflation.

Maturity	Mean			Std dev		
	Real	Nominal	Data	Real	Nominal	Data
1	0.93	4.99	4.99	0.86	1.06	2.93
4	0.83	5.12	5.37	0.91	1.12	2.93
8	0.67	5.30	5.57	0.99	1.22	2.88
12	0.50	5.50	5.74	1.08	1.32	2.81
16	0.31	5.73	5.88	1.19	1.43	2.77
20	0.08	5.97	5.97	1.30	1.55	2.72

Table 3.3: Means and standard deviations of continuously compounded zero-coupon bond yields in the model and in the data. Yields are in annual percentages. Maturity is in quarters. Data for zero-coupon bond yields is quarterly and begins in the third quarter of 1952 and ends in the last quarter of 2009.

Maturity	1 quarter	4 quarters	8 quarters	12 quarters	16 quarters	20 quarters
Skewness	-0.419	-0.419	-0.419	-0.419	-0.419	-0.419
Kurtosis	6.21	6.21	6.21	6.21	6.21	6.21

Table 3.4: Skewness and kurtosis of model-generated nominal bond yield changes across different maturities.

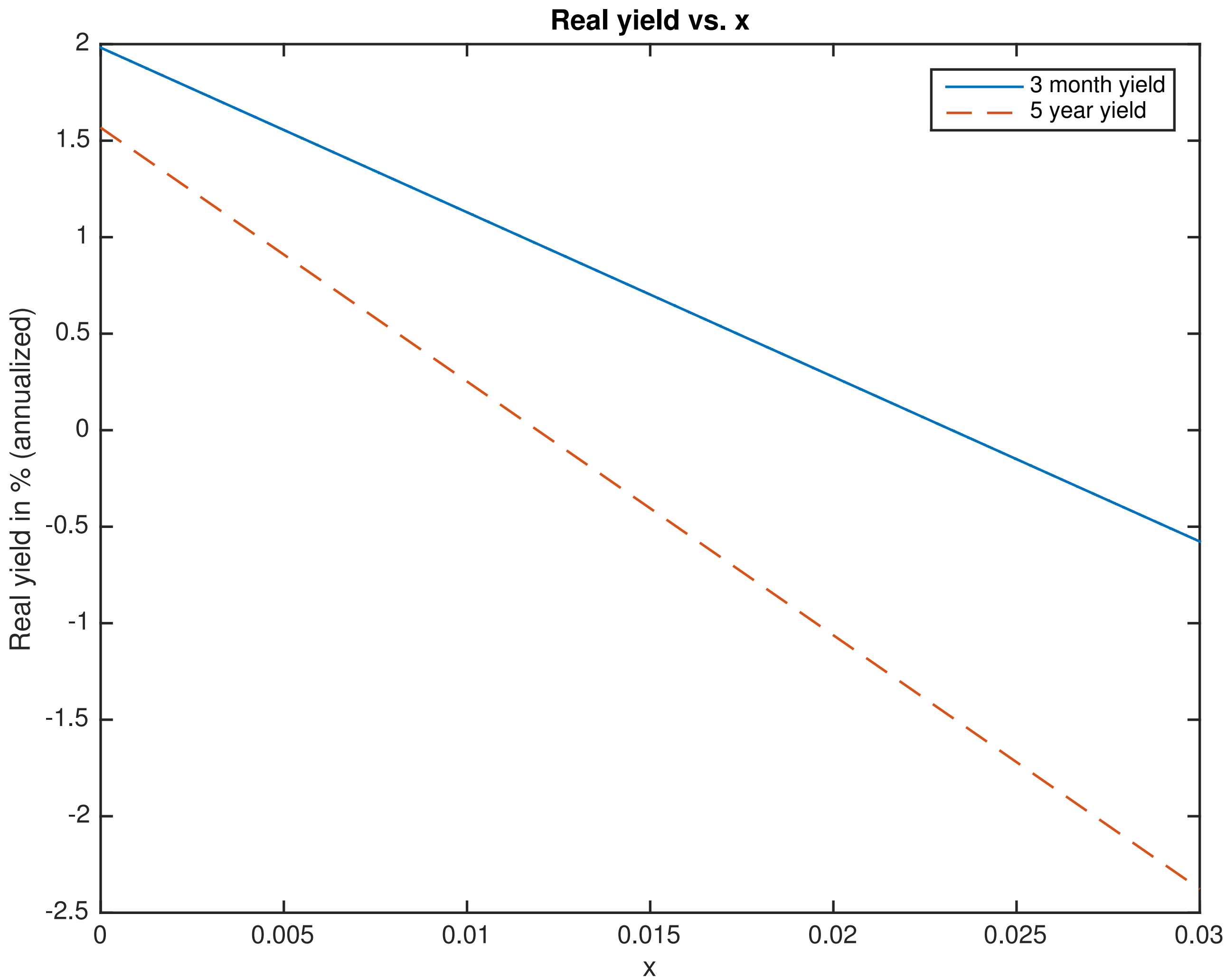

Figure 3.1: **Real yields as a function of household risk** The 3 month and 5 year real yields as a function of the state variable, x_t. Real yields and real yield spreads are procyclical.

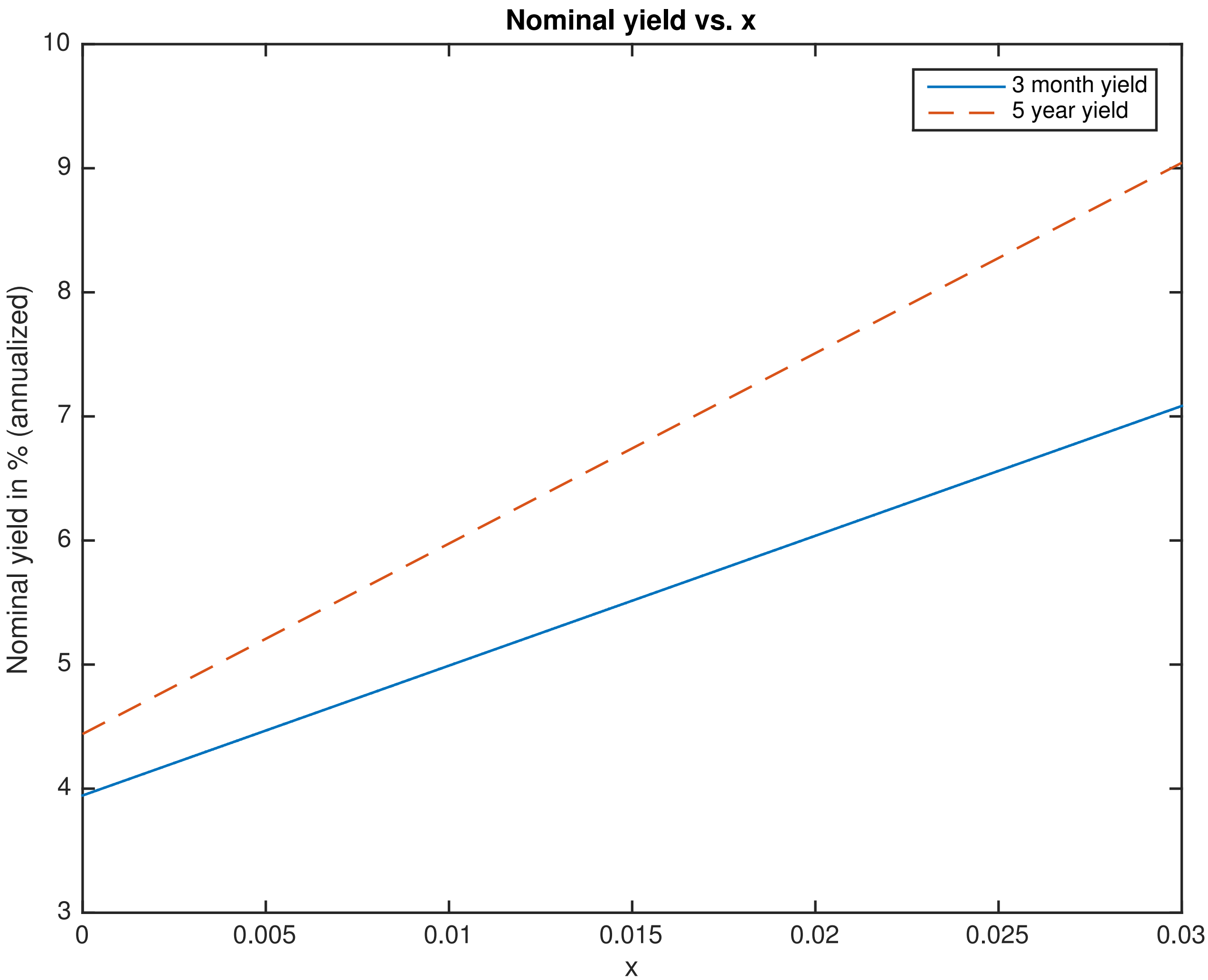

Figure 3.2: **Nominal yields as a function of household risk** The 3 month and 5 year nominal yields as a function of the state variable, x_t. Nominal yields and nominal yield spreads are countercyclical.

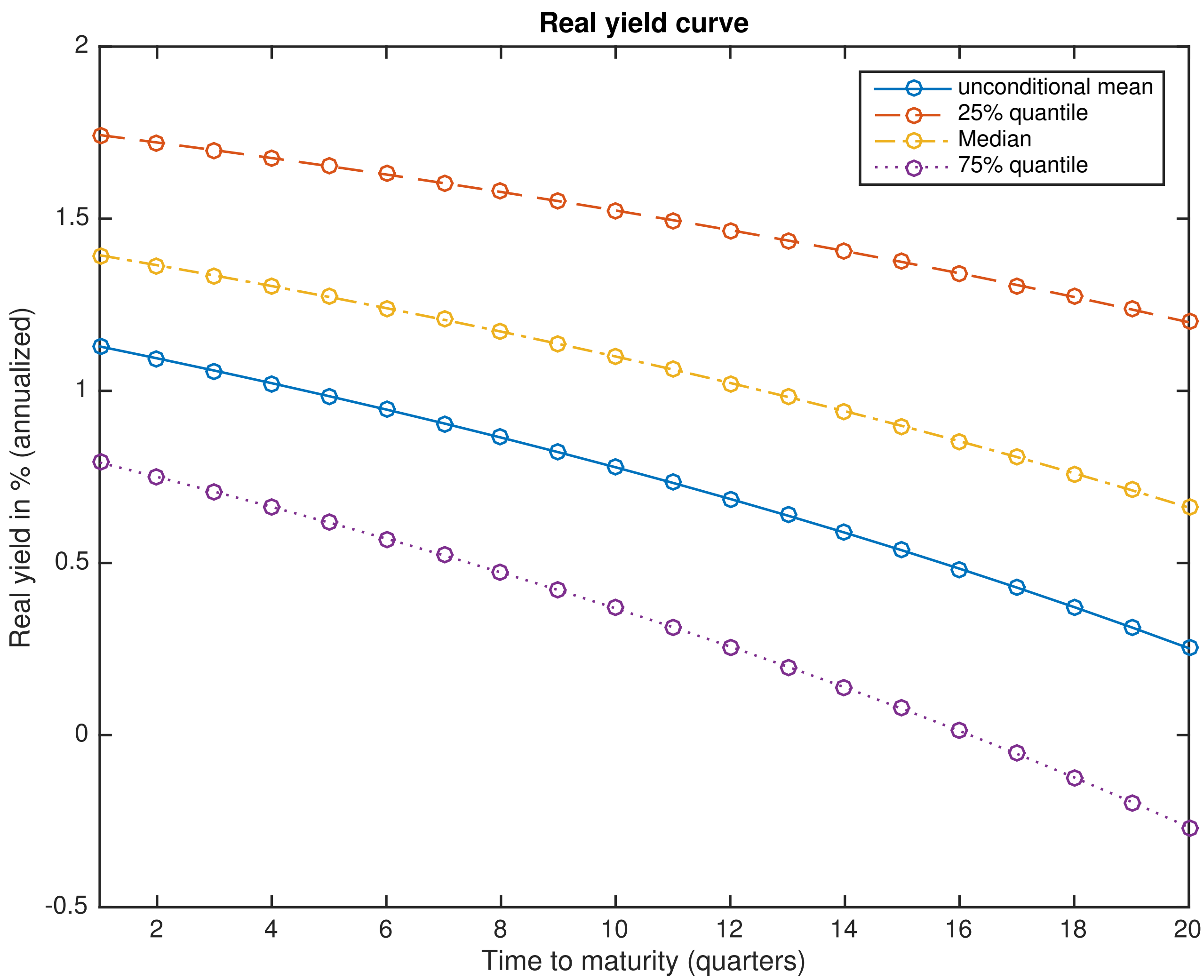

Figure 3.3: **Real yield curve** The model generated real yield curve for maturities ranging from 3 months to 5 years, plotted at the unconditional mean of x_t, $\bar{x}$, and the 25% quantile, median, and 75% quantile of the stationary distribution of x_t.

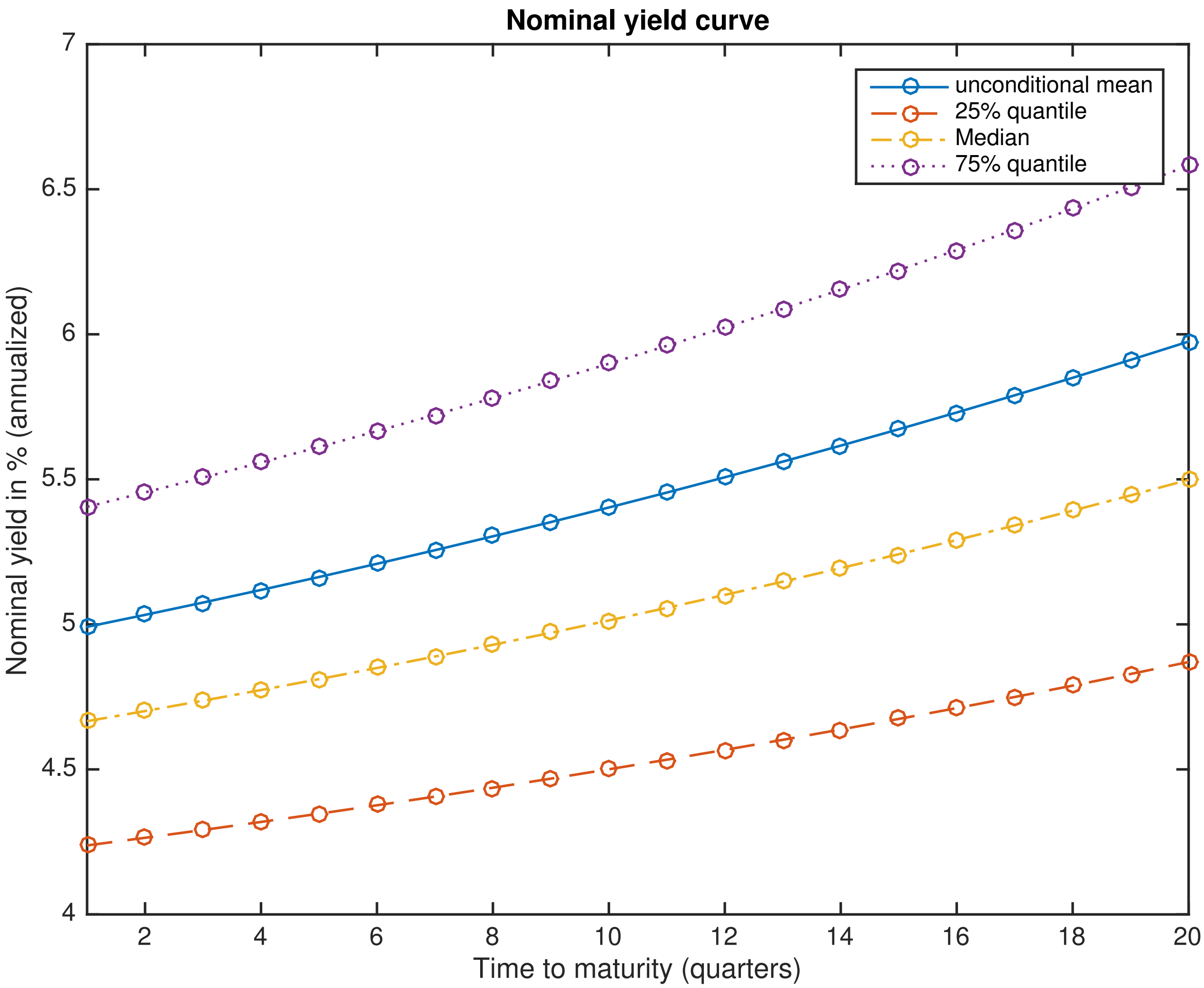

Figure 3.4: **Nominal yield curve** The model generated nominal yield curve for maturities ranging from 3 months to 5 years, plotted at the unconditional mean of x_t, $\bar{x}$, and the 25% quantile, median, and 75% quantile of the stationary distribution of x_t.

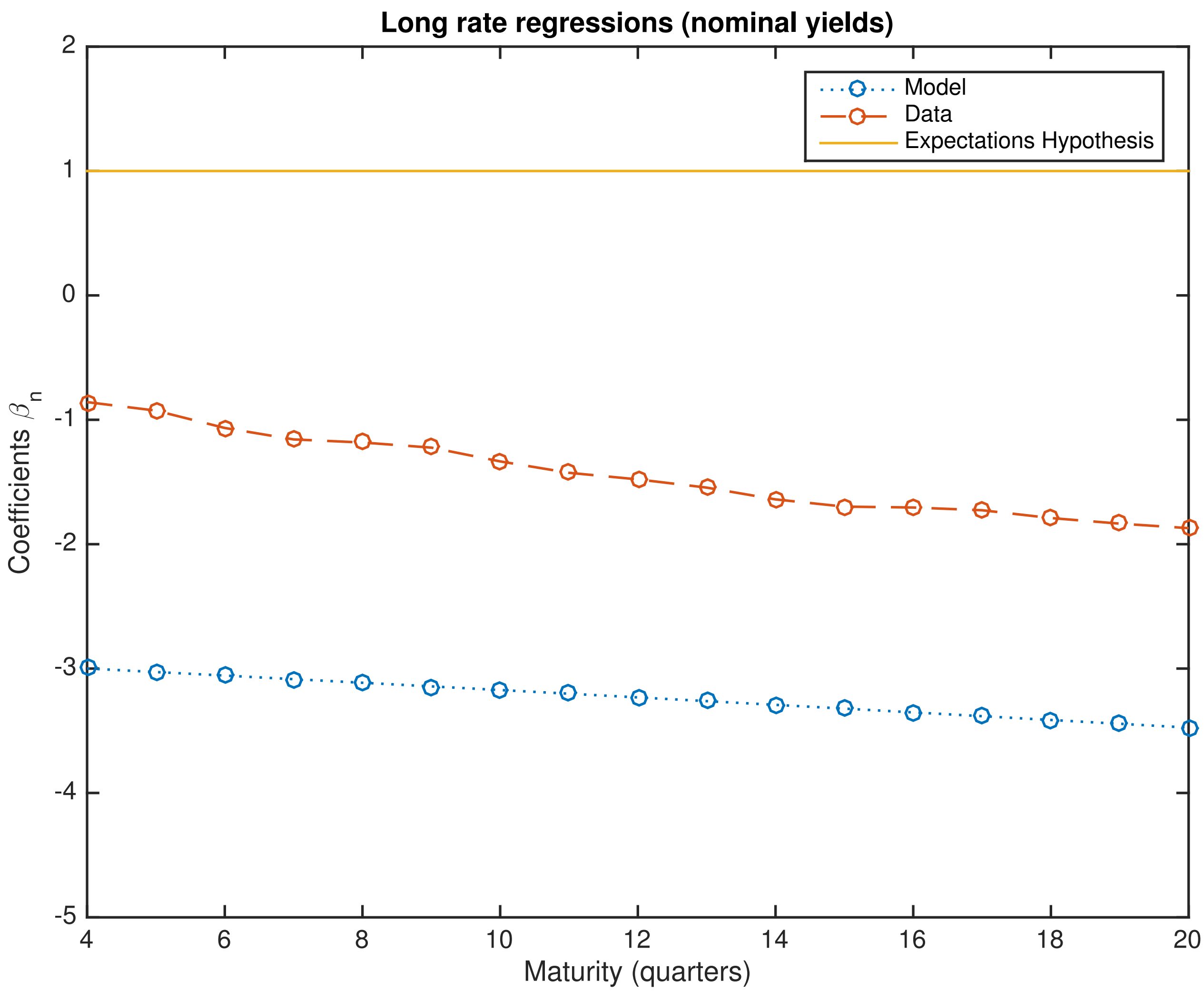

Figure 3.5: **Testing the generalized expectations hypothesis on nominal yields** The results from running the regression $y^{\$}_{n-1,t+1} - y^{\$}_{n,t} = \alpha_n + \beta_n \frac{1}{n-1}\left(y^{\$}_{n,t} - y^{\$}_{1,t}\right) + \text{error}$ using simulated data on nominal bond yields. The dotted line denote coefficients implied by the model, the dashed line denotes coefficients implied by the data, and the solid denotes the coefficients if the expectations hypothesis were to hold.

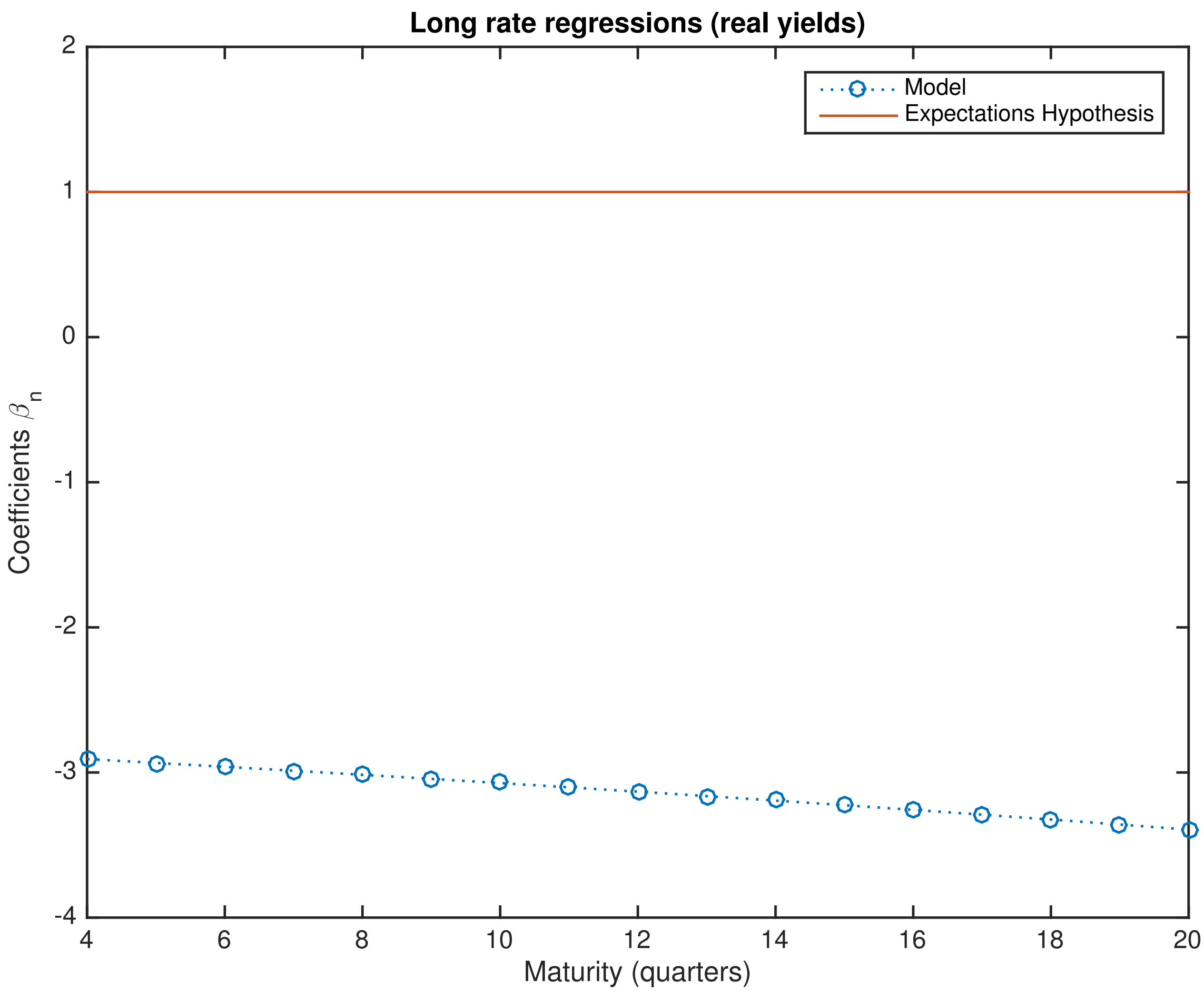

Figure 3.6: **Testing the generalized expectations hypothesis on real yields** The results from running the regression $y_{n-1,t+1} - y_{n,t} = \alpha_n + \beta_n \frac{1}{n-1}(y_{n,t} - y_{1,t}) + \text{error}$ using simulated data on real bond yields. The dotted line denote coefficients implied by the model and the solid denotes the coefficients if the expectations hypothesis were to hold.

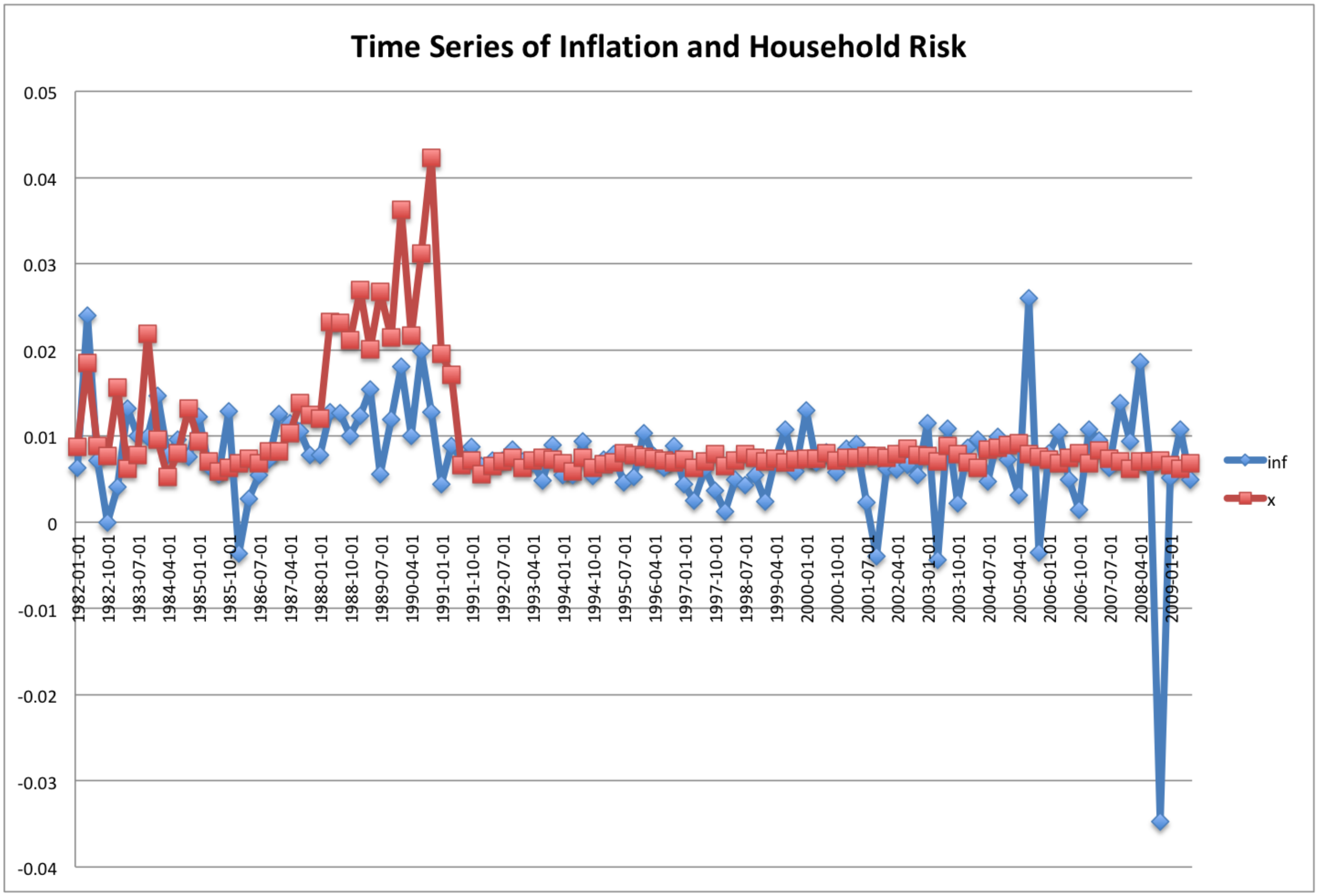

Figure 3.7: **Inflation and household risk variables (1982:Q1-2009:Q4).** Inflation is the log first order difference of the CPI and household risk is derived from the conditional moments of the distribution of household consumption growth. Each of the variables is in natural units.

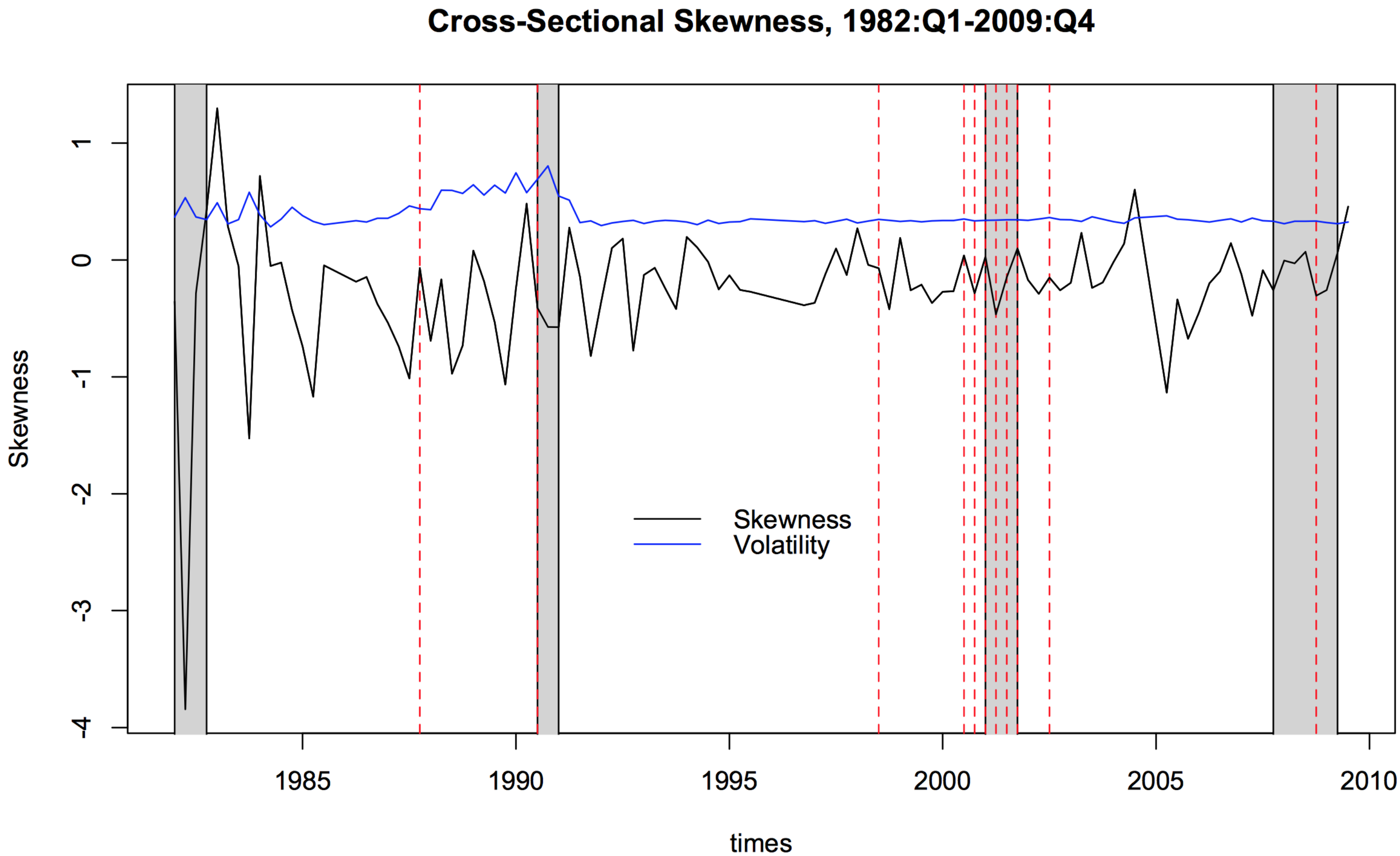

Figure 3.8: **Cross-sectional skewness and volatility of household consumption growth (1982:Q1-2009:Q4).** Calculated from the CEX index using quarterly cross-sectional data.

Appendix

C.1 Parameters of the Wealth-Consumption Ratio and λ

CG conjecture that $z_{c,t} = A_0 + A_1 x_t$ and determine that A_0 and A_1 must satisfy the following two equations:

$$\theta \log \delta + (1-\gamma)\mu + (1-\gamma)^2 \sigma_a^2/2 + \left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2} - 1\right)\hat{\omega} + \theta(h_0 + h_1 A_0 - A_0) + (1 + \theta h_1 A_1)\kappa \bar{x} = 0$$

$$-A_1\theta + (1 + \theta h_1 A_1)(1-\kappa) + (1 + \theta h_1 A_1)^2 \sigma_x^2/2 = 0.$$

Since $\bar{z}_c = A_0 + A_1 \bar{x}$, $h_0 \equiv \log\left(e^{\bar{z}_c} + 1\right) - z_c h_1$, and $h_1 \equiv \frac{e^{\bar{z}_c}}{e^{\bar{z}_c}+1}$, the parameters h_0 and h_1 are determined in terms of A_0 and A_1. A fixed point problem must be solved to determine $\bar{z}_c$, so that the parameters h_0, h_1, A_0, and A_1 can be calculated. The all-important parameter,

$$\lambda = \frac{e^{\gamma(\gamma+1)\sigma^2/2} - 1}{e^{\gamma(\gamma-1)\sigma^2/2} - 1} + (\theta - 1) h_1 A_1.$$

C.2 Derivations of Solutions for Real and Nominal Bond Prices

To solve for real bond prices, conjecture that $P^r_{n,t} = e^{W_n + V_n x_t}$ and substitute this expression into equation 3.14, and substitute the equation for the households' common SDF 3.9 into equation 3.14 as well to obtain

$$e^{W_n + V_n x_t} = \mathrm{E}_t[\exp\{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma \Delta c_{t+1} + (\theta - 1)[h_0 + h_1 A_0 - (A_0 + A_1 x_t)] + \lambda x_{t+1}\} + W_{n-1} + V_{n-1} x_{t+1}]. \quad \text{(C.28)}$$

The right hand side can be expressed as

$$\exp\left\{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) + (\theta-1)[h_0 + h_1 A_0 - (A_0 + A_1 x_t)] + W_{n-1}\right\} \\ \times \mathrm{E}_t[e^{-\gamma \Delta c_{t+1}}]\mathrm{E}_t[e^{(\lambda+V_{n-1})x_{t+1}}] \quad \text{(C.29)}$$

once the variables known at time t are taken out of the conditional expectation and the independence of x_{t+1} and Δc_{t+1} is recognized. Since $e^{(\lambda+V_{n-1})x_{t+1}}$ and $e^{-\gamma\Delta c_{t+1}}$ are both conditionally lognormal, we know that

$$\mathrm{E}_t[e^{-\gamma\Delta c_{t+1}}] = e^{-\gamma\mu + \frac{1}{2}\gamma^2\sigma_a^2} \quad \text{(C.30)}$$

and

$$\mathrm{E}_t[e^{(\lambda+V_{n-1})x_{t+1}}] = \exp\left\{(\lambda+V_{n-1})[x_t + \kappa(\bar{x} - x_t)] + \frac{1}{2}(\lambda+V_{n-1})^2\sigma_x^2 x_t\right\}. \quad \text{(C.31)}$$

Substituting equation C.30 and equation C.31 into equation C.29 and matching the coefficient of x_t and the constant with the left side of equation C.28 will yield equations 3.15 and 3.16.

To solve for nominal bond prices, conjecture that $P_{n,t}^{\$} = e^{W_n^{\$} + V_n^{\$} x_t}$. Also, note that $\frac{P_t}{P_{t+1}} = e^{-\pi_{t+1}}$. Substitute both of these expressions into equation 3.18 along with the households' common SDF equation equation 3.9 to obtain

$$e^{W_n^{\$}+V_n^{\$}x_t} = \mathrm{E}_t[\exp\left\{\theta\log\delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma\Delta c_{t+1} + (\theta-1)[h_0 + h_1 A_0 - (A_0 + A_1 x_t)] + \lambda x_{t+1}\right\} \\ - \eta_0 - \eta_1 x_{t+1} - \sigma_\pi \epsilon_{\pi,t+1} + W_{n-1}^{\$} + V_{n-1}^{\$} x_{t+1}]. \quad \text{(C.32)}$$

The right hand side can be expressed as

$$\exp\left\{\theta\log\delta+\hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2}-1\right)+(\theta-1)[h_0+h_1A_0-(A_0+A_1x_t)]+W^{\$}_{n-1}-\eta_0+\frac{1}{2}\sigma_\pi^2\right\}$$
$$\mathrm{E}_t[e^{-\gamma\Delta c_{t+1}}]\mathrm{E}_t[e^{\left(\lambda+V^{\$}_{n-1}-\eta_1\right)x_{t+1}}] \quad \text{(C.33)}$$

once the variables known at time t are taken out of the conditional expectation and the joint independence of x_{t+1} and Δc_{t+1} is recognized. Since $e^{\left(\lambda+V^{\$}_{n-1}-\eta_1\right)x_{t+1}}$ and $e^{-\gamma\Delta c_{t+1}}$ are conditionally lognormal, we can write

$$\mathrm{E}_t[e^{-\gamma\Delta c_{t+1}}]=e^{-\gamma\mu+\frac{1}{2}\gamma^2\sigma_a^2} \quad \text{(C.34)}$$

and

$$\mathrm{E}_t[e^{\left(\lambda+V^{\$}_{n-1}-\eta_1\right)x_{t+1}}]=\exp\left\{(\lambda+V^{\$}_{n-1}-\eta_1)[x_t+\kappa(\bar{x}-x_t)]+\frac{1}{2}(\lambda+V^{\$}_{n-1}-\eta_1)^2\sigma_x^2x_t\right\}. \quad \text{(C.35)}$$

Substituting equation C.34 and equation C.35 into the right hand side of equation C.32 and matching the coefficient on x_t and the constant with the left side of the equation yields equations 3.19 and 3.20.

C.3 Derivation of Long Rate Regression Coefficients

The long-rate regression coefficients of equation 3.23 are defined as follows:

$$\beta_n=\frac{cov\left(y^{\$}_{n-1,t+1}-y^{\$}_{n,t},\frac{1}{n-1}\left(y^{\$}_{n,t}-y_{1,t}\right)\right)}{var\left(\frac{1}{n-1}(y^{\$}_{n,t}-y_{1,t})\right)}.$$

First, I evaluate the numerator of the right hand side of the equation above:

$$
\begin{aligned}
&cov\left(y^{\$}_{n-1,t+1} - y^{\$}_{n,t}, \frac{1}{n-1}\left(y^{\$}_{n,t} - y_{1,t}\right)\right) = \\
&cov\left(-\frac{W^{\$}_{n-1} + V^{\$}_{n-1}x_{t+1}}{n-1} + \frac{W^{\$}_{n} + V^{\$}_{n}x_{t}}{n}, \frac{1}{n-1}\left(-\frac{W^{\$}_{n} + V^{\$}_{n}x_{t}}{n} + W^{\$}_{1} + V^{\$}_{1}x_{t}\right)\right) \\
&= cov\left(\left(-\frac{V^{\$}_{n-1}}{n-1}(1-\kappa) + \frac{V^{\$}_{n}}{n}\right)x_t, \frac{1}{n-1}\left(-\frac{V^{\$}_{n}}{n} + V^{\$}_{1}\right)x_t\right) \\
&= \left(-\frac{V^{\$}_{n-1}}{n-1}(1-\kappa) + \frac{V^{\$}_{n}}{n}\right)\frac{1}{n-1}\left(-\frac{V^{\$}_{n}}{n} + V^{\$}_{1}\right) var(x_t).
\end{aligned}
$$

The denominator is evaluated next:

$$
var\left(\frac{1}{n-1}\left(-\frac{V^{\$}_{n}}{n} + V^{\$}_{1}\right)x_t\right) = \frac{1}{(n-1)^2}\left(-\frac{V^{\$}_{n}}{n} + V^{\$}_{1}\right)^2 var(x_t).
$$

Dividing the numerator by the denominator, I arrive at equation 3.23.