On the Securitization of Student Loans and the Financial Crisis of 2007–2009

by

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À Françoise et Yvette, mes deux anges gardiens.

ABSTRACT

This dissertation contains three chapters, and each examines the securitization of student loans. The first two chapters focus on the underpricing of Asset-Backed Securities (ABS) collateralized by government guaranteed student loans during the financial crisis of 2007–2009. The findings add to the literature that documents persistent arbitrages during the crisis and doing so in the ABS market is a novelty. The last chapter focuses on the securitization of private student loans, which do not benefit from government guarantees. This chapter concentrates on whether the disclosure to investors is sufficient to prevent the selection of underperforming pools of loans. My findings have normative implications for topics ranging from the regulation of securitization to central banks' exceptional provision of liquidity during crises.

Specifically, in the first chapter, "**Near-Arbitrage among Securities Backed by Government Guaranteed Student Loans**," I document the presence of *near-arbitrage* opportunities in the student loan ABS (SLABS) market during the financial crisis of 2007–2009. I construct near-arbitrage lower bounds on the price of SLABS collateralized by government guaranteed loans. When the price of a SLABS is below its near-arbitrage lower bound, an arbitrageur that buys the SLABS, holds it to maturity and finances the purchase by frictionlessly shorting short-term Treasuries is nearly certain to make a profit. The underpricing on some SLABS relative to Treasuries exceeded 22% during the crisis.

In the second chapter, "**SLABS Near-Arbitrage: Accounting for Historically Unprecedented Macroeconomic Events**," I analyze whether the risks associated with unprecedented macroeconomic events, such as exceptionally high inflation or default by the government on its loan guarantee, could explain the large underpricing of SLABS relative to Treasuries observed during the financial crisis of 2007–2009. Using data on inflation caps, interest rate swaps and interest rate basis caps, and comparing the price dynamics of SLABS to other securities benefiting from a similar government guarantee, I find that for 90% of SLABS, the aforementioned risks explain at most 25% of the near-arbitrage gaps.

In the third chapter, "**Securitization with Asymmetric Information: The Case of PSL-ABS**" **(joint with Adam Ashcraft)**, we empirically analyze the adverse selection of loans in the private student loan (PLS) ABS market. Using loan-level data, we demonstrate the potential for an issuer of PSL-ABS to select loans in such a way that could result in materially adverse outcomes for investors (credit rating downgrades or market value losses). We find that an issuer could increase pool losses on the non-cosigned portion of securitized pools by 6%–20% among pre-crisis deals and by 16%–36% among post-crisis deals while still matching the pool characteristics disclosed to investors. The shifts in pool losses are achieved by exploiting the coarseness of the disclosure and by jointly overrepresenting unseasoned loans in the low credit score region and overrepresenting seasoned loans in the high credit score region. We present multiple additional channels for adverse selection of private student loans that could substantially increases losses without altering the

disclosed characteristics of PSL-ABS deals (e.g. overrepresenting college drop-outs, the share of which is known to the securitizer but not disclosed). The existence of such channels indicates that our estimates of ABS issuers' ability to affect pool performance via loan selection at the time of securitization should be interpreted as lower bounds.

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Contents

Chapter 1

Near-Arbitrage among Securities Backed by Government Guaranteed Student Loans

1.1 INTRODUCTION

The financial crisis of 2007-2009 presented several challenges for central banks in performing their role of liquidity provider of last resort. In the preceding decade, the origination of consumer loans became increasingly reliant on their indirect sale to investors purchasing asset-backed securities (ABS). Most ABS markets experienced sharp declines in prices during the crisis. Simultaneously, the cost of raising funds to originate loans increased. These events raise several questions. Were these declines in prices excessive? Could central banks have reduced the distress of financial intermediaries by purchasing ABS above market price, and yet be taking virtually no risk with taxpayer money? Central banks attempted to stimulate the origination of some types of consumer loans by providing non-recourse loans to ABS buyers. Were the cash-down requirements on the loans to ABS buyers sufficiently large to virtually eliminate the risk taken with taxpayer money?

I contribute to answering the above questions by documenting large underpricings among securities backed by government guaranteed student loans, henceforth SLABS, relative to Treasuries during the crisis. SLABS are unique among the universe of ABS. Holders of SLABS receive cash flows from a pool of loans that are explicitly guaranteed against borrower's default by the US federal government.[1]

[1]The guarantee on student loans issued under the Federal Family Education Loan (FFEL) program is *explicit* since it is mandated by US federal law. It is in contrast with the *implicit* guarantee that many investors expected the US government to fulfill on bonds issued by some of its government-sponsored

I proceed by first computing lower bounds on the price of SLABS. I call these bounds *near-arbitrage lower bounds*. Once the price of a SLABS is below its near-arbitrage lower bound, an arbitrageur that buys a SLABS, holds it to maturity and finances the purchase by frictionlessly shorting short-term Treasuries, is nearly certain to make a profit. Events that can cause a loss on that trade are: i) hyperinflation, ii) default by the US government on its loan guarantee or iii) the credit worthiness of the US government becoming worse than that of the average large commercial bank. During the crisis, the probabilistic assessment of market participants, revealed through derivative and bond markets, indicated that these events were extremely unlikely to occur in the next two decades.

I show that the lowest observed price of some SLABS was 8% to 22% below their near-arbitrage lower bounds during the crisis.[2] The aggregate principal of SLABS outstanding was approximately $190 billion in 2008.[3] For the majority of SLABS that presented near-arbitrage opportunities, their underpricing first exceeded 2% at the end of August 2008 and only reverted to less than 2% at the end of July 2009. Therefore, the near-arbitrage underpricings were large and persistent.

In Chapter 2, I present empirical evidence that for more than 90% of SLABS, the risks associated with historically unprecedented macroeconomic events, such as exceptionally high inflation and default by the government on its loan guarantee, explain at most 25% of their underpricing. Therefore, puzzlingly large relative mispricings remain after accounting for all sources of risk.

Some of the normative implications of my paper set it apart from the existing literature. My paper is the first to document severe relative underpricing in any ABS markets. These findings have novel normative implications for central banks' measures of liquidity provision and their attempt at stimulating the origination of loans during crises. I also propose an original reform that would reduce the costs of the guaranteed loan program for the US government. Implementing the reform without putting taxpayer money at risk requires my methodology to compute near-arbitrage lower bounds. Finally, my findings have implications for a US government asset purchase program.

The US government can issue Treasuries to finance the purchase of SLABS. The

enterprises, such as Fannie Mae and Freddie Mac.

[2] Appendix 1.7.1 contains the list of SLABS trusts that satisfied all selection criteria that makes the analytical derivation of near-arbitrage lower bounds applicable to those trusts. Among the SLABS issued by those trusts, the difference between their near-arbitrage lower bounds ($\underline{P_t}$) and the lowest observed price exceeds 8% when the senior overcollateralization ratio of the pool exceeds 1.06 and the expected paydown date of the SLABS is 2015q1 or later.

[3] In the fall of 2008, the aggregate volume of government guaranteed loans found in the securitized pools of SLM corp. alone was greater than $100 billion. SIFMA estimates the volume of SLABS outstanding to $191.9 billion for 2008.

purchase of SLABS at a price below their near-arbitrage lower bounds, but higher than their market price, would have helped reduce the financial distress of some financial intermediaries, and would have produced a profit for the government with near certainty.

Near-arbitrages among SLABS can act as a canary in the coal mine by signaling a severe need for liquidity provision. A temporary program of liquidity provision, such as the Term Asset-Backed Securities Loan Facility, would be more effective at dampening an excessive contraction of credit if implemented as soon as near-arbitrages are present. Furthermore, the near-arbitrages among SLABS allows a decomposition of the discounts on ABS collateralized by other types of loans, such as auto or credit loans, into a liquidity component and credit component. The central bank can ask for greater compensation for credit risk than the market, but little to no compensation for liquidity risk, when it sets its cash-down requirements on the collateralized loans it offers.

My findings provide insights to reduce the costs of the US federal program of guaranteed student loans. Outside of crises, near-arbitrage lower bounds could be used to establish a guaranteed price at which the government promises to repurchase SLABS in the future. In exchange for the provision of these put options, the government would reduce its supplemental interest payments.[4] As of the end of 2013, there were still more than $250 billion dollar in loans guaranteed by the US federal government, also called FFEL loans, outstanding. Therefore, small reductions in supplemental interest payments, on the order of 0.10%, would translate into savings of $250 million, just in the first year following the reform.[5]

My findings also contribute to the asset pricing literature. Classical asset pricing theory generally assumes that a sufficient number of arbitrageurs can frictionlessly short an expensive asset to raise funds to purchase a cheaper asset with identical cash flows. The trades of arbitrageurs should lead to convergence in prices between the two assets. My paper adds to a growing empirical literature that documents large mispricings during the crisis that pose a major puzzle for the classical asset pricing theory. The TIPS-Treasury arbitrage documented by Fleckenstein, Longstaff, and Lustig (2014), the convertible debenture arbitrage in Mitchell and Pulvino (2012) and the Treasury bond-Treasury note arbitrage in Musto, Nini, and Schwarz (2014) are notable examples in the literature.

Arbitrageurs generally attempt to minimize the cost of financing the purchase of an asset by pledging it as collateral for the funds lent to them. Arbitraging capital would be irrelevant for the relative price of SLABS and Treasuries if cash-down requirements on

[4]The government makes interest payments to holders of government guaranteed loans that supplement the payments made by borrowers.

[5]Assumes 100% participation rate in a voluntary loan swapping program that involves the exchange of a FFEL loan for a loan with a put option that receives smaller supplemental interest payments.

loans collateralized by SLABS were 0% when SLABS become near-arbitrage opportunities. However, the empirical work of Gorton and Metrick (2009), Copeland, Martin, and Walker (2014) and Krishnamurthy, Nagel, and Orlov (2014), suggests that cash-down requirements were at least 5% for SLABS during the crisis. This stream of empirical work partially explains the presence of near-arbitrage among SLABS.

The simultaneous occurrence of near-arbitrage among SLABS and other arbitrages during the crisis supports the hypothesis that arbitraging capital was spread too thinly across a multitude of arbitrages to eliminate them all. Thus, my findings support the slow-moving capital explanation of arbitrage persistence. I hence provide additional evidence in favor of the recent theoretical work by Gromb and Vayanos (2002), Duffie (2010), Ashcraft, Garleanu, and Pedersen (2011), Garleanu and Pedersen (2011) that stresses how arbitraging capital can be an important determinant of the relative pricing of assets.

The remainder of this Chapter is organized as follows. Section 1.2 describes the cash flows on SLABS. Section 1.3 presents benchmark no-arbitrage lower bounds on *simplified* SLABS.[6] Benchmark no-arbitrage lower bounds on simplified SLABS are analytically derived and denoted by $\underline{P_t}^{\dagger\dagger}$. Bankruptcy of the initial servicer and risks associated with unprecedented macroeconomic events are ignored to derive $\underline{P_t}^{\dagger\dagger}$. Section 1.4 presents near-arbitrage lower bounds, denoted by $\underline{P_t}$, computed by simulations for a large sample of SLABS. The computation of $\underline{P_t}$ only ignores the risks associated with unprecedented macroeconomic events.[7] The examination of the pricing and the cost of hedging risks associated with unprecedented macroeconomic events that can cause a loss on a SLABS-Treasury trade initiated at $P_t \leq \underline{P_t}$ takes place in Chapter 2. Section 1.5 examines the implications of near-arbitrages in SLABS for exceptional measures of liquidity provision to market participants and a government-run asset purchase program. A cost-saving reform of the FFEL loan program that relies on the near-arbitrage lower bounds on SLABS is also discussed. In Section 1.6, I make concluding remarks.

1.2 SOURCES OF CASH FLOW ON SLABS

In this section, I describe the sources of cash flow on pools of FFEL loans that collateralize SLABS and the rules of distribution of that cash flow among various claimholders.

[6] Section 1.3 presents the simplifying assumptions imposed to obtain a *simplified* SLABS.

[7] Conditional near-arbitrage lower bounds for SLABS, which are computed after abandoning the simplifying assumptions, but under the maintained condition that the initial servicer avoids bankruptcy and ignoring risks associated with unprecedented macroeconomic events, are denoted by $\underline{P_t}^{\dagger}$ and presented in section 1.4.2.

ABS collateralized by FFEL loans are not perfectly homogeneous and many SLABS have features that differ from the one presented in this paper. For tractability, this paper focuses on a subsample of the SLABS issued by SLM, which is the largest issuer.[8] All the institutional details that I present are accurate for that subsample and the near-arbitrage lower methodology is directly applicable to it. For brevity, I simply refer to SLABS, where it would be more accurate to use SLABS in the selected sample. The selected sample is listed in Table 1.10.[9] Also, I only document near-arbitrage among senior SLABS, although a securitized pool of FFEL loans commonly collateralizes both senior and subordinate SLABS. For brevity, I use SLABS to refer to senior SLABS.

A SLABS is an amortizing variable-rate bond. Let y_t denote the aggregate payment to holders of SLABS collateralized by a given pool of loans in period t. Throughout the paper, time periods are 3-months long, which is the frequency at which SLABS holders receive distributions and the frequency at which interest rates reset. Let ρ_t denote the aggregate principal of SLABS outstanding for a given pool of loans. SLABS promise an interest payment that is tied to the 3-month LIBOR rate,[10] plus a spread, s, that ranges from 0 to 114 basis points.[11] Throughout the paper, interest rates are described on an annualized basis in the text of the paper, and in the spread analysis of Section 1.4.1, but they must be inputted on a non-annualized basis in other equations.[12] The equation that describes the evolution of ρ_t over time is:

$$\rho_{t+1} = \rho_t \cdot (1 + (r_t^{LIBOR} + s)) - y_{t+1}, \tag{1.1}$$

where $y_{t+1} \geq 0$ and r_t^{LIBOR} denotes the LIBOR rate. Throughout the paper, I refer to the full repayment of a SLABS, which is formally defined as $\rho_t = 0$ for some t.

I present the cash flows on SLABS in two steps. First, I present cash flows from a pool of FFEL loans, as depicted in Figure 1.1. Second, I present the rules of distribution of the

[8] SLM uses the Sallie Mae brand to market its student loans. Sallie Mae was a subsidiary of SLM that lost its government-sponsored enterprise status in 2004. SLM had securitized over 50% of the SLABS outstanding in 2008.

[9] In most cases of SLABS with unusual features, a minor modification of the near-arbitrage methodology would be needed.

[10] The London Interbank Offered Rate, LIBOR, reflects an average rate charged between large banks for uncollateralized short-term loans.

[11] Two SLABS in the selected sample have negative spreads of 1–2 basis points. As shown in Section 1.4.1, there is roughly 0.40% of excess arbitrageur's spread under the simplifying assumptions and the worst assumptions that do not violate the interest rate ordering condition (C.2) of Section 1.3, in particular $r_t^{LIBOR} = r_t$. Therefore, the proof of Section 1.3 would also apply to those SLABS.

[12] For instance, the annualized interest on SLABS is equal to the annualized 3-month LIBOR rate at time t, plus an annualized spread of s%. However, in equation (1.1), the non-annualized rate must be plugged in to recover the proper law of motion.

cash flow from the pool to various claimholders, as depicted in Figure 1.2.

Let x_t denote the cash flow from a pool of FFEL loans. The pool of loans is formed prior to the issuance of SLABS.[13] As loans in the pool amortize over time, cash flows from the pool are used to pay down SLABS. The rules of distribution of the cash flow from the pool to the various claimholders lead to y_t/x_t that is generally greater than 90% and to a tight link between the amortization of the pool and the amortization of SLABS.

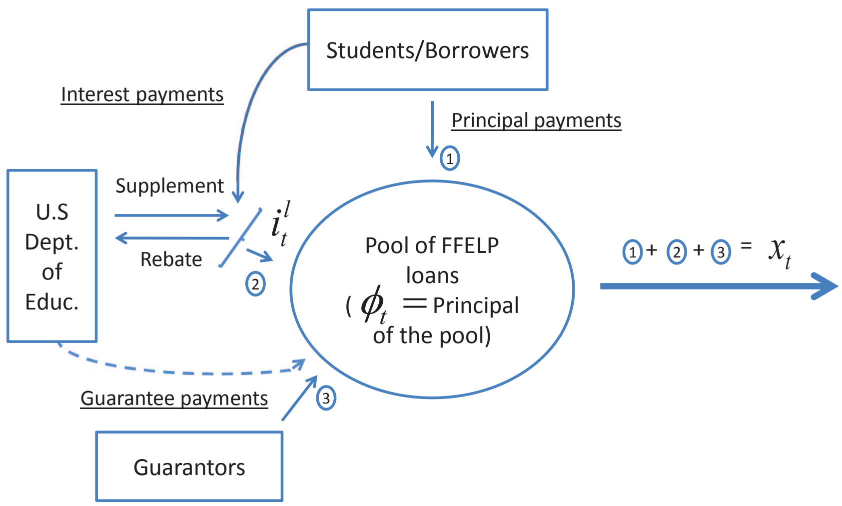

Figure 1.1: Cash flows from pool of FFEL loans: This figure shows the three sources of cash flows from a pool of FFEL loans that collateralizes a SLABS. Students/borrowers make principal payments. Students make interest payments and the U.S. Department of Education either supplements those interest payments or requires that a fraction be rebated to the government. A net interest payment i_t^l results. Upon default by a borrower, a guarantor pays a fraction of the student's debt outstanding. The fact that the guarantee is backed by the U.S. federal government is represented by a dashed line.

A pool of FFEL loans has three sources of cash flow. First, students/borrowers make principal payments. Second, students make interest payments and the federal government either supplements those interest payments or requires that a fraction be rebated to the government. A net interest payment, i_t^l, results.[14] Third, upon default by a borrower, a guarantor pays a fraction of the student's debt outstanding.

[13] From the date of issuance of the SLABS onward, no loan gets added to the pool. Using the structured product terminology, SLABS are collateralized by a *static* pool of loans. A minority of student loan ABS have a revolving pool of loans, but they are not covered in this paper.

[14] The U.S. Department of Education supplements interest payments in two ways. First, a borrower may make full interest payment at a given rate, but the government supplements those interest payments in order for the holder of the loan to receive a higher rate. These supplements are called *special allowance payments*. Second, the government pays interest on behalf of students that received *subsidized* loans, while

The formulas of the Department of Education that determine interest supplements and rebates on FFEL loans result in a net interest payment of $i_t^l = \bar{r}_{t,t+1}^{FCP} + m$, where $m \geq 1.74\%$.[15] r_t^{FCP} denotes the financial commercial paper rate with a maturity of 3-month. $\bar{r}_{t,t+1}^{FCP}$ denotes its quarterly average.[16] I model loans as accruing interest with $m = 1.74\%$.

The FFEL program relies on a network of not-for-profit agencies, called guarantors, to guarantee the student loans. Upon default by a borrower, conditional on proper origination and servicing of the loan, a guarantor pays a fraction of the student's debt outstanding. This fraction may vary with the year of origination of a FFEL loan, but it is always at least 97%.[17] There is an explicit guarantee from the government to make payments on default claims, if a guarantor becomes insolvent.[18]

Default claims filed with guarantors can be rejected because of improper servicing or improper origination. Historically, SLM's contractual obligation to repurchase loans whenever rejected default claims have a "materially adverse effect" for SLABS holders has kept write-downs due to default claims rejected below 0.03% of pool balance. Write-downs due to default claims rejected would have been less than 0.05% without the proceeds from the repurchases.[19]

Figure 1.2 presents the rules of distribution of the cash flow from the pool to various claimholders.[20] The rules of distribution are hierarchical. The cash flow is first used to pay the loan servicer and the administrator of the SLABS trust.[21] Then, if anything is left,

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they are in-school. Although the payment of interest by the government on subsidized loans is a form of credit enhancement, I conservatively treat all loans as unsubsidized in this paper. Finally, prospectuses for SLABS use *floor income rebate* to refer to the rebating of interest payments to the government.

[15] The net interest rate can differ between loans disbursed at different dates and between loan in various statuses, (such as in-school, in repayment, or in deferment.), but it is always at least $\bar{r}_{t,t+1}^{FCP} + 1.74\%$.

[16] The Federal Reserve Bank publishes a 3-month financial commercial paper rate daily (publication H.15). $\bar{r}_{t,t+1}^{FCP}$ is computed by averaging those rates over a quarter.

[17] FFEL loans are either 100%, 98% or 97% guaranteed. For my analytical analysis, if a pool of loans contains any loans that are 97% guaranteed, then I assume that the entire pool is 97% guaranteed. If a pool of loans only contains loans that are either 98% or 100% guaranteed, I assume that the entire pool is 98% guaranteed. While SLM does not explicitly disclose a balance-weighted average loan guarantee for a SLABS pool in its quarterly distribution reports, it discloses a balance-weighted average coupon and since coupon and loan guarantee have a one-to-one relation, it becomes possible to infer the balance-weighted average loan guarantee. I use inferred balance-weighted average loan guarantee when computing near-arbitrage lower bounds by simulations.

[18] See Federal Law, 20 U.S.C. §1082 (o).

[19] Based on a sample that covers the period from December 2001 to March of 2011, with an increasing number of deals in each period, reaching 60 deals by the end of the sample.

[20] I only present the rules of distribution that are relevant for the senior SLABS. For example, Figure 1.2 and the rest of the paper abstracts from the principal distribution to subordinate SLABS holder, because they only occur, if they occur at all, after all senior SLABS have been repaid in full.

[21] The trust is the entity that intermediates collection from the pool of loans and distribution to various claimholders.

the remaining cash flow is used to make a second type of payment. Then, if anything is left, a third kind of payment is made, etc. In Figure 1.2, the first type of payment is depicted at the top and each successive type of payments is placed below.[22]

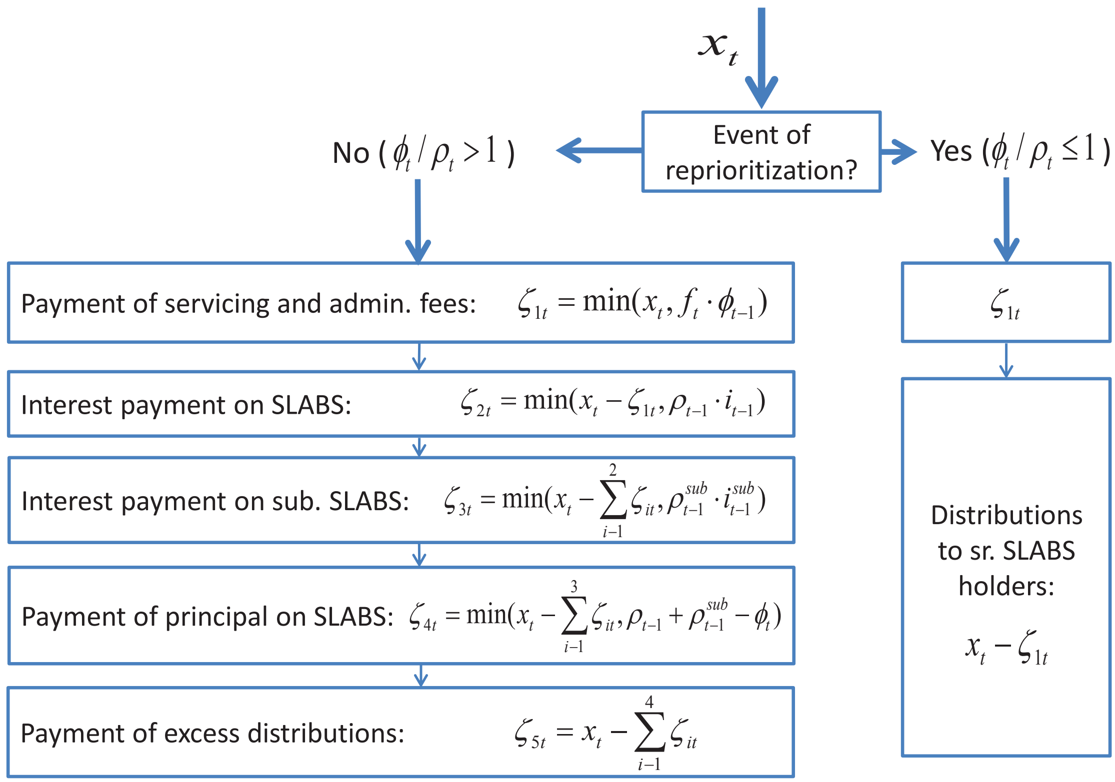


Figure 1.2: Distribution of cash flows from pool of FFEL loans: This figure shows the distribution of the cash flow from the pool, x_t, among various claimholders. f_t is expressed as a percentage of the pool balance. Specifically, f_t is obtained by dividing servicing and administrative fees in dollars, at period t, by the pool balance, at period $t-1$. i_t denotes the interest rate on SLABS. i_t^{sub} denotes the interest rate on subordinate SLABS. ρ_t^{sub} denotes the aggregate principal of subordinate SLABS collateralized by a given pool of loans.

Two sets of rules of distribution are possible. Which rules apply depend on whether an event of reprioritization has occurred. Either way, the cash flow from the pool is used to pay servicing and administrative fees first. Servicing and administrative fees are expressed as a percentage of the pool balance and denoted by f_t. Let ϕ_t denote the aggregate principal of FFEL loans in a pool. When $\phi_t/\rho_t \leq 1$, an event of reprioritization

[22]This paper focuses on SLABS issued by SLM and yet not all of them have the same rules of distribution. The SLABS that do not conform with the rules of distribution presented in this section are excluded from the selected sample.

is triggered. The senior overcollateralization ratio is computed from ϕ_t/ρ_t. For brevity, I use overcollateralization ratio to refer to the senior overcollateralization ratio.

If no event of reprioritization is triggered, the pool of loans is performing relatively well, and the cash flow from the pool is applied to interest payments on subordinate SLABS prior to being applied to principal distribution on senior SLABS. If an event of reprioritization is triggered, the entire cash flow that is left after paying the servicing and administrative fees is distributed to senior SLABS holders. Therefore, the rules of distributions are such that symptoms of underperformance by the pool of FFEL loans trigger a reprioritization that advantages senior SLABS.

Two features of SLABS that have not been presented yet will be used in later sections. First, annualized servicing fees are at most 0.90% of the pool balance. Administrative fees are at most $25,000 per quarter.[23] For SLABS in the selected sample, their individual pool balance was at least $98 millions throughout the crisis, thus initial annualized administrative fees were at most 0.11%. Second, a close look at Figure 1.2 reveals that payments of principal on SLABS under the no-event of reprioritization rules of distribution, denoted by ζ_{4t}, are constrained to be no greater than $(\rho_{t-1} + \rho_{t-1}^{sub} - \phi_t)$, where ρ_t^{sub} denotes the aggregate principal of subordinate SLABS collateralized by a given pool. I refer to this constraint as the total overcollateralization constraint because it prevents $\phi_t/(\rho_t + \rho_t^{sub}) > 1$ from occurring. The total overcollateralization constraint is removed once the pool balance is less than 10% of the initial pool balance.[24] Excess distribution certificate holders receive positive cash flows when the total overcollateralization constraint binds. The overcollateralization ratio on SLABS generally increases from their issuance onward. The total overcollateralization constraint slows down the build up of overcollateralization. If structured without the total overcollateralization constraint, SLABS would become safer sooner after issuance.

I sum up the information presented in this section with a set of equations. The equations abstract from minor features of SLABS that will be accounted for in the final computation of near-arbitrage lower bounds in Section 1.4. Let x_t^k denote the cash flow on an individual FFEL loans and ϕ_t^k denote its principal. Incorporating the minimum loan guarantee and re-arranging the law of motion for the principal of an individual loan,

[23] One would plug in $f_t = 25,000/\phi_{t-1} + \frac{0.90\%}{4}$ in Figure 1.2, and equations (1.10) and (1.12).

[24] For clarity, initial pool balance refers to the balance of a pool of loans at time of issuance of the SLABS.

gives:

$$x_t^k = \begin{cases} \phi_{t-1}^k \cdot (1 + i_{t-1}^l) - \phi_t^k, & \text{without default,} \\ 0.97 \cdot \phi_{t-1}^k \cdot (1 + i_{t-1}^l), & \text{with default and default claim paid,} \\ rec^k \cdot \phi_{t-1}^k \cdot (1 + i_{t-1}^l), & \text{with default and default claim rejected,} \end{cases} \tag{1.2}$$

where i_t^l denotes the net interest rate on FFEL loans and rec^k the recovery on loan k that would follow the rejection of its default claim. Note that for the case with default claim paid, cash flow from the loan can be re-written as $x_t^k = 0.97 \cdot (\phi_{t-1}^k \cdot (1 + i_{t-1}^l) - \phi_t^k)$, since $\phi_t^k = 0$. Note that, making worst case assumption on recovery, $rec^k = 0$, the cash flow from the loan for the case with default claim rejected can be rewritten as $x_t^k = 0.97 \cdot (\phi_{t-1}^k \cdot (1 + i_{t-1}^l) - \phi_t^k) - 0.97 \cdot (\phi_{t-1}^k \cdot (1 + i_{t-1}^l))$, since $\phi_t^k = 0$. Let $\mathbb{1}_{\{\text{def. reject}\}}^k$ denote the indicator function that takes value 1 if loan k entered default and the default claim was rejected by the guarantor, and value 0 otherwise. Therefore, the following inequality holds:

$$x_t^k \geq 0.97 \cdot \big(\phi_{t-1}^k \cdot (1 + i_{t-1}^l) - \phi_t^k\big) - 0.97 \cdot \mathbb{1}_{\{\text{def. reject}\}}^k \big(\phi_{t-1}^k \cdot (1 + i_{t-1}^l)\big). \tag{1.3}$$

For a pool with N borrowers, let the write-downs in period t, w_t be given by:

$$\mathrm{w}_t = \sum_{k=1}^{N} \mathbb{1}_{\{\text{def. reject}\}}^k \cdot \phi_{t-1}^k \cdot (1 + i_{t-1}^l), \tag{1.4}$$

and let write-downs as a percentage of pool balance be denoted by ω_t.

Therfore, for a pool with N borrowers, the following inequalities hold:

$$\begin{aligned} \sum_{k=1}^{N} x_t^k \geq& 0.97 \cdot \Big(\sum_{k=1}^{N} \phi_{t-1}^k \cdot (1 + i_{t-1}^l) - \sum_{k=1}^{N} \phi_t^k \Big) - 0.97 \cdot \mathrm{w}_t, & (1.5) \\ \sum_{k=1}^{N} x_t^k \geq& 0.97 \cdot \Big(\sum_{k=1}^{N} \phi_{t-1}^k \cdot (1 + i_{t-1}^l) - \sum_{k=1}^{N} \phi_t^k \Big) - 0.97 \cdot \omega_t \cdot \phi_{t-1}, & (1.6) \\ x_t \geq& 0.97 \cdot \Big(\phi_{t-1} \cdot \big(1 + (i_{t-1}^l - \omega_t)\big) - \phi_t\Big), & (1.7) \\ x_t \geq& 0.97 \cdot \big(\phi_{t-1} - \phi_t + \phi_{t-1} \cdot (i_{t-1}^l - \omega_t)\big). & (1.8) \end{aligned}$$

Consider the cash flow from a pool consistent with the issuance of SLABS at time 0 and with first distribution date at time 1. Let T_ϕ denote the termination date of the pool, meaning the first period when $\phi_t = 0$ occurs. Re-arranging and aggregating over time, gives:

$$\sum_{t=1}^{T_\phi} x_t \geq 0.97 \cdot \big(\phi_0 + \sum_{t=0}^{T_\phi - 1} \phi_t \cdot (i^l_{t-1} - \omega_t)\big). \tag{1.9}$$

And, the aggregate cash flow on the SLABS, y_t, is given by:

$$y_t = \begin{cases} \zeta_{2t} + \zeta_{4t}, & \text{if } \phi_t/\rho_t > 1, \\ x_t - \zeta_{1t}, & \text{if } \phi_t/\rho_t \leq 1. \end{cases} \tag{1.10}$$

where equations that explain the ζ_{it} terms can be found in Figure 1.2. ζ_{1t} is at most $f_t \cdot \phi_{t-1}$. Let T_ρ denote the termination date of the SLABS, meaning the earlier of the termination date of the pool and the date of full repayment of the SLABS. The special case with $\rho_0 = \phi_0$ and $\phi_t/\rho_t \leq 1$ for all $t \geq 1$, gives:

$$\sum_{t=1}^{T_\rho} y_t \geq \sum_{t=1}^{T_\rho} x_t - \sum_{t=1}^{T_\rho} f_t \cdot \phi_{t-1} \geq 0.97 \cdot \big(\phi_0 + \sum_{t=0}^{T_\rho - 1} \phi_t \cdot (i^l_{t-1} - \omega_t)\big) - \sum_{t=1}^{T_\rho} f_t \cdot \phi_{t-1}, \tag{1.11}$$

$$\geq 0.97 \cdot \big(\rho_0 + \sum_{t=0}^{T_\rho - 1} \phi_t \cdot (i^l_{t-1} - \omega_t)\big) - \sum_{t=1}^{T_\rho} f_t \cdot \phi_{t-1}. \tag{1.12}$$

In Section 1.3, I place these cash flows in an environment with an arbitrageur that can frictionlessly short Treasuries. I identify weak conditions on $\{i^l_t\}_{t=0}^{T_\rho}$, $\{r^{LIBOR}_t\}_{t=0}^{T_\rho}$, and the 3-month Treasury rate $\{r_t\}_{t=0}^{T_\rho}$, and stronger conditions on $\{f_t\}_{t=0}^{T_\rho}$ and $\{\omega_t\}_{t=0}^{T_\rho}$, such that an arbitrageur that buys a SLABS at a sufficiently low price is guaranteed to make a profit on a trade that goes long SLABS and short Treasuries. In Section 1.4, I present near-arbitrage lower bounds that yield at least 99.9% probability of profit on the SLABS-Treasury trade, when weak conditions on interest rates are maintained and no default by the government on loan guarantees is assumed. The near-arbitrage lower bounds are computed after relaxing the condition on $\{f_t\}_{t=0}^{T_\rho}$ and $\{\omega_t\}_{t=0}^{T_\rho}$ by setting them equal to empirically derived upper bounds.

1.3 BENCHMARK NO-ARBITRAGE LOWER BOUNDS ON THE PRICE OF SIMPLIFIED SLABS

In this section, I analytically derive benchmark no-arbitrage lower bounds for *simplified* SLABS under the assumption that the initial servicer avoids bankruptcy and ignoring risks associated with historically unprecedented macroeconomic events. The no-arbitrage lower bounds, denoted by $\underline{P_t}^{\dagger\dagger}$ serve as a reference point that provides intuition for the near-arbitrage lower bounds that are computed by simulations in section 1.4.

Three simplifying assumptions characterize a simplified SLABS:

Simplifying assumption 1 (SA.1): All supplemental interest payments by the government and payments by guarantors upon default are paid without delay.

SA.2: The net interest rate on FFEL loans is at least $r_t^{FCP} + 1.74\%$.

SA.3: Administrative fees are 0.20% of the pool balance or less.[25]

Next, I explicitly state an assumption that is standard in the financial literature when frictionless no-arbitrage exercises are conducted:

Modelling assumption 1 (MA.1): Investors can frictionlessly short Treasuries to finance their purchase of SLABS. There are no transaction costs.

The analytical no-arbitrage lower bounds only apply to SLABS that meet the following conditions, thus I refer to them as selection criteria:

Selection criterion 1 (SC.1): The rules of distribution of the cash flow from a securitized pool of loans among various claimholders is as presented in Figure 1.2.

SC.2: The SLABS trust receives offsetting payments from the servicer for reductions in interest rate or principal offered to borrowers.

SC.3: The interest rate spread over LIBOR promised on SLABS is positive, $s \geq 0$.

SC.4: None of the SLABS collateralized by the pool are auction-rate securities.[26]

SC.1 is partly to reiterate that not all ABS collateralized by government guaranteed student loans are structured the same way. However, all SLABS in the selected sample, which are listed in Table 1.10, satisfy SC.1 as well as SC.2 to SC.4.[27]

The following two conditions have a very low probability of being violated and I assume that they are met in order to derive analytical no-arbitrage lower bounds:

[25] Both administrative fees and servicing fees are annualized for easy comparison with the annualized interest on SLABS and pools of FFEL loans.

[26] For an analysis of the collapse of the auction rate securities market, see Han and Li (2010).

[27] There are a few exceptions of SLABS with negative spreads of no more than a few basis points. As shown in Section 1.4.1, there is roughly 0.40% of excess arbitrageur's spread on a simplified SLABS when worst case assumptions that do not violate the conditions of this section are used, meaning $r_t^{LIBOR} = r_t$ and $r_t^{FCP} = r_t$. Therefore, Proposition 1 would also apply to those SLABS. But, it might not apply to out-of-sample SLABS that would have negative spreads smaller or equal to -0.40%.

C.1: The U.S. federal government does not default on its guarantee on FFEL loans.[28]

C.2: $r_t^{FCP} \geq r_t$ and $r_t^{LIBOR} \geq r_t$ in every time period.[29]

The following two conditions have low probability of being violated and I use them to derive the benchmark no-arbitrage lower bounds of this section:

C.3: Servicing fees are 0.90% of the pool balance.

C.4: Default claims rejected cause write-downs of no more than 0.05% of the pool balance, per quarter.

A sufficient condition for C.3 to hold is that SLM, which is under contract to service the underlying loans of all SLABS in the selected sample, avoids bankruptcy. Historically, SLM's contractual obligation to repurchase loans whenever default claims rejected have a "materially adverse effect" for SLABS holders has kept write-downs due to default claims rejected below 0.03% of pool balance. Write-downs due to default claims rejected would have been less than 0.05% without the proceeds from the repurchases. Both conditions C.3 and C.4 are guaranteed to hold as long as SLM avoids bankruptcy. Conditions C.3 and C.4 could also hold despite the bankruptcy of SLM, but this would require that the SLABS trust finds a successor servicer that accepts the terms of SLM's servicing contract which is uncertain.

Let P_t denote the price of a SLABS with a principal of \$100. Proposition 1 establishes a benchmark no-arbitrage lower bound for SLABS:

Proposition 1: If conditions C.1 to C.4 hold, then buying a simplified SLABS when $P_t <$\$97 and the overcollateralization ratio is greater or equal to 1 ($\phi_t/\rho_t \geq 1$), and financing the purchase by shorting 3-month Treasuries leads to a positive cash flow $97 - P_t$ at time t and non-negative cash flows in every subsequent period. Thus, the *simplified SLABS-Treasury* trade is an arbitrage.

Proof See Appendix 1.7.2.

The series of equations presented at the end of Section 1.2 provides intuition about the asset side of an arbitrageur's balance sheet. The shorting of Treasuries creates a liability

[28]The full statement of the condition would end with the qualifier "between the date a SLABS is purchased and its termination". The qualifier is intuitive and is omitted for brevity. The qualifier is also omitted from conditions C.2, C.3 and C.4.

[29]In other words, the risk of default by the U.S. federal government is perceived as lower than the risk of default of financial institutions that issue commercial paper, which determines the r^{FCP} rate, and the risk of default on inter-bank loans, which determine the r^{LIBOR} rate.

for the arbitrageur. The proof shows that the cash flow from a simplified SLABS with a principal of $100, which is purchased when its overcollateralization ratio is greater than one, is certain to repay the arbitrageur's debt that has face value of $97 or less and accrues interest at the 3-month Treasury rate.

Proposition 1 can be modified and generalized in two ways. Let $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1}$ denote the benchmark no-arbitrage lower bound on the price of a simplified SLABS with overcollateralization ratio greater or equal to one. For SLABS collateralized by pools of FFEL loans that only contain loans that are at least 98% guaranteed, $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1} =$ \$98. If a SLABS has an overcollateralization ratio below one, then an analytical no-arbitrage lower bound can easily be computed by scaling $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1}$ by a factor of ϕ_t/ρ_t.

Taking as given that conditions C.1 and C.2 hold, $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1}$ can be interpreted in two ways. First, no matter how high the default rates on the pool of FFEL loans, if SLM avoids bankruptcy, then a SLABS-Treasury trade initiated when $P_t < \underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1}$ and $\phi_t/\rho_t \geq 1$ will be profitable. If SLM goes bankrupt, but a successor servicer accepts SLM's original servicing contract, then again, a SLABS-Treasury trade initiated when $P_t < \underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1}$ and $\phi_t/\rho_t \geq 1$ will be profitable.

1.4 NEAR-ARBITRAGE LOWER BOUND ON THE PRICE OF SLABS

In this section, I make two kinds of adjustments on the benchmark no-arbitrage lower bounds derived in the previous section. On the one hand, all simplifying assumptions on SLABS are abandoned to compute near-arbitrage lower bounds. Furthermore, the near-arbitrage lower bounds do not rely on the survival of SLM or on the successor servicer accepting the terms of the original servicing contract. These changes open up the possibility of a loss on a SLABS-Treasury trade initiated at $P_t =$\$97 when $\phi_t/\rho_t = 1$. On the other hand, all SLABS in the selected sample had overcollateralization ratio greater than 1.03 throughout the crisis. I tighthen the lower bounds on the price of SLABS by giving them credit for their overcollateralization ratio in excess of 1.

1.4.1 SIMULATIONS, OVERCOLLATERALIZATION AND RELATION WITH ANALYTICAL LOWER BOUNDS

The benchmark no-arbitrage lower bound of Proposition 1 ($\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1}$) did not give credit to SLABS for their overcollateralization ratio in excess of one ($\phi_t/\rho_t > 1$). Giving full credit for the overcollateralization of a SLABS is important in order to compute near-arbitrage lower bounds that are tight. Figure 1.3 plots a pair of points for every SLABS found in Table 1.10. The overcollateralization ratios range between 1.034 and 1.28 on January 2008 and they increase over time.

The simulation model allows a decomposition of the near-arbitrage lower bounds into two components. $\underline{P_t}|_{\phi_t/\rho_t=1}$ denotes the near-arbitrage lower bound obtained after abandoning all simplifying assumptions and relaxing condition C.3 and C.4, but assuming a counterfactual overcollateralization ratio of 1. Let $\gamma(\phi_t/\rho_t)$ be a scaling function that depends on overcollateralization. Near-arbitrage lower bounds can be decomposed as:

$$\underline{P_t}|_{\phi_t/\rho_t}(\theta_t) = \underline{P_t}|_{\phi_t/\rho_t=1}(\theta_t) \cdot \gamma(\phi_t/\rho_t, \theta_t), \tag{1.13}$$

where θ_t is a vector of parameters that includes the pool balance, ϕ_t, the interest rate level, r_t, the interest rate spread on the subordinate SLABS, s^{sub}, and a few other variables. Holding $\underline{P_t}|_{\phi_t/\rho_t=1}(\theta_t) < 100$ constant, and starting from an overcollateralization ratio such that $\underline{P_t}|_{\phi_t/\rho_t}(\theta_t) < 100$, increases in ϕ_t/ρ_t lead to increases in γ. Past a certain threshold, increases in ϕ_t/ρ_t no longer lead to increases in γ, but they increase the payment of excess distributions, as defined in Figure 1.2. This excess cash flow can help insure against losses due to risks associated with historically unprecedented macroeconomic events.

Figure 1.3 depicts an important relation between two initial parameters used in the simulations: SLABS that are collateralized by a pool with a low balance have high levels of overcollateralization. If one focuses on the downward adjustment needed to go from the benchmark no-arbitrage lower bound, $\underline{P_t}^{++}|_{\phi_t/\rho_t\geq 1}$, to the lower bound $\underline{P_t}|_{\phi_t/\rho_t=1}$ obtained after abandoning all simplifying assumptions and relaxing condition C.3 and C.4, then the downward adjustment would be larger on SLABS with a smaller pool balance.[30] However, once proper credit is given for overcollateralization, the near-arbitrage lower bounds for the sample of SLABS found in Table 1.10 become more similar and close to $100.

POSITIVE ARBITRAGEUR'S SPREAD: SUFFICIENT BUT NOT NECESSARY

This subsection introduces the arbitrageur's spread and explains how abandoning all simplifying assumptions and relaxing servicing fee condition C.3 and write-down condi-

[30] The smaller pool balance is correlated with a smaller average principal per borrower. Therefore, the presence of fixed administrative fees, after abandoning simplifying assumption SA.1, and the introduction of servicing fees per borrower, as a consequence of relaxing condition C.4, cause a larger downward adjustment on SLABS with smaller pool balance.

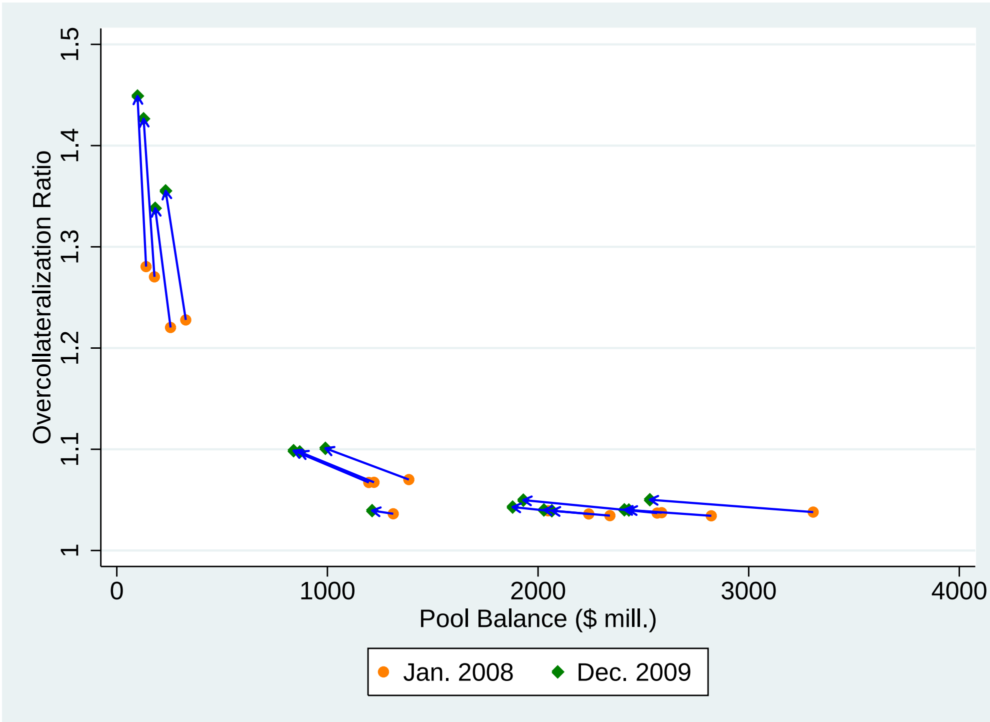


Figure 1.3: Overcollateralization Ratio and Pool Balance. This figure plots the overcollateralization ratio, ϕ_t/ρ_t, and the pool balance for all SLABS found in Table 1.10. There is a pair of points for each SLABS: one for January 2008 and one for December 2009. There is an arrow showing the dynamic between the two points.

tion C.4 can lead to quarters of negative arbitrageur's spread. The occurence of quarters of negative arbitrageur's can easily be handle by the simulation model, but would be challenging to handle analytically.

I define the SLABS spread, ς_t^{SLABS}, by combining cash flows from the pool and rules of distribution of the cash flow to SLABS:

$$\varsigma_t^{SLABS} = \begin{cases} \phi_{t-1} \cdot (i_{t-1}^l - \omega_t - f_t) - \rho_{t-1} \cdot (r_{t-1}^{LIBOR} + s) - \rho_{t-1}^{sub} \cdot (r_{t-1}^{LIBOR} + s^{sub}), & \text{if } \phi_t/\rho_t > 1, \\ \phi_{t-1} \cdot (i_{t-1}^l - \omega_t - f_t) - \rho_{t-1} \cdot (r_{t-1}^{LIBOR} + s), & \text{if } \phi_t/\rho_t \leq 1. \end{cases}$$

Assume an arbitrageur that buys the aggregate principal of SLABS collateralized by a pool and finances the purchase by shorting 3-month Treasuries frictionlessly. Let d_t^{arb} denote the arbitrageur's debt, where $d_t^{arb} = P_t/100 \cdot \rho_t$ and let it evolve over time according to:

$$d_t^{arb} = d_{t-1}^{arb} \cdot (1 + r_{t-1}) - y_t. \tag{1.14}$$

Let the arbitrageur's spread, ς_t^{arb}, be given by:

$$\varsigma_t^{arb} = \begin{cases} \phi_{t-1} \cdot (i_{t-1}^l - \omega_t - f_t) - d_{t-1}^{arb} \cdot r_{t-1} - \rho_{t-1}^{sub} \cdot (r_{t-1}^{LIBOR} + s^{sub}), & \text{if } \phi_t/\rho_t > 1, \\ \phi_{t-1} \cdot (i_{t-1}^l - \omega_t - f_t) - d_{t-1}^{arb} \cdot r_{t-1}, & \text{if } \phi_t/\rho_t \leq 1. \end{cases}$$

And let the worst case arbitrageur spread, $\underline{\varsigma_t^{arb}}$ be given by:

$$\underline{\varsigma_t^{arb}} = \begin{cases} 0.97 \cdot \phi_{t-1} \cdot (i_{t-1}^l - \omega_t) - \phi_{t-1} \cdot f_t - d_{t-1}^{arb} \cdot r_{t-1} - \rho_{t-1}^{sub} \cdot (r_{t-1}^{LIBOR} + s^{sub}), & \text{if } \phi_t/\rho_t > 1, \\ 0.97 \cdot \phi_{t-1} \cdot (i_{t-1}^l - \omega_t) - \phi_{t-1} \cdot f_t - d_{t-1}^{arb} \cdot r_{t-1}, & \text{if } \phi_t/\rho_t \leq 1. \end{cases}$$

The simplifying assumptions and conditions imposed to derive $\underline{P_t}^{++}|_{\phi_t/\rho_t \geq 1} = \$97$ in Section 1.3 form a set of sufficient conditions that guarantees that, for a SLABS-Treasury trade initiate at time 0 with $d_0^{arb}/\rho_0 \leq 0.97$ and $\phi_0/\rho_0 \geq 1$, the worst case arbitrageur's spread is positive whenever $\phi_t/\rho_t \leq 1$. This can be illustrated by considering the special case with $d_0^{arb}/\phi_0 = 0.97$, $\phi_0/\rho_0 = 1$ and $\phi_t/\rho_t \leq 1$ for all $t \geq 1$, which gives:

$$\underline{\varsigma_t^{arb}} = 0.97 \cdot \phi_{t-1} \cdot (i_{t-1}^l - \omega_t) - \phi_{t-1} \cdot f_t - d_{t-1}^{arb} \cdot r_{t-1}. \tag{1.15}$$

Then, plugging in $f_t = 1.10\%$, and the worst case interest rate under condition C.2, $i_{t-1}^l = r_{t-1} + 1.74\%$ gives:

$$\underline{\varsigma_t^{arb}} = d_0^{arb} \cdot (r_0 + 1.74\% - 0.20\%) - \frac{1}{0.97} \cdot d_0^{arb} \cdot 1.10\% - d_0^{arb} \cdot r_0 \approx 0.40\% \cdot d_0^{arb} \tag{1.16}$$

in the first period. The "interest payment" portion of the cash flow from the pool, net of write-downs, $0.97 \cdot \phi_{t-1} \cdot (i_{t-1}^l - \omega_t)$, is more than sufficient to i) pay the servicing and administrative fees and iii) pay the interest on the arbitrageur's debt.[31] Therefore, the entire "principal payment" portion of the cash flow from the pool is available to make principal payment on the SLABS, resulting in:

$$d_1^{arb} \leq d_0^{arb} - 0.97 \cdot (\phi_0 - \phi_1) \tag{1.17}$$

$$d_1^{arb} \leq 0.97 \cdot \phi_1 \tag{1.18}$$

[31] The inequality that relates cash flow from the pool with interest rates and changes in pool balance, equation (1.8), contains a $\phi_{t-1} \cdot i_t^l$ term that can be interpreted as an "interest payment" term. In the background, we may have a situation where lots of borrowers are making interest and principal payments, other borrowers making no payments, resulting in a constant pool balance. From the point of view of the equation at the pool level, it looks as if all borrowers are making their interest payments and none are making principal payment and the entire cashflow from the pool is categorized as "interest payment".

By iteration, with positive arbitrageur's spread in every period, $d_t^{arb} \leq 0.97 \cdot \phi_t$ holds for all t, and $d_t^{arb} = 0$ occurs for some t no greater than the date at which $\phi_t = 0$. Therefore, the arbitrageur's debt is guaranteed to be repaid by the cash flow from the SLABS.

The special case above, and it's generalized version, as summarized by Proposition 1, is achieved by making worst case assumption that does not violate condition C.2, meaning $r_t^{FCP} = r_t$. However, it relies on: i) simplifying assumption SA.3 on administrative fees and condition C.3 on servicing fees to bound f_t at 1.10%, ii) simplifying assumption SA.2 that interest rate on FFEL loans is linked to r_t^{FCP} instead of the actual $\bar{r}_{t,t+1}^{FCP}$, and iii) condition C.4 to bound write-downs due to rejections of default claims. Abandoning all simplifying assumptions and relaxing condition C.3 and C.4 allows for negative arbitrageur's spread in some periods.

Positive arbitrageur's spreads in every period, while sufficient to guarantee the profitability of a SLABS-Treasury trade initiated at \$97, is not necessary for the SLABS-Treasury trade to be profitable. The statement is valid even if we focus on worst case scenario of defaults and start from a counterfactual level of overcollateralization $\phi_t/\rho_t = 1$. There are strictly positive arbitrageur's spreads of at least $0.40\% \cdot d_t^{arb}$ in every quarters in the environment of Section 1.3 for $P_t \leq \$97$, but the analytical no-arbitrage lower bound of $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1} = \$97$ did not give credit to the SLABS for it.

In this section, simulations give proper credit to quarters of positive arbitrageur's spread, which helps offset the effect of some quarters of negative spreads and derive near-arbitrage lower bounds that are not excessively loose. This is especially important with respect to the replacement of r_t^{FCP} by the actual $\bar{r}_{t,t+1}^{FCP}$ because of path dependencies. For example, recurrent quarters of negative spreads may occur during a long period of declining interest rates. However, because interest rates were low during the crisis, a long period of declining interest rates must be preceded by a long period of increasing interest rates, which creates positive spreads that either increase SLABS cash flow directly or contribute to overcollateralization build up.

Recurrent quarters of negative SLABS spreads can occur when $f_t > (i_t^l - r_t)$ in the tail of the amortization of the pool. However, thanks to positive arbitrageur's spreads early in the life of a SLABS or thanks to overcollateralization, the arbitrageur's debt can be much smaller than the the pool balance once SLABS spreads become negative and the cash flow from the SLABS may nonetheless finish to pay down the arbitrageur's debt.

My simulation model allows taking the complicated path dependencies and dynamics described above into account and gives proper credit to SLABS for quarters with positive arbitrageur's spread. It also allows to check whether immediate full default by all borrowers or alternative scenarios lead to the largest downward adjustment relative to

$\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1}$.

SIMULATIONS AND POOLS WITH MULTIPLE TRANCHES OF SENIOR SLABS

There is one additional reason for using simulations. Up to this point, I have assumed that every pool of FFEL loans collateralizes a single senior tranche of SLABS. However, empirically, the majority of pools collateralize multiple tranches of senior SLABS. SLM 2007-2 is a representative deal with four senior tranches collateralized by the same pool: there is tranche A-1, all the way to A-4. When multiple senior tranches are outstanding and no reprioritization event has been triggered, the principal distribution to senior SLABS holders is entirely applied to the top tranche of a deal, until it is paid down.

Following the triggering of an event of reprioritization, the distribution of principal payments among senior tranches of SLABS can either continue to be sequential or become pro rata. When distributions are pro rata, the benchmark no-arbitrage lower bound of $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1} = \$97$ applies to all tranches of a deal. Relative to pro rata distributions among senior tranches, sequential distributions are detrimental to the bottom tranche and beneficial to all other senior tranches. Therefore, $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1} = \$97$ is invalid for the bottom tranche and too loose for the other tranches. The simulation model can compute near-arbitrage lower bounds precisely for all cases.

The vector of inputs for cases with multiple tranches and overcollateralization strictly greater than 1 is larger. For all tranches in the deal, the aggregate principal, ρ_t^{Aj}, and interest rate spread over LIBOR, s^j, of every tranche are determinants of the near-arbitrage lower bounds.

1.4.2 ABANDONING THE SIMPLIFYING ASSUMPTIONS

FIXED ADMNISTRATIVE FEES

Administrative fees on securitized pools are at most \$25,000 per quarter. On a percentage basis, the pool balance needs to be smaller than \$50 million for administrative fees to exceed the 0.20% annualized fees assumed under simplifying assumption SA.3. Figure 1.3 shows that securitized pools either had balances 25 times greater than \$50 million or overcollateralization ratio greater than 1.10.

Furthermore, the total overcollateralization constraint presented in Section 1.2 no longer applies when the pool balance is less than 10% of the initial pool balance.[32] Once

[32] For clarity, initial pool balance refers to the balance of a pool of loans at time of issuance of the SLABS.

the total overcollateralization constraint no longer applies, the payment of principal on SLABS becomes accelerated. SLABS directly benefit from accelerated payment of principal in quarters with positive spread. Since all pools in the selected sample have initial principal balance greater than \$1.3 billion, the total overcollateralization constraint does not apply when there is less than \$130 million left in the pool, at the latest. This occurs at a much earlier date than the point in time at which the pool balance falls below \$50 million, which helps build up overcollateralization.

By simulation, I combine administrative fees of \$25,000 per quarter with a staircase scenario of amortization of the pool that maximizes the impact of default and fixed administrative fees on the downward adjustment from $\underline{P_t}^{++}|_{\phi_t/\rho_t \geq 1}$ to $\underline{P_t}|_{\phi_t/\rho_t=1}(\theta_t)$. This leads to a downward adjustment of at most \$2, with largest adjustments among SLABS with a low pool balance. Because pools with a low balance benefit from significant overcollateralization, once overcollateralization is taken into account, switching from administrative fees on a percentage basis to fixed administrative fees has either no effect on the near-arbitrage lower bounds or trivial effects of at most \$0.10.

$\bar{r}^{FCP}_{t,t+1}$ INSTEAD OF r^{FCP}_t

The Department of Education computes the net interest rate on FFEL loans by using the quarterly average of the 3-month financial commercial paper rate. The interest payment at time $t+1$ on most consumer loans is based on the interest rate that was realized at time t. The computation of interest payments on FFEL loans is unusual: interest rate payments that occur at time $t+1$ are computed by averaging realized rates between t and $t+1$.[33] My notation attempts to reflect this peculiar feature of FFEL loans.

If interest rates during a quarter are lower than the interest rate at the beginning of the quarter, then $r^{FCP}_t > \bar{r}^{FCP}_{t,t+1}$ occurs. The simulation method uses the worst case assumption that does not violate the interest rate condition C.2, meaning that $r^{FCP}_t = r_t$ is used. Thus, in my simulations, it is the difference between $\bar{r}_{t,t+1}$ and r_t that determines the arbitrageur's spread. Furthermore, $r^{LIBOR}_t \geq r_t$ is required to prevent situations where a SLABS is repaid in full, but the arbitrageur's debt, incurred to purchase SLABS with $P_t = 100$, is not repaid in full. $r^{LIBOR}_t > r_t$ creates slack that is beneficial to the SLABS

[33] For example, the payment of an interest rate supplement to a FFEL loan holder on March 30th, 2008 would be based on the principal of the FFEL loan on January 1st, 2008, the average of the 3-month financial commercial paper published daily from January 1st, 2008 to March 30th, 2008, plus a margin ranging between 1.74% and 2.64%. The holder of a FFEL loan must partially rebate interest payments rather than receive interest supplement when a borrower's interest payment is in excess of the net interest promised by the government to the holder of a FFEL loan. Whether the government pays an interest supplement or the holder of the FFEL loan must rebate interest to the government, the computation is the same.

holder. Therefore, the worst case assumption under condition C.2 implies $r_t^{LIBOR} = r_t$ as well as $r_t^{FCP} = r_t$.

How does one makes sure that the near-arbitrage is initiated at a sufficiently low price to be robust to quarters with $r_t > \bar{r}_{t,t+1}$? I check that near-arbitrage lower bounds are robust to a wide universe of interest rate paths by simulations.

I use the regime-switching stochastic volatility model of Kalimipalli and Susmel (2004). Interest rate paths produced by the model have a tendency to revert to a long-run mean, a common feature of models of interest rate with short maturity. The model has shocks that can counterweigh the tendency of interest rate to mean revert. The volatility of these shocks is stochastic. For a given interest rate, the mean of the stochastic process for volatility can either be high or low and switches between low-volatility and high-volatility regimes can occur when interest rate paths are simulated. The following set of equations describes the interest rate model that I use:

$$
\begin{gathered}
r_t - r_{t-1} = a_0 + a_1 r_{t-1} + \sqrt{h_t r_{t-1}^{2\alpha}}\epsilon_t, \\
ln(h_t) - \mu_t = \psi(ln(h_{t-1}) - \mu_{t-1}) + \sigma_\eta \eta_t, \\
\mu_t = \beta + \nu\lambda_t, \text{ where } \nu > 0, \qquad (1.19) \\
P[\lambda_t = \lambda_j | \lambda_{t-1} = \lambda_i] = p_{ij}, \text{ where } \lambda_t = \{0, 1\}, \\
\text{where } \epsilon_t \text{ and } \eta_t \text{ are independently distributed } \sim N(0, 1).
\end{gathered}
$$

I estimate parameters $a_0, a_1, \psi, \sigma_\eta, \beta, \nu, p_{01}, p_{10}$ using the Monte Carlo Markov Chain (MCMC) approach of Kalimipalli and Susmel. The model is estimated on data for the period 01/04/54 to 07/31/08. Therefore, the data contains periods with switches from low to high volatility regimes and the period of high volatility and high inflation of the early 80s. Details of the estimation method and parameter estimates can be found in Appendix 1.7.3.

In addition, in my simulation model, I abandon simplifying assumption SA.1 that supplemental interest payments by the government and payments by guarantors upon default are paid without delay. Abandoning the assumption of no delays in government and guarantor payments pushes near-arbitrage lower bounds to trivially lower levels.

1.4.3 UPPER BOUND ON SERVICING FEES

In my simulations, servicing fees are set equal to the maximum of i) the initial servicing fee and ii) fees on a delinquency-robust marginal servicing contract.

Table 1.1: Intermediate near-arbitrage lower bounds

Interest rate		
Initial rate (r_t)	Type of path	$\underline{P_t}^{\dagger}\|_{\rho_t/\phi_t=1}$
1%	Constant	96.98
2%	Constant	96.84
15%	Constrant	95.09
1%	Stochastic	96.94
2%	Stochastic	96.75
15%	Stochastic	94.30

Simplifying assumption SA.1 of payments without delays is abandoned and loans accrue interest at the $\bar{r}^{FCP}_{t,t+1}$ rate, meaning that SA.2 is abandoned as well. The frictionless shorting assumption MA.1 is maintained and conditions C.1 to C.4 hold. The near-arbitrage lower bounds are robust to 1000 interest rate paths combined with extremely high rates of default (cumulative default rate of 100% within 5–8 quarters of the initiation of a SLABS-Treasury trade), as well as another 1000 interest rate paths combined with varying rates of default. Lower bounds guaranteeing profitability of the trade obtained with the former set of scenario are lower than the lower bounds obtained with the latter set of scenarios. The "representative SLABS" used to set parameter values is the bottom tranche of pool 2003-3, with a counter-factually low minimum loan guarantee of 97% and a counter-factually low overcollateralization ratio of 1.

Table 1.2: Servicing fees under initial contract with SLM

SLABS collateralized by non-consolidation loans:	0.90%
SLABS collateralized by consolidation loans:	0.50%

Table 1.3: Fees on a delinquency-robust marginal servicing contract

Fee to service a delinquent borrower (annualized):	\$70
Fee to service a non-delinquent borrower (annualized):	\$40
Upfront fee per borrower:	\$10
Default claim filing cost:	\$23
Default claim filing cut:	0.50%

Table 1.4 shows the main source of data used to derive fees for the delinquency-robust marginal servicing contract. I use the fees bid by servicer to obtain contract from the Department of Education to bound the cost of servicing a non-delinquent loan. I use hand collected data, found in Appendix 1.7.4, to estimate the difference in the cost of servicing a delinquent loan and a non-delinquent loan. I use the servicing fee on the contract between Goal Financial and ACS to infer an upper bound on the profit margin for non-delinquent borrower of approximately $17 or 70%. Applying the same profit margin to the cost of servicing a delinquent loan, I obtain a fee on delinquent loan of approximately $70. I add an upfront fee of $10 to insure against pre-payment risk. I abstract from the fee charged to file default claims by servicer in Table 1.4, but they are part of the servicing contract between ACS and Goal Financial. I add those fees to the package of fees found in Table 1.3. This package of fees is sufficient to secure a new servicer, regardless of the delinquency rate of in a pool.

Table 1.4: Servicing fees per borrower

Borrower's Status	DoE-Big 4	MOHELA-PHEAA	Goal-GL	Goal-ACS
In School:	13	N/A	15	22
In Grace:	26	N/A	37	45
Current:	26	36	39	43
Deferment/Forbearance:	25	36	39	45
Delinquent 0-30 days:	26	36	39	45
Delinquent 30+ days:	20	36	39	45
Duration:	5 years	Life of loan	5 year	5 year
Borrower count (approx.):	1,000,000	100,000	5000-10000	500-2000

This table reports the terms of third party servicing contracts. The first column reports annualized fees that constituted the winning bids from four large servicers (Big 4) for large servicing contracts from the Department of Education (DoE). The second column reports the terms of a medium size contract between MOHELA and PHEAA. The third and fourth columns report the terms of a very small contract between Goal Financial and Great Lakes, and a marginal contract between Goal Financial and ACS. Servicing fees are reported in dollar, maximum value is reported when a contract included a range of values and fees are rounded up to the nearest dollar.

In addition, the servicing fees are indexed to inflation. The universe of inflation paths considered for the purpose of deriving near-arbitrage lower bounds are those consistent with the interest rate paths drawn from the regime-switching stochastic volatility model. Assuming a real rate of 0%, inflation rate is set equal to the nominal interest rate. The near-arbitrage lower bounds are robust to the indexing of servicing fees to inflation and the inflation paths produced by the regime-switching stochastic volatility model of interest rate. However, inflation paths that are abnormally high, combined with a scenario of bankruptcy of the initial servicer, followed by servicing fees indexed to inflation, could be a source of loss on a SLABS-Treasury near-arbitrage initiated when the price of a SLABS is at its near-arbitrage lower bound or below. Therefore, we had the following conditions to conditions C.1 and C.2 for the near-arbitrage lower bounds computed in this Chapter:

C.3.B: Servicing fees following the initiation of a SLABS-Treasury trade are set equal to the periodic maximum of the initial fee (0.90% of pool balance on an annualized basis) and the sum of the delinquency-adjusted fee of a marginal servicing contract (unit fee per borrower that depend on borrower's status). Servicing fees are indexed to inflation. Inflation paths are limited to those consistent with the interest rate paths drawn from the estimated regime-switching and stochastic-volatility model of interest rate.

The hedging of inflation consistent with paths that fall outside those covered by condition C.3.B is discussed in Chapter 2. The consequent downward revision of near-arbitrage lower bounds on some SLABS, to reflect the cost of purchasing inflation caps, is also presented in Chapter 2.

BOUNDING WRITE-DOWNS DUE TO THE REJECTION OF DEFAULT CLAIMS

Guarantors are allowed to refuse to make loan guarantee payment if they determine that a loan was improperly serviced. SLM's historical ratios of aggregate default claims over aggregate principal across its securitized pools have never exceeded 0.05%.[34] Aggregate write-downs due to default claims rejected represent an even smaller percentage of the aggregate principal of securitized pools: they have never exceeded 0.03%. SLM's contractual obligation to repurchase loans whenever default claims rejected have a "materially adverse effect" on SLABS holders has three implications. First, it mechanically explains the 0.02% difference between the first and second measure. Second, it justifies the validity of conservatively bounding write-downs to 0.05% of the pool as long as SLM avoids bankruptcy, as was done under condition C.4. Third, SLM's obligation to repurchase loans provides incentives that may contribute to the historically low levels of default claims.

Should SLM go bankrupt and reject the initial servicing contract in bankruptcy, the SLABS trust might find a successor servicer that accepts the same servicing contract as

[34] Based on a sample that covers the period from December 2001 to March of 2011, with an increasing number of deals in each period, reaching 60 deals by the end of the sample.

SLM and condition C.4 would continue to hold. However, a successor servicer may not agree to such a low write-down threshold for the repurchase of default claims rejected when a pool is expected to have an abnormally high level of default. In this section, I propose a contract that possesses several desirable features. First, it is designed to be appealing to a successor servicer, no matter how high the default rate on the pool of loans. Second, it jointly incentives the successor servicer to properly service loans and generously compensates him for providing insurance against abnormally high ratios of default claims rejected over default claims submitted.

The ratio of default claims rejected over the pool balance can be decomposed into two components: 1) the ratio of total default claims over the pool balance, and 2) the ratio of default claims rejected over total default claims. My near-arbitrage methodology already allows 1) to reach 100%. My near-arbitrage methodology draws 2) from a distribution that is estimated from historical data. The distribution has no upper bound, thus it allows for draws that have no historical precedents.

Figure 1.4 shows the rate of rejection of default claims in SLM's aggregate portfolio of securitized FFEL loans.[35] I fit a gamma distribution to the data. I compute the maximum likelihood estimators and their 80% confidence intervals. I assume that the successor servicer draws rates of rejection of default claims from a gamma distribution with shape and scale parameter set equal to the upper bound of the 80% confidence intervals estimated. This constitutes a conservative distribution for the successor servicer since the likelihood ratio statistics indicates that the probability that SLM's data was truly drawn from a gamma distribution with parameters this large is less than 1 in 250.

The contract with the successor servicer is designed to punish high rates of rejection of default claims and to reward low rates. I force the servicer to repurchase default claims rejected when they represent more than 2.85% of default claims. The repurchase threshold represents the 99th percentile of the conservative gamma distribution, meaning that only 1% of draws trigger a repurchase.[36] Whenever the fraction of default claims rejected is less than 2.85%, part of the cash flow received from loans is directed into a bonus pool according to the formula:

$$\text{bonus}_t = 2.85\% \cdot \text{default claims}(\$)_t - \text{default claims rejected}(\$)_t. \tag{1.20}$$

[35]I take the sum of default claims rejected ($) and divide it by the sum of all default claims ($) across all pools of loans that collateralize SLABS for which SLM provides disclosure.

[36]I use the pre-crisis data. Using the full sample data would yield a repurchase threshold of 2.70%. Which data to use depends on the question that is asked: since my current objective is to show that the predictions of the frictionless no-arbitrage approach failed during the crisis, I use pre-crisis data. If the objective is to determine near-arbitrage lower bounds to be used by the government for a future asset purchase program, then the full sample should be used.

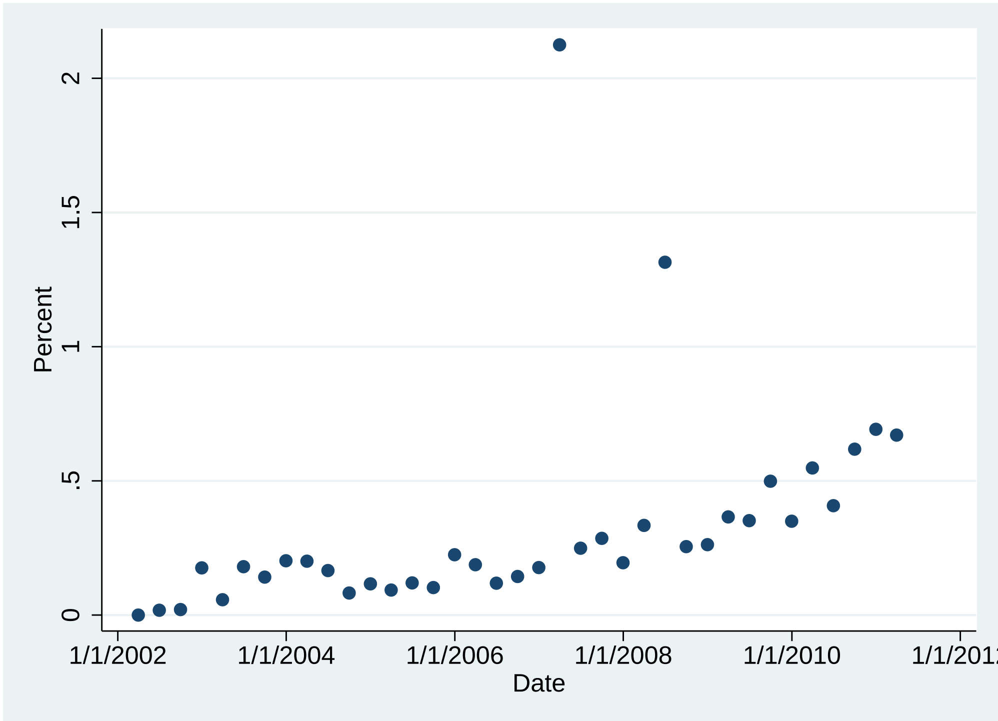


Figure 1.4: Rate of rejection of default claims. This figure plots the time series of the aggregate rate of rejection of default claims, which is computed from (default claims rejected($))/(default claims submitted($)) x 100, using all of SLM's securitized pools.

The formula yields a negative bonus when the fraction of default claims rejected exceeds 2.85%, which is exactly how I compute the repurchase obligation of the servicer. The successor servicer does not need capital in order to cushion against repurchases: the SLABS trust allows the successor servicer to carry a liability that the servicer should be able to repay by drawing lower rejection rates in future periods. The bonus, when positive, is not paid immediately to the servicer, but it accumulates in a bonus pool where it accrues interest at the risk-free rate. In other words,

$$\text{bonus pool}_{t+1} = \text{bonus pool}_t \cdot (1 + r_t) + \text{bonus}_t. \tag{1.21}$$

The bonus is paid at the termination of the pool of loans.

In my simulations, I draw the rejection rate on default claims from the conservative gamma distribution and use the bonus/repurchase scheme to compute cash flows to/from the successor servicer. The successor servicer obtains a negative bonus in less than 1% of simulations. Furthermore, since the servicing fee paid to the successor servicer produces a profit of $20 per loan annually, total profits are positive more than 99.9% of the time.

Table 1.5: Gamma distribution fitted to SLM's rejection rate data

Panel A: 80% C.I. of the ML Estimators(MLE)

	80% C.I. of MLE	
	κ (shape)	θ (scale)
Pre-crisis	[0.461, 0.838]	[0.281, 0.674]
Full sample	[0.618, 1.034]	[0.287, 0.577]

Panel B: Testing joint hypothesis on parameters of gamma distribution

	$H_0 : \kappa_0 = ub_{0.80}$ and $\theta_0 = ub_{0.80}$	
	Likelihood ratio statistics	p-value
Pre-crisis	11.180	0.0037
Full sample	13.351	0.0013

Panel A of this table reports the 80% confidence interval of the Maximum Likelihood Estimators(MLE) for the pre-crisis sample and full sample. The probability density function of the gamma distributions is given by: $f(y) = \frac{y^{\kappa-1}exp(-y/\kappa)}{\Gamma(\kappa)\theta^{\kappa}}$. Panel B of this table reports likelihood ratio statistics and p-value for the joint test that rates of rejection of default claims for SLM were drawn from a gamma distribution with shape and scale parameters set equal to the upper bound of the 80% confidence interval of the Maximum Likelihood Estimators.

This is despite worst case assumptions that the recovery on loans whose default claim was rejected is 0% and the cure rate of default claims rejected is 0%.[37] Thus, my analysis suggests that the contract would have no difficulty attracting a servicer willing to succeed SLM and SLABS holder can be almost certain that the successor servicer will be able to make good on its promise to repurchase default claims rejected whenever they exceed 2.85% of default claims.

The vast majority of simulated scenarios require write-downs due to default claims rejected and payment to the bonus pool of the servicer that are equivalent to write-downs of 2.85% of default claims in every quarter. Combined with the worst case scenario of cumulative borrower's default rate of 100%, the write-downs due to default claims rejected and bonus payment are significantly greater than the 0.05% of the pool balance

[37]The assumption is extremely conservative for several reasons. First, between 2005 and 2007, SLM was able to cure between 50% and 73% of default claims rejected and obtained close to full reimbursement on them. Second, the recovery rate of guarantors on regular default claims are roughly 40%. For example, USA funds, a guarantor, had recovery rates ranging between 38% and 45% from 2006 to 2008. So, the expected recovery rate of a servicer on uncured default claims rejected should be close to 40%. Finally, I abstract from an endogenous adjustment in servicing effort and/or technology: the contract produces incentives for the successor servicer to adjust is effort and/or technology in response to lower recovery rate in order to maximize profit. If the successor servicer truly faced recovery rate of 0% on uncured default claims rejected, the servicer would likely make adjustments to his effort and/or technology to reduce the fraction of default claims that get rejected and increase his profit.

assumed by condition C.4. Therefore, the near-arbitrage lower bounds obtained by simulations are robust to a severe relaxation of condition C.4.

1.4.4 EXAMPLES OF SLABS-TREASURY NEAR-ARBITRAGE

If we only maintain assumptions C.1 (no government default on loan guarantees), C.2 (the interest rate ordering $r_t^{FCP} \geq r_t$ and $r_t^{LIBOR} \geq r_t$, for all t), and C.3.B (no historically unprecedented inflation paths), what is the largest amount of arbitrageur's debt that can be repaid with near certainty from the cash flow on SLABS? What are the near-arbitrage lower bounds now that we allow the bankruptcy of SLM and allow servicing fees to be as high as the upper bound derived in Section 1.4.3? I answer those questions and present the size of the gap between price observed during the crisis and near-arbitrage lower bounds for the SLABS collateralized by two pools of loans.

SLM 2003-3 A4: CASE WITH A SINGLE SENIOR TRANCHE OUTSTANDING

From December 15th, 2005 onward, SLM 2003-3 A4 was the only senior tranche outstanding on a pool of FFEL loans that initially collateralized four senior tranches of SLABS.[38] Thus, the intuition of Section 1.3 applies to SLM 2003-3 A4 throughout the crisis. To build on the intuition of Section 1.3, I first abstract from the overcollateralization ratio in excess of 1 and examine the near-arbitrage lower bound under a counterfactual overcollateralization ratio of 1. Table 1.6 reports a counterfactual near-arbitrage lower bound, $\underline{P_t}|_{\phi_t/\rho_t=1}$, of \$93.00 for SLM 2003-3 A4. All the FFEL loans that collateralize SLM 2003-3 are at least 98% guaranteed. How does the near-arbitrage bound of \$93.00 relates to the no-arbitrage benchmark of Section 1.3? Proposition 1 states that if the difference between interest rate on FFEL loans and interest on 3-month Treasuries exceeds servicing and administrative fees in every period, then the SLABS-Treasury trade initiated at $P_t \leq \$98.00$ is always profitable. The near-arbitrage lower bound is pushed down for several reasons.

The lowest break-even price on the SLABS-Treasury trade is obtained when two events occur the day after the trade is initiated: i) all borrowers stop making payments on their loans, and ii) SLM goes bankrupt. In addition to the 2% write-down due to default, there are two other sources of write-downs: i) with 100% default, 2.85% of the principal is paid to the successor servicer as bonus to properly service loans under the new servicing contract and ii) 0.50% of the principal is paid to the servicer for filing default claims. If,

[38] I only discusses the case of SLM 2003-3-A4 in the text of this paper, but tables report data on SLM 2003-8 A4 as well. The two SLABS are nearly identical.

Table 1.6: Near-arbitrage among SLABS

Pool	Tranche	Rules of dist. post event of reprioritization	Loan guarantee (min.)	$\underline{P_t}\|_{\phi_t/\rho_t=1}$ (min.)	Overcoll. ratio (min.)	$\underline{P_t}$ (min.)	P_t (min.)	Near-arb. ($P_t < \underline{P_t}$)	$slack_t$ (min.)	Agg. pr. tranche (max.)
2003-3	A4	pro rata (act.)	98%	93	1.23	100	91.56	Y	0.14	140
2003-3	B					60				38
2003-8	A4	pro rata (act.)	98%	93	1.23	100	91.03	Y	0.14	267
2003-8	B					60				61
2007-2	A1	sequential (act.)	97.8%	100	1.04	100	97.81	Y	3.14	693
		pro rata (cfact.)	97.8%	92	1.04	97	N/A			
2007-2	A2	sequential (act.)	97.8%	100	1.04	100	88.44	Y	0.40	1349
		pro rata (cfact.)	97.8%	92	1.04	96	N/A			
2007-2	A3	sequential (act.)	97.8%	100	1.04	100	77.44	Y	0.15	446
		pro rata (cfact.)	97.8%	92	1.04	92	N/A			
2007-2	A4	sequential (act.)	97.8%	61	1.04	82	69.50	Y	N/A	486
		pro rata (cfact.)	97.8%	92	1.04	93	N/A			

This table reports the minimum near-arbitrage lower bound for various SLABS during the crisis, $\underline{P_t}$, and their minimum observed price, P_t. SLABS that meet the condition $P_t < \underline{P_t}$ presented near-arbitrage opportunities during the crisis. The minimum (balance-weigthed average) loan guarantee among the FFEL loans that collateralize a SLABS observed during the crisis is reported. The near-arbitrage lower bound at a counterfactual level of overcollateralization, $\phi_t/\rho_t = 1$, is reported. Cases with $\underline{P_t}|_{\phi_t/\rho_t=1} < 97.8$ are observed because all simplifying assumptions of section 1.3 are abandoned and servicing fees and write-downs due to default claims rejected are set equal to their upper bounds. The minimum overcollateralization ratio during the crisis and the type of rules of distribution that follows an event of reprioritization are reported; they are both important determinants of the near-arbitrage lower bound, $\underline{P_t}$. The near-arbitrage lower bounds under both actual, abbreviated to *act.*, and couterfactual, abbreviated *cfact.*, rules of distribution post event of reprioritization are reported. The *slack* variable indicates how many additional dollars of SLABS can be repaid from the cash flow on the pool after a given SLABS is paid down. The slack variable is reported in the table and the formula to compute it is provided in equation (1.22). Finally, maximum aggregate principal of the tranche (ρ_t^j) during the crisis, which produces the minimum slack value, is reported. Prices are per $100 principal of SLABS and aggregate principal is reported in millions of dollars. Near-arbitrage lower bounds are rounded down to the nearest dollar.

Table 1.7: Near-arbitrage among SLABS - Larger sample

Pool	Tranche	Rules of dist. post event of reprioritization	Loan guarantee (min.)	Overcollat. ratio (min.)	Loan type	Interest rate margin (m)	$\underline{P_t}$ (min.)	P_t (min.)	Near-arb. ($\underline{P_t} > P_t$)
2003-3	A4	pro rata	98%	1.230	Non-consolidation	1.74%	100	91.56	Y
2003-6	A4	pro rata	98%	1.276	Non-consolidation	1.74%	100	91.28	Y
2003-8	A4	pro rata	98%	1.227	Non-consolidation	1.74%	100	91.03	Y
2003-9	A4	pro rata	98%	1.221	Non-consolidation	1.74%	100	86.78	Y
2004-4	A4	pro rata	98%	1.175	Non-consolidation	1.74%	100	90.31	Y
2004-6	A4	pro rata	98%	1.175	Non-consolidation	1.74%	100	97.78	Y
2004-6	A5	pro rata	98%	1.175	Non-consolidation	1.74%	100	86.91	Y
2004-7	A4	pro rata	98%	1.153	Non-consolidation	1.74%	100	96.50	Y
2004-7	A5	pro rata	98%	1.153	Non-consolidation	1.74%	100	85.44	Y
2004-9	A4	pro rata	98%	1.134	Non-consolidation	1.74%	100	95.22	Y
2004-9	A5	pro rata	98%	1.134	Non-consolidation	1.74%	100	83.47	Y
2005-1	A2	pro rata	98%	1.130	Non-consolidation	1.74%	100	88.72	Y
2005-2	A4	pro rata	98%	1.126	Non-consolidation	1.74%	100	94.75	Y
2005-2	A5	pro rata	98%	1.126	Non-consolidation	1.74%	100	81.97	Y
2005-10	A2	pro rata	98%	1.076	Non-consolidation	1.74%	100	99.72	Y
2005-10	A3	pro rata	98%	1.076	Non-consolidation	1.74%	100	96.66	Y
2005-10	A4	pro rata	98%	1.076	Non-consolidation	1.74%	100	86.31	Y
2005-10	A5	pro rata	98%	1.076	Non-consolidation	1.74%	99.7	87.44	Y
2006-1	A2	pro rata	98%	1.072	Non-consolidation	1.74%	99.9	99.69	N
2006-1	A3	pro rata	98%	1.072	Non-consolidation	1.74%	99.8	97.50	Y
2006-1	A4	pro rata	98%	1.072	Non-consolidation	1.74%	99.7	85.28	Y
2006-1	A5	pro rata	98%	1.072	Non-consolidation	1.74%	98.3	79.63	Y
2006-3	A2	pro rata	98%	1.072	Non-consolidation	1.74%	99.9	99.66	N
2006-3	A3	pro rata	98%	1.072	Non-consolidation	1.74%	99.7	96.34	Y
2006-3	A4	pro rata	98%	1.072	Non-consolidation	1.74%	99.6	85.56	Y
2006-3	A5	pro rata	98%	1.072	Non-consolidation	1.74%	98.0	73.41	Y

This table shows the minimum quoted market price during the crisis, P_t, and minimum near-arbitrage lower bounds during the same period, $\underline{P_t}$, for a larger sample of SLABS with rules of distribution following an event of reprioritization that are pro rata among tranches of senior SLABS. Other variables in the table are as presented in Table 1.6. Near-arbitrage lower bounds are rounded down to the nearest one tenth of a dollar. The bounds are sufficiently low to lead to profitable SLABS-Treasury trade, 100% of the time, over 1,000 simulations with stochastic interest rate paths drawn with the estimated regime-switching and stochastic-volatility model and extremely high levels of default rate leading to cumulative loan default rate of 100% within 5-8 quarters of the initiation of a SLABS-Treasury trade.

Table 1.8: Near-arbitrage among SLABS - Larger sample (*continued*)

Pool	Tranche	Rules of dist. post event of reprioritization	Loan guarantee (min.)	Overcollat. ratio (min.)	Loan type	Interest rate margin (m)	$\underline{P_t}$ (min.)	P_t (min.)	Near-arb. ($\underline{P_t} > P_t$)
2007-2	A1	sequential	97.8%	1.041	Non-consolidation	1.74%	99.9	97.81	Y
2007-2	A2	sequential	97.8%	1.041	Non-consolidation	1.74%	100	88.44	Y
2007-2	A3	sequential	97.8%	1.041	Non-consolidation	1.74%	100	77.44	Y
2007-2	A4	sequential	97.8%	1.041	Non-consolidation	1.74%	82	69.50	Y
2007-3	A1	sequential	97.8%	1.039	Non-consolidation	1.74%	99.9	97.38	Y
2007-3	A2	sequential	97.8%	1.039	Non-consolidation	1.74%	100	88.16	Y
2007-3	A3	sequential	97.8%	1.039	Non-consolidation	1.74%	100	77.50	Y
2007-3	A4	sequential	97.8%	1.039	Non-consolidation	1.74%	81	69.19	Y

This table shows the minimum quoted market price during the crisis, P_t, and minimum near-arbitrage lower bounds during the same period, $\underline{P_t}$, for a larger sample of SLABS with rules of distribution following an event of reprioritization that are sequential among tranches of senior SLABS. Other variables in the table are as presented in Table 1.6.

for $P_t = \$98$, arbitrageur's spreads would have been positive in every period without the additional write-downs, then the no-arbitrage lower bound would become $94.65 with the additional write-downs. However, the arbitrageur's spread is not positive in every period: in the worst case scenario, the sum of the upfront servicing fee charged by the successor servicer, the fee for a delinquent loan, and the fee for default filing, the arbitrageur's spread for $P_t = \$98$ is -0.44% in the first year.[39] In the simulation model, when a loan defaults, the SLABS trust continues to pay servicing fees until a payment is received from the guarantor. There is a delay of 60 days before the submission of a default claim and its payment, and no interest accrues on the loan during this delay. Under the worst case scenario of default, the delay in payment requires a downward adjustment of 0.30%.

SLM 2003-3 A4 enjoyed a large overcollateralization ratio during the crisis, so its actual near-arbitrage lower bound was $100.18. The simulations are performed with the difference between the LIBOR rate and the 3-month T-bill rate set to zero, the smallest difference that does not violate condition C.2. Thus, although SLM 2003-3 A4 holders are promised an interest rate of LIBOR plus 0.22%, in the simulations receiving what they were promised means receiving the 3-month T-bill rate, plus 0.22%. The overcollateralization ratio is so large that SLM 2003-3 A4 holders always receive what they were promised, which leads to a near-arbitrage lower bound of $100.18. In the case of SLM 2003-3 A4, an overcollateralization ratio of 1.09 would be sufficient to obtain a near-arbitrage bound of $100.18. An overcollateralization ratio above 1.09 represents insurance against violations of conditions C.1, C.2 or C.3.B.

Any fully informed investors would view SLM 2003-3 A4 as an asset that, if purchased below its near-arbitrage lower bound and held to maturity, is almost certain to outperform a roll-over investment in 3-month Treasuries. Yet, during the crisis of 2007-2009, SLM 2003-3 A4 was transacted for prices ranging between $92.50 and $96.06 on two occasions by insurance companies.[40] Since insurance companies only represent a fraction of market participants on the SLABS market, it is very likely that other transactions occurred at a significant discount to the near-arbitrage lower bound of $100.18. Quoted prices obtained from the Bloomberg system are below $98.00 from late August 2008 to July of 2009. Quoted prices reach a minimum of $91.60 in late December of 2008. Figure 1.5 combines near-arbitrage lower bound with quoted and transaction prices and shows the magnitude

[39] All loans are delinquent, so a servicing fee of $10 + $70 + $23 = $103 per borrower is charged when the average principal per borrower is $4,720, producing a servicing fee of 2.18% that is charged on loans that accrue interest at the 3-month Treasury rate plus a spread that can be as low as 1.74%. The simulation model assumes the minimum spread value of 1.74% for all loans.

[40] I collected data provided by the National Association of Insurance Commissioners on a Bloomberg terminal.

of the underpricings observed during the crisis.

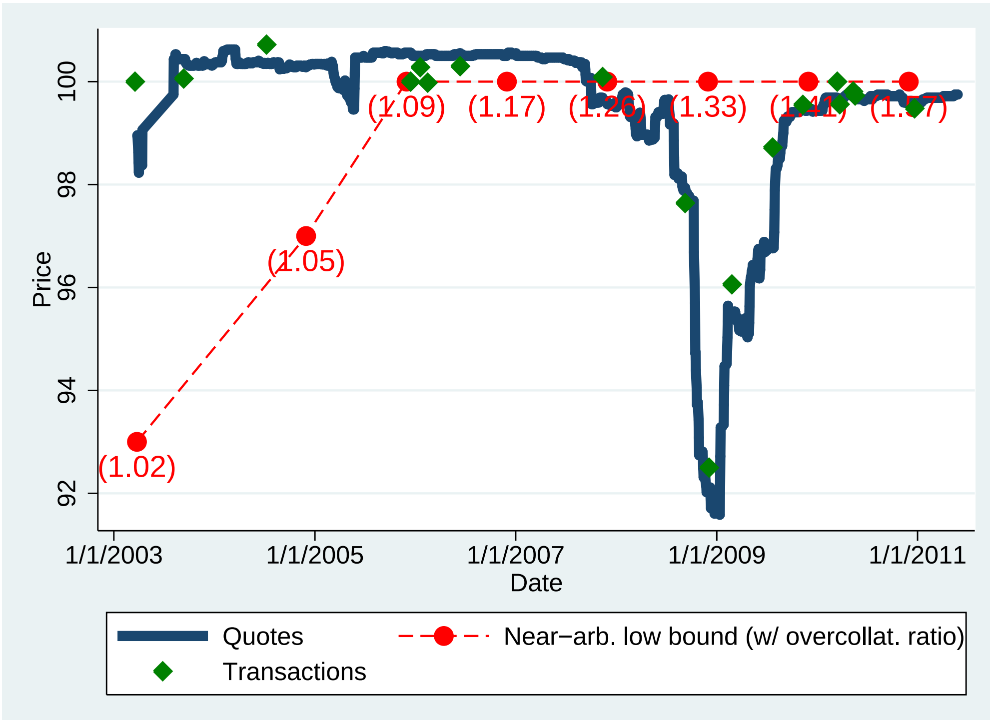


Figure 1.5: Near-arbitrage lower bounds and prices on a SLABS. This figure plots the quoted prices and near-arbitrage lower bounds for SLM 2003-3 A4. Corresponding overcollateralization ratio is presented for each point. The prices of seventeen transactions are added to the figure to show that transactions at significant discount to the near-arbitrage lower bounds occurred.

SLM 2007-2: CASE WITH SEVERAL SENIOR TRANCHES OF SLABS OUTSTANDING

In this subsection, I analyze near-arbitrage lower bounds for the four tranches of senior SLABS (A1, A2, A3 and A4) collateralized by the pool SLM 2007-2.[41]

Table 1.6 reports four types of near-arbitrage lower bounds for SLABS collateralized by the pool SLM 2007-2. The first type of near-arbitrage lower bounds, which is easiest to relate to the benchmark no-arbitrage lower bound of Section 1.3, $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1}$, assumes a counterfactual overcollateralization ratio of 1 and counterfactual pro rata rules of distribution of the cash flow from the pool among senior SLABS following an event of reprioritization. In that case, all tranches have near-arbitrage lower bound of \$92 after rounding down to the nearest dollar. The downward adjustment from $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1} = \$97.8$

[41] There is also a subordinate SLABS or a B tranche. The principal and spread over LIBOR paid on the subordinate SLABS are inputted to compute the near-arbitrage lower bounds on senior SLABS. However, I do not report on the subordinate tranche in Table 1.6.

to $\underline{P_t}|_{\phi_t/\rho_t=1} = \$92$ results from abandoning all simplifying assumption from Section 1.3 and setting servicing fees and write-downs due to default claims rejected equal to their upper bounds. The second type of near-arbitrage lower bounds, uses the actual overcollateralization ratio of 1.04, but maintains the counterfactual assumption of pro rata rules of distribution of the cash flow from the pool among senior SLABS following an event of reprioritization. The overcollateralization increases the near-arbitrage lower bounds on the A1 and A2 tranche by \$5 and \$4 respectively. There is a small increase in the near-arbitrage lower bound on the A3 tranche, but it is too small to show when results are rounded down to the nearest dollar. The overcollateralization increases the near-arbitrage lower bounds on the A4 tranche by \$1.

Empirically, the cash flow from pool SLM 2007-2 is distributed sequentially among senior SLABS after an event of reprioritization. Abandoning the counterfactual assumption of pro rata rules of distribution and using sequential rules benefits tranches A1 to A3, but harms tranche A4. With rules of distribution that continue to be sequential after an event of reprioritization, the near-arbitrage lower bounds on tranche A1 to A3, whether using a counterfactual level of overcollateralization ratio of 1 or the actual overcollateralization ratio of 1.04, are \$100. The near-arbitrage lower bound on the A4 tranche is \$82 under sequential rules, instead of \$93 under pro rata rules. The A4 tranche benefits significantly from the overcollateralization under sequential rules: its near-arbitrage lower bound increases from \$61 to \$82 when the overcollateralization ratio increases from 1 to 1.04.

Table 1.6 reports a variable that quantifies how safe SLABS can be beyond a near-arbitrage lower bound of \$100. The slack variable is given by:

$$slack_t^j = \frac{\sum_{\text{all } jj} \rho_t^{jj} \cdot \underline{P_t}}{\sum_{jj \leq j} \rho_t^{jj} \cdot 100} - 1, \tag{1.22}$$

where $j = 1$ is assigned to the A1 tranche, $j = 2$ is assigned to the A2 tranche, etc. The slack variable indicates how many additional dollars of SLABS can be repaid from the cash flow on the pool after a given SLABS is paid down. For example, the slack of 0.40 on tranche A2 of pool 2007-2 means that for every dollar of A2 SLABS paid down by the pool, an additional \$0.40 of SLABS can be paid down from the cash flow on the pool afterward. This level of slack roughly means that the SLABS-Treasury near-arbitrage initiated with 2007-2 A2 when $P_t = 100$ would be profitable despite a 100% default rate, default by the government on its loan guarantee and recovery on the loans of 72% instead of the 97.8% obtained without default by the government on its loan guarantee.

Finally, Table 1.9 reports data on the persistence of near-arbitrage opportunities among

SLABS. For the majority of SLABS, near-arbitrage underpricings of 2% or more first appear in August 2008, at the latest, and disappear in July 2009, at the earliest. However, there are exceptions, such as the A1 tranche of 2007-2, which had an extremely large level of slack and was expected to be repaid within a 2-year horizon from the crisis, that have shorter period of significant underpricings.

Table 1.9: Near-arbitrage persistence

Pool	Tranche	Distribution post event of reprioritization	Overcollat. ratio	$\underline{P_t}$	$P_t < 98$ Begin	End
2003-3	A4	pro rata	1.23	100	Aug-08	Jul-09
2003-8	A4	pro rata	1.23	100	Jul-08	Jul-09
2007-2	A1	sequential	1.04	100	Nov-08	Jan-09
2007-2	A2	sequential	1.04	100	Oct-07	Aug-09
2007-2	A3	sequential	1.04	100	Sep-07	Apr-12

This table reports the first date at which the price of SLABS dropped below $98 and the last date at which the price of SLABS remained below $98 for a sample of SLABS with a near-arbitrage lower bound of $100 from the fall of 2007 to the fall of 2009. The overcollateralization ratio and the type of rules of distribution that follows an event of reprioritization are important determinants of the near-arbitrage lower bounds and they are also reported. The minimum overcollateralization ratio during the crisis is reported.

1.5 NORMATIVE IMPLICATIONS

This section examines the implications of near-arbitrages in SLABS for a government-run asset purchase program and presents a cost-saving reform of the FFEL loan program that relies on near-arbitrage lower bounds on SLABS.

1.5.1 CENTRAL BANKS' EXCEPTIONAL MEASURES OF LIQUIDITY PROVISION

The Term Asset-Backed Securities Loan Facility (TALF) was announced on November 25, 2008 and began operation in March 2009 (Ashcraft, Malz, and Pozsar (2012). The facility lent on a non-recourse basis to investors that collateralized their borrowing with ABS that had been pre-approved as TALF-eligible. Issuers of ABS would apply for TALF eligibility and obtain it if their ABS met a list of eligibility criteria, notably a high proportion of loans collateralizing the ABS originated no earlier than 2007. The goal of the program was to lower originators' cost of funding recently disbursed consumer and small business loans and to stimulate the origination of such loans.

The near-arbitrage gaps among SLABS that began in September 2007 and became more important in August 2008 were an early signal that the cost of funding the origination of consumer loans was becoming excessive. For policy-makers looking for an early signal that there might be a need for an exceptional measure of liquidity provision, the near-arbitrage gaps among SLABS can provide such a signal. The origination of FFEL loan stopped in June 2010 after the Department of Education decided to abandon the government guaranteed loan program and switched entirely to loans directly funded by the government. The outstanding balance of FFEL has since been shrinking. While ABS collateralized by FFEL loans are disappearing, their high correlation with other arbitrages, such as the TIPS-Treasury arbitrage documented by Fleckenstein, Longstaff, and Lustig (2014), means that these other arbitrages could become signals for exceptional intervention.

Also, should a future crisis occur before the disappearance of SLABS or should the US government re-instate a guaranteed student loan program in the future, then the near-arbitrage gaps could guide the guide the setting of terms (haircut, interest rate) at a facility that would complement a TALF-like facility and lend against ABS that are more seasoned. If near-arbitrage lower bounds on SLABS indicate that financial markets demand excessively large haircuts and interest rate given the price of SLABS (e.g. a haircut that is greater than necessary to make a loan collateralized by a SLABS nearly-riskless given the price of the SLABS), this can be a starting point to analyze whether a central bank might want to extend ABS-collateralized loan to market participants on more favorable terms than those offered by other market participants. By lending against ABS that are more seasoned, the injection of capital would not necessarily translate into the stimulation of the origination of loans that collateralized the ABS, but helping financial market participants releverage their informational advantage into whichever asset class they find most attractive would likely trickle down to some form of increase in real investment or lending that would lead to increases in consumption.

1.5.2 ASSET PURCHASE PROGRAM

During the crisis, the private arbitraging capital was spread too thinly over several arbitrages to eliminate them all and this provided an opportunity for the US federal government to take advantage of some arbitrages. How should one analyze the situation where the US government incurs a loss on the SLABS-Treasury near-arbitrage because of default on the loan guarantee?

From the point of view of the US government, a loss on the SLABS-Treasury near-arbitrage caused by a default on the guarantee on FFEL loans means that every dollar of loss on the near-arbitrage implies a dollar saved by the government in payment of guarantee. Is this kind of loss relevant when analyzing whether a trade constitute an arbitrage for the government? The short answer is yes, but not as much as for a non-governmental arbitrageur. Suppose that the government and a hypothetical non-governmental arbitrageur that can short Treasuries frictionlessly execute the same trade. If the non-governmental arbitrageur loses $10 on the trade, then the government would look back on the trade and regret initiating it because a valuable option to default on creditors was forgiven. Ex-post, and assuming rational default, the option to default is worth $10 minus the cost of default and this difference is positive.

If the government defaults on 3-month Treasuries at the same time that it defaults on its guarantee on FFEL loans, then the hypothetical frictionless arbitrageur does not lose money on the SLABS-Treasury arbitrage and neither does the government. If the government defaults on its guarantee on FFEL loans, but not on 3-month Treasuries, then the frictionless arbitrageur could lose money on the trade. Every dollar of shortfall to the arbitrageur caused by the default on the guarantee means a dollar saved for the government. If the government is the arbitrageur, then default on the guarantee does not save any money. If the government defaults on the guarantee on FFEL loans held by non-governmental investors, such as the non-governmental investors in ABS collateralized by FFEL loans, then there is a cost to default, in the form of higher future borrowing costs on all forms of government debt. Rational default occurs when the money saved by defaulting on the guarantee on FFEL loans exceeds the present value of the cost of default. It is unlikely that the government would find itself in a state of the world where default on the guarantee is rational. And even less likely that the government would find itself in a state of the world where default on the guarantee on FFEL loans, without default on short-term Treasuries would be rational. It is even unclear whether the US government would have the freedom to choose to default on the guarantee on FFEL loans without default on other forms of debt. But, assuming the worst case, assuming that it is possible to default on the guarantee on FFEL loans without defaulting on 3-month Treasuries, then only in those rare state of the world where it is rational to choose both of these actions at the same time, might the government look at the loss on the SLABS-Treasury trade negatively and regret, ex-post, having initiated it.

Based on 1) the low probability that the US government defaults on any kind of debt or guarantee, 2) the even lower probability that the government defaults on the guarantee on FFEL loans without defaulting on 3-month Treasuries, 3) the cost of a loss on the SLABS-Treasury near-arbitrage should the government default on the guarantee on FFEL loans and not on 3-month Treasuries is less than for a hypothetical non-governmental frictionless arbitrageur, 4) the low probability that all borrowers default on their loan as assumed to perform my cash flow simulations, I conclude that purchasing SLABS when they sell at or below their near-arbitrage lower bounds provides generous compensation for the small risk taken with taxpayers' money.

1.5.3 FIRE-SALE INSURANCE

The merit of the proposal found in Gorton (2010)[42] that the government should offer fire sale insurance on some ABS can be analyzed for the special case of SLABS. More precisely, what would happen if the government initiated a program today, or at any date in the future outside of crisis, that would allow investors to exchange their SLABS for their near-arbitrage lower bound price at any time in the future? My analysis suggests that investors would very rarely, and possibly never, exercise this option. This prediction relies on my findings that once SLABS sell for less than their near-arbitrage lower bound, a SLABS that is held to maturity is almost certain to outperform a roll-over investment in 3-month Treasuries. This should make SLABS attractive for money market funds that roll-over trillions of dollars in Treasuries and similar assets. Money market funds, (MMF), did not prevent the price of SLABS from dropping below their near-arbitrage lower bounds during the crisis for two reasons. The first and straightforward reason is that MMF cannot invest in assets with maturity of more than 397 days. A second and deeper reason is that MMF are given a clear mandate by their investors to buy assets that present virtually no risk of declining in value over their holding period, which is not the case for a SLABS, even when it trades below its near-arbitrage lower bound. However, if the government offers a guarantee on SLABS, eliminating the risk that a SLABS purchased at the near-arbitrage lower bound could decline further in value, then MMF would provide support for the price of SLABS at the near-arbitrage lower bound.

My near-arbitrage methodology provides a trigger point for the initiation of the SLABS-Treasury arbitrage. The trigger point depends on a set of state variables that includes the interest rate, the average principal per borrower and the overcollateralization ratio. Near-arbitrage lower bounds can decrease over time has the state variables change. For the government to face minimal risk, it would have to be allowed to reset the strike price on the put it offers on SLABS periodically. If the government resets the strike price

[42] See p.55 where Gorton prescribes a policy package in which "Senior tranches of securitizations of approved asset classes should be insured by the government."

for puts every 90 days, and announces the strike price for the put one day before it is reset, then 1) should the put option be exercised, the government is exposed to a minimal risk of paying a price superior to the near-arbitrage lower bound because the state variables move relatively little over a 90 days period, and 2) MMF do not face any price risk inside 90-day windows and they would support the price of SLABS above the strike price.

What would be the benefits for the government of offering fire-sale insurance? In the case of SLABS, the government could make smaller special allowance payment on FFEL loans without causing a reduction in the supply of FFEL loans. Institutional investors demand a higher interest rate on SLABS if the security can decrease in price below its near-arbitrage bounds during financial crisis, a time when they are more likely to need to trade their asset for cash in order to shrink their balance sheet or to satisfy investors' withdrawals. Once the fire-sale risk is removed, once a SLABS is guaranteed to never fall below its near-arbitrage lower bound, investor do not demand as high an interest rate. Originators of FFEL loans could sell securities collateralized by pool of FFEL loans that promise a lower interest rate and obtain the same proceeds from the sale, because these securities would no longer bear the fire-sale risk. As of the end of 2013, there were still more than $250 billion dollar in FFEL loans outstanding. Therefore, small reductions in supplemental interest payments, on the order of 0.10%, would translate into savings of $250 million, just in the first year following the reform.[43] To sum up, the government could achieve the same level of financial support for students at a lower cost, thanks to fire-sale insurance.

1.6 CONCLUSION

In this paper, I show that ABS collateralized by government guaranteed student loans that benefit from significant overcollateralization are nearly riskless variable-rate bonds. The frictionless no-arbitrage framework predicts a market price of $100 for a riskless variable-rate bond with a principal of $100. I quantify underpricings on some SLABS that exceeded $20 per $100 principal during the crisis.

While other mispricings of a similar magnitude have been documented since I began work on this project, most notably by Fleckenstein, Longstaff, and Lustig (2014), several of the normative implications of my findings are novel. To my knowledge, this is the first paper to document large underpricings in the ABS market that would present nearly certain opportunities for profit for the US government. Other arbitrages have limited

[43] Assumes 100% participation rate in a voluntary loan swapping program that involves the exchange of a FFEL loan for a loan with fire-sale insurance that receives smaller supplemental interest payments.

implications for US central bank's liquidity provision facility and temporary programs of liquidity provision. SLABS can be used as a canary in the coal mine, meaning that near-arbitrages signal an extraordinary need for liquidity provision. To nearly eliminate risk on a non-recourse loan collateralized by SLABS, the Fed could set cash-down requirements to the greater of 0% and the premium between market prices and near-arbitrage lower bounds.

The are also normative implications that arise from the co-occurrence of the underpricing of SLABS relative to Treasuries and those of TIPS relative to Treasuries. Since FFEL loans stopped being originated on June 30, 2010, SLABS collateralized by FFEL loans will gradually amortize over the next twenty-five years or so. Since the underpricing of SLABS is correlated with other types of large mispricings, the latter could become the signal for extraordinary need for liquidity provision in a future crisis.

The US government could save hundreds of millions of dollar by providing fire-sale insurance on SLABS collateralized by FFEL loans. FFEL loans with fire-sale insurance on SLABS could be an alternative to the current policy of direct origination of all federal loans by the government. Through the adoption of the current origination policy, the government has given up a fiscal hedging option. The US government would not give up its fiscal hedging option by providing fire-sale insurance on SLABS: defaulting jointly on the loan guarantee and the fire-sale insurance would have similar impact on future borrowing costs as a default on the loan guarantee alone. It would be interesting to see whether a FFEL program with fire-sale insurance on SLABS could compete with the cost-saving of the switch from FFEL loans to direct loans estimated by Lucas and Moore (2010).

The release of data on haircuts for finer categories of asset classes than is currently published in Gorton and Metrick (2009), Copeland, Martin, and Walker (2014) and Krishnamurthy, Nagel, and Orlov (2014), would allow a detailed empirical analysis of the link between the price of SLABS and the time variation of haircuts. This could help disentangle whether time-varying haircuts or another mechanism, such as shocks to the balance-sheet of financial intermediaries combined with differential capital requirements, is the more likely cause of the dynamic of underpricings of SLABS relative to Treasuries during the crisis. The same data could help us better understand the drastically different price dynamic between SLABS and variable-rate ABS guaranteed by the Small Business Administration. More research is needed in the area.

1.7 APPENDIX

1.7.1 SLABS THAT SATISFY ALL SELECTION CRITERIA

Table 1.10: SLABS that satisfy all selection criteria. This table lists the SLABS trusts that satisfy all selection criteria used to derive benchmark no-arbitrage lower bounds in Section 1.3. Selection criterion SC.1 requires that the rules of distribution of the cash flows from the pool among various claimholders be as presented in Figure 1.2. SC.2 requires that the SLABS trust receives offsetting payments from the securitizer for reductions in interest rate or principal offered to borrowers, which are also called borrower's incentive programs. SC.3 requires that the interest rate spread over LIBOR promised on a SLABS is positive, $s \geq 0$. SC.4 requires that none of the SLABS collateralized by a pool are auction-rate securities, which are denoted by ARS in the table.

Pool	Minimum loan guarantee	Rules of distrib. (SC.1)	Borrowers' incentive programs (SC.2)	Positive interest rate spread (SC.3)	ARS (SC.4)	Loan type
SLM 2003-3	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2003-6	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2003-8	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2003-9	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2004-4	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2004-6	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2004-7	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2004-9	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2005-1	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2005-2	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2005-10	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2006-1	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2006-3	98%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2007-2	97%	Fig. 1.2	Offset	Yes	No	Non-consol.
SLM 2007-3	97%	Fig. 1.2	Offset	Yes	No	Non-consol.

Table 1.11: SLABS that satisfy all selection criteria, except for a stepdown date. This table lists the SLABS trusts that satisfy selection criteria SC.2 to SC.4 used to derive benchmark no-arbitrage lower bounds in Section 1.3. The selection criteria are as defined in Table 1.10. Selection criterion SC.1 is not satisfied because of the presence of a stepdown date after which distribution of principal to subordinate SLABS become pro rata with the distribution of principal to senior SLABS, under some conditions. For distributions to be pro rata among subordinate SLABS and senior SLABS, the total overcollateralization ratio from the stepdown date onward must be non-decreasing and there must be no event of reprioritization. Otherwise, senior SLABS receive a disproportionate fraction of the distribution of principal or all of it.

Pool	Minimum loan guarantee	Rules of distribution (SC.1)	Stepdown date	Borrowers' incentive programs (SC.2)	Positive interest rate spread (SC.3)	ARS (SC.4)	Loan type
SLM 2005-3	98%	Fig. 1.2, but stepdown	Apr-11	Offset	Yes	No	Consolidation
SLM 2005-4	98%	Fig. 1.2, but stepdown	Jul-11	Offset	Yes	No	Consolidation
SLM 2006-2	98%	Fig. 1.2, but stepdown	Jan-12	Offset	Yes	No	Consolidation
SLM 2006-8	97%	Fig. 1.2, but stepdown	Jul-12	Offset	Yes	No	Consolidation
SLM 2006-9	97%	Fig. 1.2, but stepdown	Jan-13	Offset	Yes	No	Consolidation

1.7.2 PROOF OF PROPOSITION 1

I prove Proposition 1 for the special case where an arbitrageur buys the aggregate principal of SLABS collateralized by a given pool of FFEL loans. Since distributions among SLABS holders for a given pool are pro rata to their ownership of SLABS in Sections 1.2 and 1.3, the generalization from this special case to the case where a SLABS holder owns a fraction of the SLABS collateralized by a pool is immediate.

Consider an arbitrageur who purchases at time 0 the aggregate principal of SLABS, ρ_0, that is collateralized by a pool of FFEL loans with balance of ϕ_0, where $\phi_0/\rho_0 \geq 1$. The arbitrageur pays $P_0/100 \cdot \rho_0 < 0.97 \cdot \rho_0$ for the SLABS. The arbitrageur shorts Treasuries and raises $\$0.97 \cdot \rho_0$. In other words, the arbitrageur's debt is $\$0.97 \cdot \rho_0$.

Recall that the principal of the SLABS evolves according to:

$$\rho_{t+1} = \rho_t \cdot (1 + (r_t^{LIBOR} + s)) - y_{t+1}, \tag{1.23}$$

subject to the constraint $\rho_{t+1} \geq 0$. The full repayment of the SLABS means that $\rho_t = 0$ for some $t > 0$.

Every sequence of cash flows on a SLABS, $\{y_t\}_{t \geq 1}$, can be categorized into one of two groups:

- Cash flows that repay the SLABS in full;
- Cash flows that do not repay the SLABS in full.

The proof of Proposition 1 is presented in two parts.

Proof of Proposition 1, Part 1: This part shows that the SLABS-Treasury trade produces a strictly positive cash flow for the arbitrageur at time 0 and non-negative cash flows at time $t \geq 1$ when cash flows on the SLABS repay the SLABS in full.

The arbitrageur pockets $0.97 \cdot \rho_0 - P_0/100 \cdot \rho_0 > 0$ at time 0. The arbitrageur's debt evolves over time according to:

$$d_{t+1}^{arb} = d_t^{arb} \cdot (1 + r_t) - y_{t+1}, \tag{1.24}$$

subject to $d_{t+1}^{arb} \geq 0$.

To show that cash flows at time $t \geq 1$ are non-negative, it is sufficient to show that the cash flows from the SLABS, $\{y_t\}_{t \geq 1}$, can repay the arbitrageur's debt in full (i.e. $d_t^{arb} = 0$ holds for some $t > 0$).

I use a proof by induction that shows that ρ_t is an upper bound for d_t^{arb} and that since ρ_t converges to 0, so must d_t^{arb}. At time 1, the following inequality holds:

$$\rho_1 = \rho_0 \cdot (1 + (r_0^{LIBOR} + s)) - y_1 > d_0^{arb} \cdot (1 + r_0) - y_1 = d_1^{arb}. \tag{1.25}$$

Inequality (1.25) results from interest rate condition C.2, $r_t^{LIBOR} \geq r_t$ for all t, selection criterion SC.3, $s \geq 0$, and from $\rho_0 > d_0^{arb}$ (initial conditions $\rho_0 > 0$, $d_0^{arb} > 0$ and $0.97 \cdot \rho_0 = d_0^{arb}$ imply $\rho_0 > d_0^{arb}$).

Assuming that $\rho_t > d_t^{arb} \geq 0$ holds for a given t, the following inequality holds at $t + 1$:

$$\rho_{t+1} = \rho_t \cdot (1 + (r_t^{LIBOR} + s)) - y_{t+1} > d_t^{arb} \cdot (1 + r_t) - y_{t+1} = d_{t+1}^{arb}, \tag{1.26}$$

again, because of $r_t^{LIBOR} \geq r_t$ and $s \geq 0$.

Thus, by induction, starting from $\rho_0 > d_0^{arb} \geq 0$, $\rho_t > d_t^{arb}$ holds for $t \geq 1$, whenever $d_{t-1}^{arb} \geq 0$. Let (*) denote this intermediate result. By the definition of a cash flow that repays a SLABS in full, $\rho_t = 0$ holds for some $t \geq 1$. Therefore, either $d_t^{arb} = 0$ occurs prior to $\rho_t = 0$ or $d_t^{arb} = 0$ occurs at the same time as $\rho_t = 0$ (and the $d_t^{arb} \geq 0$ constraint binds). Any other outcome is in contradiction with (*). ■

Proof of Proposition 1, Part 2: This part shows that the SLABS-Treasury trade produces a strictly positive cash flow for the arbitrageur at time 0 and non-negative cash flows at time $t \geq 1$, even when the cash flow on a SLABS, $\{y_t\}_{t \geq 1}$, is not sufficient to repay the SLABS in full.

Again, the arbitrageur pockets $0.97 \cdot \rho_0 - P_0/100 \cdot \rho_0 > 0$ at time 0. I show that the cash flows from the SLABS, $\{y_t\}_{t \geq 1}$, can repay the arbitrageur's debt in full in order to show that cash flows at time $t \geq 1$ are non-negative.

Let τ denote the time period when $\phi_t = 0$ first occurs. Note that when the cash flow on a SLABS is not sufficient to repay the SLABS in full, an event of reprioritization is triggered for some $t \leq \tau$. To understand why, consider the following: if the cash flow on a SLABS is not sufficient to repay the SLABS in full, then we have a positive principal of SLABS outstanding, $\rho_\tau > 0$, when the pool balance is zero, $\phi_\tau = 0$. Since $\rho_t > \phi_t$ triggers an event of reprioritization, as mentioned in Section 1.2, then an event of reprioritization is triggered at time τ, at the latest. I let $\hat{t}$ denote the first time period when an event of reprioritization is triggered.

From time 0 to $\hat{t} - 1$, since no event of reprioritization has been triggered, the inequality $\phi_t \geq \rho_t$ holds. Starting from $d_0^{arb} = 0.97 \cdot \rho_0$, intermediate result (*) of Part 1 can be strengthened in two steps. First, at time 1, the following inequalities hold:

$$\begin{aligned}
0.97 \cdot \rho_1 &= 0.97 \cdot (\rho_0 \cdot (1 + (r_0^{LIBOR} + s)) - y_1) && (1.27)\\
&\geq 0.97 \cdot (\rho_0 \cdot (1 + r_0) - y_1) && (1.28)\\
&\geq 0.97 \cdot \rho_0 \cdot (1 + r_0) - y_1 && (1.29)\\
&= d_0^{arb} \cdot (1 + r_0) - y_1 \\
&= d_1^{arb}. && (1.30)
\end{aligned}$$

Inequality (1.28) results from interest rate condition C.2, $r_t^{LIBOR} \geq r_t$ for all t, and selection criterion SC.3, $s \geq 0$. Inequality (1.29) results from $-0.97 \cdot y_1 \geq -y_1$ since $y_1 \geq 0$. All equalities either result from the initial condition $0.97 \cdot \rho_0 = d_0^{arb}$ or the laws of motion for ρ_t and d_t^{arb}.

Second, assuming that $0.97 \cdot \rho_t \geq d_t^{arb} \geq 0$ holds for a given t, the following inequalities hold at $t + 1$:

$$\begin{aligned}
0.97 \cdot \rho_{t+1} &= 0.97 \cdot (\rho_t \cdot (1 + (r_t^{LIBOR} + s)) - y_{t+1}) && (1.31)\\
&\geq 0.97 \cdot (\rho_t \cdot (1 + r_t) - y_{t+1}) \\
&\geq 0.97 \cdot \rho_t \cdot (1 + r_t) - y_{t+1} \\
&\geq d_t^{arb} \cdot (1 + r_t) - y_{t+1} \\
&= d_{t+1}^{arb}. && (1.32)
\end{aligned}$$

All equalities and inequalities hold for the same reasons given for the case at $t = 1$, except that initial condition $0.97 \cdot \rho_t \geq d_t^{arb}$ replaces $0.97 \cdot \rho_0 = d_0^{arb}$. Therefore, by induction, we have that $d_t^{arb} \leq 0.97 \cdot \rho_t$ for all t. Let (**) denote this intermediate result.

By definition of an event of reprioritization, the inequality $\phi_t \geq \rho_t$ holds at any time $t \in [0, \hat{t} - 1]$. Combining this inequality with intermediate result (**), the inequality $d_t^{arb} \leq 0.97 \cdot \phi_t$ holds holds at any time $t \in [0, \hat{t} - 1]$.

Recall that the following inequality, which was derived in Section 1.2, holds for the cash flow from the pool, x_t:

$$x_t \geq 0.97 \cdot \big(\phi_{t-1} - \phi_t + \phi_{t-1} \cdot (i_{t-1}^{l} - \omega_t)\big). \tag{1.33}$$

Following an event of reprioritization, the law of motion for the arbitrageur's debt becomes:

$$d_t^{arb} = d_{t-1}^{arb} \cdot (1 + r_{t-1}) - (x_t - f_t \cdot \phi_{t-1}). \tag{1.34}$$

Combining equations (1.33) and (1.34), the following inequalities hold at $t = \hat{t}$:

$$\begin{aligned} d^{arb}_{\hat{t}-1} - d^{arb}_{\hat{t}} &= x_{\hat{t}} - f_{\hat{t}} \cdot \phi_{\hat{t}-1} - d^{arb}_{\hat{t}-1} \cdot r_{\hat{t}-1}, \\ &\geq 0.97 \cdot \left[\phi_{\hat{t}-1} - \phi_{\hat{t}} + \phi_{\hat{t}-1} \cdot (i^{l}_{\hat{t}-1} - \omega_{\hat{t}})\right] - f_{\hat{t}} \cdot \phi_{\hat{t}-1} - d^{arb}_{\hat{t}-1} \cdot r_{t-1}, \\ &\geq 0.97 \cdot \left[\phi_{\hat{t}-1} - \phi_{\hat{t}}\right] + 0.97 \cdot \phi_{\hat{t}-1} \cdot (i^{l}_{\hat{t}-1} - \omega_{\hat{t}}) - f_{\hat{t}} \cdot \phi_{\hat{t}-1} - d^{arb}_{\hat{t}-1} \cdot r_{t-1}, \\ &\geq 0.97 \cdot (\phi_{\hat{t}-1} - \phi_{\hat{t}}). \qquad (1.35) \end{aligned}$$

The following steps show the validity of inequality (1.35). Let $\underline{\varsigma_t^{arb}}$ denote the term $(0.97 \cdot \phi_{\hat{t}-1} \cdot (i^{l}_{\hat{t}-1} - \omega_{\hat{t}}) - f_{\hat{t}} \cdot \phi_{\hat{t}-1} - d^{arb}_{\hat{t}-1} \cdot r_{t-1})$.[44] Using the initial condition, $0.97 \cdot \phi_{\hat{t}-1} \geq d^{arb}_{\hat{t}-1}$, and plugging in values for $i^{l}_{\hat{t}-1}, f_{\hat{t}-1}, \omega_{\hat{t}}$, the following inequalities hold:

$$\begin{aligned} \underline{\varsigma_t^{arb}} &= 0.97 \cdot \phi_{\hat{t}-1} \cdot (i^{l}_{\hat{t}-1} - \omega_{\hat{t}}) - f_{\hat{t}} \cdot \phi_{\hat{t}-1} - d^{arb}_{\hat{t}-1} \cdot r_{\hat{t}-1}, \\ &\geq 0.97 \cdot \phi_{\hat{t}-1} \cdot (i^{l}_{\hat{t}-1} - \omega_{\hat{t}}) - f_{\hat{t}} \cdot \phi_{\hat{t}-1} - 0.97 \cdot \phi_{\hat{t}-1} \cdot r_{\hat{t}-1}, \qquad (1.36) \\ &\geq 0.97 \cdot \phi_{\hat{t}-1} \cdot (r^{FCP}_{\hat{t}-1} + 1.74\% - 0.20\%) - 1.10\% \cdot \phi_{\hat{t}-1} - 0.97 \cdot \phi_{\hat{t}-1} \cdot r_{\hat{t}-1}, \qquad (1.37) \\ &\geq 0.97 \cdot \phi_{\hat{t}-1} \cdot (r_{\hat{t}-1} + 1.74\% - 0.20\%) - 1.10\% \cdot \phi_{\hat{t}-1} - 0.97 \cdot \phi_{\hat{t}-1} \cdot r_{\hat{t}-1}, \qquad (1.38) \\ &= 0.97 \cdot (1.74\% - 0.20\%) \cdot \phi_{\hat{t}-1} - 1.10\% \cdot \phi_{\hat{t}-1}, \qquad (1.39) \\ &\geq 0.39\% \cdot \phi_{\hat{t}-1} \\ &\geq 0. \qquad (1.40) \end{aligned}$$

Inequality 1.36 holds because of the initial condition, $0.97 \cdot \phi_{\hat{t}-1} \geq d^{arb}_{\hat{t}-1}$ (and $d^{arb}_{\hat{t}-1} \geq 0$). Inequality 1.37 results from plugging in $i^{l}_{t} \geq r^{FCP}_{t} + 1.74\%$, along with $f_t = 1.10\%$ and $\omega_t = 0.20\%$.[45] Inequality 1.38 holds because of interest rate condition C.2, which imposes $r^{FCP}_{t} \geq r_t$ for all t.

Therefore, $d^{arb}_{\hat{t}-1} - d^{arb}_{\hat{t}} \geq 0.97 \cdot (\phi_{\hat{t}-1} - \phi_{\hat{t}})$ holds. This can be re-arranged to obtain:

$$0.97 \cdot \phi_{\hat{t}} \geq 0.97 \cdot \phi_{\hat{t}-1} - d^{arb}_{\hat{t}-1} + d^{arb}_{\hat{t}} \geq d^{arb}_{\hat{t}} \qquad (1.41)$$

Note that all the steps followed to obtain $0.97 \cdot \phi_{\hat{t}} \geq d^{arb}_{\hat{t}}$ rely on the initial condition $0.97 \cdot \phi_{\hat{t}-1} \geq d^{arb}_{\hat{t}-1}$ and on conditions on i^{l}_{t}, f_t and ω_t that hold for every t. It follows that by assuming that $0.97 \cdot \phi_t \geq d^{arb}_{t}$ holds for some $t \in [\hat{t}, \tau - 1]$, it is straightforward to show that $0.97 \cdot \phi_{t+1} \geq d^{arb}_{t+1}$ holds in the following time period. Thus, by induction, starting

[44] $\underline{\varsigma_t^{arb}}$ here is a special case of the worst case arbitrageur's spread that will be introduced formally in Section 1.4.

[45] Note that annualized values for $f_{\hat{t}-1}, i^{l}_{\hat{t}-1}, \omega_{\hat{t}}$ are inputted here, since for the purpose of showing that $\underline{\varsigma_t^{arb}} > 0$, it is without loss of generality. However, one would want to input the non-annualized values in equations that represent laws of motion to recover the proper dynamic.

from $0.97 \cdot \phi_{\hat{t}-1} \geq d^{arb}_{\hat{t}-1}$, it follows that $d^{arb}_t \leq 0.97 \cdot \phi_t$ for $t \in [\hat{t}, \tau]$. Let (***) denote this intermediate result. Therefore, by (***) either $d^{arb}_t = 0$ occurs prior to $\phi_t = 0$ or $d^{arb}_t = 0$ occurs at the same time as $\phi_t = 0$. Any other outcome, in particular $d^{arb}_\tau > 0$ when $\rho_\tau = 0$, is in contradiction with (***). ■

The proof presented above covers the case where a single senior tranche is issued from a given pool of FFEL loans, which is the only type of SLABS presented in section 1.2 and 1.3. Can this result be applied to empirical cases where there are multiple senior tranches of SLABS issued on a given pool, which are first introduced in section 1.4.1? If the distributions among senior tranches are *pro rata* following an event of reprioritization, then a nearly identical proof to the one presented above, which would simply require the introduction of additional notation, could be constructed. I take a close look at SLM 2003-3 in Table 1.6 and in Figure 1.5. In Table 1.6, the near-arbitrage lower bound computed by simulations on the bottom tranche of SLM 2003-3, the A4 tranche, under a counter-factually low overcollateralization ratio of 1, $\underline{P_t}|_{\phi_t/\rho_t=1}$ is \$93, which is lower than the \$98 that would be consistent with the analytical proof and the 98% minimum guarantee on the FFEL loans that collateralize pool SLM 2003-3. This is because the lower bound computed by simulations are obtained after relaxing conditions C.3 on servicing fees and C.4 on write-downs due to default claim rejected. A similar reason explain why in Figure 1.5, at issuance of SLABS on pool SLM 2003-3, despite an overcollateralization ratio of 1.02, the near-arbitrage lower bound under factual overcollateralization of 1.02 $\underline{P_t}$ is significantly below \$100. The downward adjustment from \$100 to \$93 at issuance of the SLABS is needed to insure against worst case scenario of default immediately after SLABS purchase, as well as bankruptcy of the initial servicer, resulting in a worst case servicing contract with higher fees to service the loans of delinquent borrower and retention of small fraction of the loan guarantee payment by the new servicer (to incentivize proper servicing). After accounting for these adjustments, the near-arbitrage lower bounds are similar for the A1, A2, A3 and A4 tranche because of the pro rata distributions after the triggering of an event of reprioritization and the fact that the worst case scenario of default quickly triggers pro rata distribution. This is illustrated by looking at 2007-2 A1, A2, A3 and A4 in Table 1.6, under pro rata rules of distribution after an event of reprioritization (this is especially apparent in the counter-factual case with $\phi_t/\rho_t = 1$, because there are very few quarters during which the A1 tranche is the only senior tranche to receive distribution of principal).

I do not attempt a derivation of a proof that covers the case of SLABS trusts, such as SLM 2007-2, with multiple senior tranches of SLABS and distributions among senior tranches that continue to be *sequential* following an event of reprioritization. At the very least, the no-arbitrage lower bound of $\underline{P_t}^{\dagger\dagger}|_{\phi_t/\rho_t \geq 1} = \$97$ is valid for the top tranche of SLM 2007-2 and similar SLABS trust. However, this is a loose lower bound, as shown by the near-arbitrage lower bounds computed by simulations $\underline{P_t}$ for SLM 2007-2 A1 in Table 1.6.

1.7.3 ESTIMATION OF PARAMETERS OF THE INTEREST RATE MODEL

The data used to estimate the model consists of annualized yields on Treasuries with a maturity of 3-months for the period 01/04/54 to 07/31/08, sampled at a daily frequency.[46] h and λ are latent variables that we do not observe and MCMC is a popular method for the estimation of models with latent variables. I begin by estimating a_0 and a_1 by an ordinary least square (OLS) regression. Like Kalimipalli and Susmel, I set $\alpha = 0.5$ and estimate the other parameters of the model by feeding the residuals of the OLS regression into an MCMC algorithm. Table 1.12 presents the parameter estimates used to simulate interest rate paths. Table 1.13 provides priors and posterior of the MCMC estimation:

Table 1.12: Parameters of the interest rate model.

Parameter	Estimate
a_0	0.0030
a_1	$-5.82 \cdot 10^{-4}$
ψ	0.915
ψ	0.915
σ_η^2	0.234
β	-7.887
ν	1.573
p_{01}	0.0839%
p_{10}	0.456%

My estimates imply a long-run mean for interest rates of $-a_0/a_1 = 5.15\%$ and an half-life for volatility shock of $-ln(2)/ln(\psi) = 7.8$ days.[47] When the economy is in a low-volatility regime, it can be expected to remain in that regime for $1/p_{01} = 1192$ days.[48] The expected duration of high-volatility regime is shorter at $1/p_{10} = 219$ days or $219/250 \approx 0.88$ year. β represents the long-run mean for $ln(h)$ when in a low-volatility regime, thus the average daily standard deviation on interest rate is given by $\sqrt{e^{\beta}} = 0.0194$, which is 0.0194 percentage point; when in a high-regime, the same statistics is computed from $\sqrt{exp^{\beta+\nu}} = 0.0426$.

[46]The Treasury bills (secondary market) data published by the Federal Reserve System can be found at http://www.federalreserve.gov/releases/h15/data.htm.

[47]I only consider business days, thus my estimates imply a half-life of 7.8 business days or 7.8 days out of a calendar with 250 days, which corresponds roughly to half-life of 11.4 days for a 365 days calendar.

[48]Let X denote the number of trials needed for a transition from state 0 to state 1 to occur. Then, X has a geometric distribution and $E[X] = 1/p_{01}$.

Table 1.13: Estimation of parameters of the interest rate model. The data used to estimate the model consists of annualized yields on Treasuries with a maturity of 3-months for the period 01/04/54 to 07/31/08, sampled at a daily frequency. The sample size is 13635. Prior distribution of σ_η^2 is improper. See Kalimipalli and Susmel (2004) for details about the model estimation. Parameters estimates reported here for β^* differs from those used in simulations: $\beta = \beta^* - ln(10000)$. The difference arises because residuals from the OLS regression are scaled by a factor of 100 before they are feed into the MCMC algorithm.

Parameters	Prior values		Posterior values		
	Mean	Standard deviation	Mean (std. error)	Standard deviation	95% C.I.
ψ	0	1	0.9147 (0.0012)	0.0067	[0.89,0.93]
σ_η^2	-	-	0.2340 (0.0025)	0.0142	[0.21, 0.26]
β^*	0	50	1.3231 (0.0069)	0.0495	[1.23,1.41]
ν	1	50	1.5734 (0.0216)	0.1964	[1.10,1.96]
p_{01}	0.2	0.16	$8.4 \cdot 10^{-4}$ ($5.3 \cdot 10^{-5}$)	$4.5 \cdot 10^{-4}$	[$(2.1, 190.3) \cdot 10^{-4}$]
p_{10}	0.2	0.16	0.0046 ($1.5 \cdot 10^{-4}$)	0.0014	[$(2.2, 8.2) \cdot 10^{-3}$]

1.7.4 DATA AND COMPUTATION OF THE SERVICING COST DIFFERENCE BETWEEN DELINQUENT AND CURRENT BORROWER

The main additional tasks that must be performed by a servicer on a delinquent loan is to contact a delinquent borrower by phone a minimum of 4 times and to send a minimum of 6 collection letters between the moment that a borrower becomes delinquent and the point at which the borrower enters default. In addition, when an address or phone number for a delinquent borrower is found to be invalid, a servicer must perform skip-tracing activities an attempt to obtain valid contacts for the borrower.

Table 1.14 presents the data I used to compute a cost differential between current and delinquent borrowers.

Table 1.14: Data used to compute servicing cost difference between delinquent and current borrowers.

Unit cost of collection letters	$1.50
Average duration of phone contacts with delinquent borrowers	2 minutes
Median hourly wage of customer representative within financial service	$14.56
Median of salaries as a % of op. exp. in the "for-profit services" sector	50%
Fraction of borrowers who default that require skip-tracing	12%
Fraction of delinquent borrowers who require skip-tracing	6%
Skip-tracing fee	$28

I obtained quotes from mailing company that handle the task of printing and mailing collection letters: the unit cost of sending collection letters, including postage fee, is approximately $1.50. A small-scale servicer of FFELP loans provided the data on the average duration of phone contact with delinquent borrowers. Contacting borrowers by phone is labor-intensive and the Bureau of Labor Statistics reports that the median hourly wage for customer representative within the banking industry is $14.56.[49] The Society for Human Resource Management reports that the median of salaries as a percentage of operating expenses in the "for-profit services" sector is 50%. A small-scale servicer provided data on fraction of borrower who default that require ski-tracing. The skip-tracing fee is from the servicing contract between MOHELA and PHEAA.

I estimate the cost of the additional tasks that must be performed on deliquent loans to be $17.

$$6 * \$1.50 + 4 * \$14.56/60min * 2min * 1/0.5 + 0.12 * \$28 \approx \$17 \tag{1.42}$$

[49]SLM pays its customer service representative $12 an hour at the entry-level position.

1.7.5 Near-arbitrage lower bounds on SLABS deals with a stepdown date

For deals issued by SLM that contains consolidation loans, the rules of distribution are not as presented in Figure 1.2. After the stepdown date is reached, the distribution of principal to subordinate SLABS becomes pro rata with the distribution of principal to senior SLABS under some conditions. For distributions to be pro rata among subordinate SLABS and senior SLABS, the total overcollateralization ratio from the stepdown date onward must be non-decreasing and there must be no event of reprioritization. Otherwise, senior SLABS receive a disproportionate fraction of the distribution of principal or all of it.

Table 1.15 presents the minimum market price during the crisis, P_t, and minimum near-arbitrage lower bounds during the same period. On the one hand, the presence of a stepdown date reduces the build-up of senior overcollateralization over time in pools collateralized by consolidation loans. On the other hand, the net interest rate on consolidation loans is the quarterly average of the financial commercial paper, plus an annualized margin of 2.64%, instead of the minimum annualized margin of 1.74% for non-consolidation loans.

Table 1.15: Near-arbitrage among SLABS deals with a stepdown date. This table shows the minimum quoted market price during the crisis, P_t, and minimum near-arbitrage lower bounds during the same period, $\underline{P_t}$, for the sample of SLABS found in Table 1.11. Other variables in the table are as presented in Table 1.6.

Pool	Tranche	Rules of dist. post event of reprioritization	Mininimum loan guarantee	Overcollat. ratio (min.)	Loan type	Interest rate margin (m)	$\underline{P_t}$ (min.)	P_t (min.)	Near-arb. ($\underline{P_t} > P_t$)
2005-3	A3	pro rata	98%	1.037	Consolidation	2.64%	98	96.10	Y
2005-3	A4	pro rata	98%	1.037	Consolidation	2.64%	98	88.41	Y
2005-3	A5	pro rata	98%	1.037	Consolidation	2.64%	97	75.63	Y
2005-3	A6	pro rata	98%	1.037	Consolidation	2.64%	97	53.60	Y
2005-4	A1	pro rata	98%	1.040	Consolidation	2.64%	99	99.07	N
2005-4	A2	pro rata	98%	1.040	Consolidation	2.64%	99	91.57	Y
2005-4	A3	pro rata	98%	1.040	Consolidation	2.64%	98	72.38	Y
2005-4	A4	pro rata	98%	1.040	Consolidation	2.64%	97	51.72	Y
2006-2	A2	pro rata	98%	1.038	Consolidation	2.64%	100	99.25	Y
2006-2	A3	pro rata	98%	1.038	Consolidation	2.64%	99	96.31	Y
2006-2	A4	pro rata	98%	1.038	Consolidation	2.64%	98	88.03	Y
2006-2	A5	pro rata	98%	1.038	Consolidation	2.64%	98	77.56	Y
2006-2	A6	pro rata	98%	1.038	Consolidation	2.64%	97	60.75	Y
2006-8	A2	sequential	97%	1.036	Consolidation	2.64%	100	97.22	Y
2006-8	A3	sequential	97%	1.036	Consolidation	2.64%	100	91.88	Y
2006-8	A4	sequential	97%	1.036	Consolidation	2.64%	100	84.28	Y
2006-8	A5	sequential	97%	1.036	Consolidation	2.64%	100	72.06	Y
2006-8	A6	sequential	97%	1.036	Consolidation	2.64%	88	54.81	Y
2006-9	A2	sequential	97%	1.036	Consolidation	2.64%	100	97.69	Y
2006-9	A3	sequential	97%	1.036	Consolidation	2.64%	100	92.81	Y
2006-9	A4	sequential	97%	1.036	Consolidation	2.64%	100	84.56	Y
2006-9	A5	sequential	97%	1.036	Consolidation	2.64%	100	72.50	Y
2006-9	A6	sequential	97%	1.036	Consolidation	2.64%	90	53.91	Y

Chapter 2

SLABS Near-Arbitrage: Accounting for Historically Unprecedented Macroeconomic Events

2.1 INTRODUCTION

In this Chapter, I analyze whether the risks associated with historically unprecedented macroeconomic events, such as exceptionally high inflation and default by the government on its loan guarantee, could explain the large underpricings observed on SLABS during the financial crisis of 2007–2009 that was documented in Chapter 1. Let *near-arbitrage gaps* consist of the difference between the minimum near-arbitrage lower bounds on SLABS during the crisis and their minimum market price (as they are reported in Tables 1.7 and 1.8). I present evidence that for more than 90% of SLABS the aforementioned risks explain at most 25% of the SLABS-Treasury near-arbitrage gaps. I proceed in three steps.

First, the rules of distribution of the cash flow from the securitized pools of guaranteed loans that I analyze prioritize payments of servicing fees over payments to SLABS holders. The servicing fee under the initial servicing contract is a constant percentage of the pool balance and tight lower bounds on the price of SLABS can be derived when the initial servicer remains solvent and no unprecedented macroeconomic event occurs. However, following the bankruptcy of the initial servicer and his rejection of the initial servicing contract, a SLABS trust would need to find a new servicer or to renegotiate with the initial servicer. In this context, the new servicing contract would need to pay servicing fees that can cover the costs of servicing the loans over their remaining life. It could become necessary to index the servicing fees to inflation and exceptionally high inflation rates could ultimately lead to a loss on a SLABS-Treasury near-arbitrage without hedging of inflation risk. However, it is relatively inexpensive to buy inflation protection via inflation caps that would insure against this scenario: for 90% of SLABS, the cost of hedging inflation risk shrinks the near-arbitrage gaps by at most 10%.

Second, in the case of SLABS, I use *basis risk* to refer to the risk that the uncollateralized costs of US government borrowing exceed those of the average commercial bank.[1] Interest rate swaps data suggests that, in the process of hedging the basis risk on SLABS, arbitrageurs should have been able to lock in a positive spread that would have been incremental to their profits from capital gains. When near-arbitrage lower bounds are computed, basis risk is ignored, but the extremely conservative assumption that the LIBOR rate equals the Treasury rate is used. As a result of jointly i) adding the basis risk, ii) letting the arbitrageur enter interest rate swaps, and iii) allowing the LIBOR rate to be as expected from swap traders, meaning generally greater than the Treasury rate, a SLABS with hedging of basis risk becomes more valuable than a SLABS under the original near-arbitrage methodology.

Third, I price the risk of default on the government guarantee. I match SLABS to other variable-rate securities that benefit from government guarantees: Pool Certificates guaranteed by the US Small Business Administration (henceforth, SBA PCs). The price change among SBA PCs was less than one fifth of the near-arbitrage underpricings in SLABS. Identifying the causes of the difference in the price dynamics of SLABS and SBA PCs is beyond the scope of this Chapter. However, there are two features that could lead to larger discounts on SLABS during a period of fire-sale of financial assets: SLABS are more complex securities, with guarantees covering the collaterals instead of the bond payments, and their risk-based capital requirements are higher.

2.2 INSURING AGAINST INFLATION RISK

Cash flow from a pool of securitized government-guaranteed student loans must be used to first pay servicing fees before they can be applied to the payment of interest or principal on SLABS. Under the initial servicing contract, the servicing fee consists of a fixed percentage of the pool balance and it is not indexed to inflation.[2] One can imagine scenarios where the initial servicer goes bankrupt, the servicing fees paid to the successor servicer are indexed to inflation and inflation is exceptionally high. I refer to

[1] In general, basis risk simply refers to the risk associated with changes in relative interest rates.

[2] The statement is true for all SLABS in my selected sample (which is listed in Appendix 1.7.1). Other SLABS issued by smaller securitizers have servicing fees indexed to inflation in the initial servicing contract.

these scenarios as *high-inflation scenarios* in the rest of this section.

If inflation is sufficiently high, high-inflation scenarios can lead to a loss on a SLABS-Treasury near-arbitrage. Near-arbitrage lower bounds computed by simulations in Chapter 1 were already robust to inflation paths consistent with interest paths drawn from a model estimated on historical data.[3] Given the relatively modest inflation paths created by the interest rate model and the slow build up of inflation when nominal interest are initially low, it is the scenarios involving extremely high levels of borrower default and extremely fast amortization of the pool, leaving little opportunity for inflation to have much of an impact, that would lead to cash flow with the smallest capacity to repay the arbitrageur's debt and thus, the near-arbitrage lower bound. However, as shown by Table 2.2 under the near-arbitrage-breaking constant inflation rate (CIR) column, for most SLABS, a combination of a slowly amortizing pool (9 or more years) and an extreme scenario of inflation (annual rate of 15% or more) that co-occur with the bankruptcy of the initial servicer and the indexing of servicing fees to inflation could lead to a loss on a SLABS-Treasury near-arbitrage (i.e. a SLABS-Treasury trade initiated at the original near-arbitrage lower bound).

For example, a SLABS-Treasury near-arbitrage initiated at $100 on SLM 2003-3 A4 in November 2007, when the pool that collateralizes it could conservatively be expected to fully amortize within a 9-year horizon, combined with a scenario that includes a 0% rate of default and a 0% rate of prepayment to exacerbate the effect of a ramp up in servicing fees via inflation, makes a constant inflation rate of 21% over 9 years sufficient to generate a loss on the SLABS-Treasury near-arbitrage. The SLABS-Treasury near-arbitrage would have been profitable under a high-inflation scenario with a CIR of 20%. SLM 2003-3 A4 already had an overcollateralization ratio of 1.23 in November 2007 and this contributes to the high level of near-arbitrage-breaking CIR. Scenarios with a 0% rate of default and a 0% rate of prepayment to exacerbate the effect of a ramp up in servicing fees caused by inflation are used to derive the near-arbitrage breaking CIR for all SLABS in table 2.2.

Inflation caps are derivative instruments that can be used by an arbitrageur to protect a SLABS-Treasury trade against inflation risk at a low cost. Two factors contribute to these low costs. First, as revealed by the maximum (risk-adjusted) expected rate of inflation observed during the crisis and reported in Table 2.1, inflation was expected to remain under an annualized rate of 3% over horizons of 20 years or less. The maximum

[3] This meant drawing nominal interest rate paths with the estimated regime-switching and stochastic volatility interest rate model of Kalimipalli and Susmel (2004) and inferring upper bound on inflation consistent with the interest paths (specifically, inflation rate given by the nominal interest rate minus a real rate of 0%). These periodic upper bounds on inflation rates were used to create a cumulative inflator that was then applied to scale up the servicing fees over time.

expected rate of inflation occurred close to the August 22, 2008 date when the historical data on inflation caps begins on the Bloomberg system for the maturities found in table 2.1. Expected rate of inflation were lower later in the crisis. Second, inflation, even at the high levels of CIR found in Table 2.2, can be visualized as slowly eating away the senior overcollateralization of a SLABS pool by scaling up the servicing fees. Eventually, servicing fees can become greater than the cash flow generated by the pool of loans, but this occurs late in the life of the pool. Therefore, it is generally sufficient to buy inflation caps, the derivative instrument offering insurance against inflation, for a notional that is a small fraction of the principal of the SLABS purchased in the SLABS-Treasury trade. This would be true even if the strike of the inflation cap was close to the near-arbitrage-breaking CIR, but this is amplified by the following factor. Data on inflation caps on the Bloomberg system is available up to a strike value of 6%. For simplicity, I conduct my entire analysis on the cost of insuring against inflation risk using inflation caps with a strike value of 4.5%. Using inflation caps with a strike value that is much lower than the near-arbitrage breaking level of CIR contributes to the low levels of notional needed on most SLABS to insure against high-inflation scenarios.

In exchange for the price paid to acquire a year-over-year inflation cap with maturity of T years, an investor receives T independent caplets that pay at the end of year 1,2,...,T. Every caplet pays according to:

$$\max\left(0, \frac{\text{CPI}_{m-2}}{\text{CPI}_{m-14}} - (1+\text{strike})\right) \times \text{Notional}, \tag{2.1}$$

where m is the maturity of the caplet, in months, and CPI stands for the consumer price index.

To illustrate, assume that the realized inflation rate over the next ten years is 5%. An investor who purchases the cap with a strike of 4.5% and maturity of 10 years would be entitled to a cash flow of $(\max(0, 1.05 - 1.045) \times \text{Notional})$ at the end of year 1 to T. The maximum price that had to be paid to insure \$100 notional against annual inflation rate exceeding 4.5% ranged from \$0.72 for the two-year maturity to \$17.05 for the 30-year maturity.[4]

In addition to showing the CIR levels that would lead to a loss on SLABS-Treasury near-arbitrage in absence of protection against inflation risk, Table 2.2 shows the revision

[4] Fleckenstein, Longstaff, and Lustig (2013) validate the quality of the zero-coupon inflation caplet and floorlet quoted on the Bloomberg system. They check the quality of the data by ensuring that the caplet and floorlet prices satisfy standard option pricing bounds such as those described in Merton (1973) including put-call parity, monotonicity, intrinsic value lower bounds, strike price monotonicity, slope, and convexity relations. They do so on data ranging from October 5, 2009 to October 5, 2012.

Table 2.1: Inflation caps and risk-adjusted expected inflation

Maturity (year)	Cap Price ($) (max)	Expected inflation (max)
2	0.72	2.78%
3	1.26	2.74%
5	2.53	2.71%
10	6.70	2.82%
20	11.87	2.87%
30	17.05	3.13%

This table reports the maximum price, observed between August 25, 2008 and June 30, 2009, for year-over-year inflation caps with a strike of 4.5%, for the indicated maturities. The prices are expressed in dollars per $100 notional. Maturity is expressed in years. The last column reports on the zero-coupon inflation swap rate, a risk-adjusted measure of expected inflation. The maximum swap rate over the August 25, 2008 to June 30, 2009 is reported.

(if any) to the near-arbitrage lower bound that makes the SLABS-Treasury trade robust to a CIR of 4.5% and ultimately, incorporating the discount needed to purchase inflation caps, a near-arbitrage lower bound on the price of SLABS that enables a SLABS-Treasury-and-inflation-cap near-arbitrage.

The low costs of insuring against inflation risk support two points. First, they reflect the unlikeliness of the inflation paths needed to produce losses on SLABS-Treasury near-arbitrages. Second, they demonstrate that inflation risk cannot explain the large gaps observed between actual prices of SLABS and their near-arbitrage lower bounds.

For example, an arbitrageur could acquire SLM 2003-3 A4 bonds with a principal of $100 for as little as $92 during the crisis. I showed in Section 1.4 that, setting aside historically unprecedented inflation scenarios, maintaining the no government default condition (C.1) and the interest rate ordering condition (C.2), the cash flow from the SLABS was nearly certain to repay a debt of $100 that accrues interest at the 3-month Treasury rate. Thus, under frictionless shorting of Treasuries, an arbitrageur could borrow $100 at the 3-month Treasury rate, use $92 to purchase SLM 2003-3 A4 bonds with a principal of $100, pocket $8, then let the cash flow from the SLABS repay the $100 that he borrowed with near certainty. Insuring against inflation risk with caps is relatively inexpensive, thus the change that results from the addition of high-inflation scenarios is small. The arbitrageur borrows $100 at the 3-month Treasury rate, uses $92 to purchase SLM 2003-3 A4 bonds with a principal of $100 and uses $0.21 to purchase a 10-year inflation cap on \$3 of notional with strike at 4.5%.[5] The arbitrageur pockets \$7.79 and then lets the combined cash flow from the SLABS and the cap repay the \$100 that he borrowed and some more. In other words, the inflation-robust near-arbitrage lower bound on SLM 2003-3 A4 is \$99.79 and market prices below \$99.79 represent near-arbitrage opportunities.

To sum up, for more than 90% of SLABS, inflation risk explains less than 15% of the near-arbitrage gaps. Hedging against inflation risk is cheap and inflation risk alone cannot explain the large discounts below their near-arbitrage lower bounds observed on SLABS during the crisis. Once inflation risk is hedged, default on government payments on the (government-guaranteed) FFEL student loans and violation of the interest rate ordering conditions remain as the only possible events that could lead to a loss on the SLABS-Treasury trade initiated when the price of the SLABS is at its near-arbitrage lower bound or below.

2.3 BASIS RISK

Basis risk refers to the risk associated with changes in relative interest rates. The relationships between three interest rates determine whether a SLABS-Treasury near-arbitrage is profitable. The net interest rate that is paid by the government and the borrower on FFEL loans is tied to the quarterly average of the 3-month financial commercial paper rate $\bar{r}^{FCP}_{t,t+1}$. The SLABS promises interest rate payments that are tied to the 3-month LIBOR rate, r_t^{LIBOR}. The arbitrageur's debt, which was contracted to finance the purchase of SLABS, accrues interest at the 3-month Treasury rate r_t. The interest rate ordering condition (C.2), $r_t^{LIBOR} \geq r_t$ and $r_t^{FCP} \geq r_t$ for all t, was used to derive benchmark near-arbitrage lower bounds in Chapter 1. It places weak restrictions on changes in the relative interest rates, essentially assuming that the credit worthiness of the US government for the repayment of its nominal debt, with its capacity to "print money", is always superior to that of the average bank that contributes to the r_t^{LIBOR} and r_t^{FCP} rate indexes. For the near-arbitrage lower bounds established via simulations in Chapter 1, the worst conditions that do not violate the interest rate ordering condition were assumed, meaning $r_t^{FCP} = r_t$ and $r_t^{LIBOR} = r_t$ for all t. Absent a violation of the interest rate ordering condition, and absent

[5] The ex-post paydown factor of SLM 2003-3 A4 between September 2008 and February 2015 was 100%. This amortization was highly predictable. Therefore, there is little need to insure against inflation beyond a 10-year horizon. A more involved hedging strategy would include inflation caps at less than 10-year horizon on \$2.60 notional, inflation caps at 10-year horizon on \$0.50 notional and inflation caps at 20-year horizon on \$0.10 notional. The cost of this portfolio of inflation caps would be of the same order of magnitude as the 10-year cap on \$3 notional that I use for simplicity of exposition.

Table 2.2: Near-arbitrage-breaking CIR and near-arbitrage after hedging of inflation risk (Part 1 of 2)

Tranche	Tranche O/C (min)	Yrs to pool amortiz (max)	Breaking CIR	$\underline{P_t}$ Orig.	$\underline{P_t}$ 4.5%-infl. robust	$\underline{P_t}$ Infl. robust	Cap Notional	Cap Cost	P_t (min)	Remain. arb. gap ($)
2003-3 A4	1.230	9	21%	100	100	99.79	3	0.21	91.56	8.23
2003-6 A4	1.276	9	24%	100	100	99.79	3	0.21	91.28	8.51
2003-8 A4	1.227	9	21%	100	100	99.79	3	0.21	91.03	8.76
2003-9 A4	1.221	9	20%	100	100	99.79	3	0.21	86.78	13.01
2004-4 A4	1.175	9	19%	100	100	99.79	3	0.21	90.31	9.48
2004-6 A4	1.175	10	95%	100	100	99.93	1	0.07	97.78	2.15
2004-6 A5	1.175	10	18%	100	100	99.79	3	0.21	86.91	12.88
2004-7 A4	1.153	10	47%	100	100	99.93	1	0.07	96.50	3.43
2004-7 A5	1.153	10	15%	100	100	99.66	5	0.34	85.44	14.22
2004-9 A4	1.134	10	28%	100	100	99.93	1	0.07	95.22	4.71
2004-9 A5	1.134	10	13%	100	100	99.59	6	0.41	83.47	16.12
2005-2 A4	1.126	10	31%	100	100	99.93	1	0.07	94.75	5.18
2005-2 A5	1.126	10	12%	100	100	99.59	6	0.41	81.97	17.62
2005-10 A3	1.076	11	42%	100	100	99.93	1	0.07	96.66	3.27
2005-10 A4	1.076	11	10%	100	100	99.53	7	0.47	86.31	13.22
2005-10 A5	1.076	11	7%	99.7	99.3	98.02	19	1.28	87.44	10.58

This table reports SLABS-Treasury near-arbitrage that remain after accounting for the cost of hedging inflation risk. The second column reports minimum tranche level overcollateralization observed during the crisis (November 2007 to November 2009) and the third column reports a conservative estimate of the years remaining until full amortization of the pool as of November 2007. Both are determinants of the near-arbitrage-breaking constant inflation rate (CIR) reported in the fourth column. The near-arbitrage-breaking CIR is the minimum level of CIR that, when combined with a scenario with 0% default rate, a 0% prepayment rate and constant borrower count from the purchase of a SLABS to the full amortization of the pool (conditions that exacerbate the adverse effect of inflation), leads to a loss on a SLABS-Treasury near-arbitrage when inflation risk is unhedged. The eighth and ninth columns report the notional of year-over-year inflation cap with a strike of 4.5% that is sufficient to hedge inflation risk on a SLABS-Treasury near-arbitrage and its corresponding cost. The fifth, sixth and seventh columns respectively report i) the original near-arbitrage lower bounds computed by ignoring risks associated with historically unprecedented macroeconomic events, which are robust to the implied inflation paths produced by the interest rate model used to compute the original bounds, ii) near-arbitrage lower bounds robust to an exogenously set rate of inflation of 4.5%, iii) near-arbitrage lower bounds robust to any level of inflation, which incorporate the cost of hedging inflation risk. The tenth column reports the minimum market price observed on a SLABS during the crisis and the remaining near-arbitrage gaps when near-arbitrage lower bounds that are robust to inflation risk are used.

Table 2.3: Near-arbitrage-breaking CIR and near-arbitrage after hedging of inflation risk (Part 2 of 2)

Tranche	Tranche O/C	Yrs to pool amortiz	Breaking CIR	$\underline{P_t}$			Cap		P_t	Remain. arb. gap
	(min)	(max)		Orig.	4.5%-infl. robust	Infl. robust	Notional	Cost	(min)	($)
2006-1 A3	1.072	11	31%	99.8	99.7	99.63	1	0.07	97.50	2.13
2006-1 A4	1.072	11	11%	99.8	99.7	99.23	7	0.47	85.28	13.95
2006-1 A5	1.072	11	7%	98.3	98.2	96.86	20	1.34	79.63	17.24
2006-3 A3	1.072	11	39%	99.7	99.7	99.63	1	0.07	96.34	3.29
2006-3 A4	1.072	11	11%	99.6	99.6	99.13	7	0.47	85.56	13.57
2006-3 A5	1.072	11	7%	98.0	97.7	96.22	22	1.48	73.41	22.82
2007-2 A1	4.466	13	106%	99.9	99.9	99.83	1	0.07	97.81	2.02
2007-2 A2	1.516	13	22%	100	100	99.76	3.5	0.24	88.43	11.33
2007-2 A3	1.244	13	16%	100	100	99.06	14	0.94	77.44	21.62
2007-2 A4	1.041	13	10%	82	82	80.12	28	1.88	69.50	10.62
2007-3 A1	3.733	13	75%	99.9	99.9	99.83	1	0.07	97.38	2.46
2007-3 A2	1.489	13	22%	100	100	99.73	4	0.27	88.16	11.58
2007-3 A3	1.217	13	16%	100	100	99.06	14	0.94	77.50	21.56
2007-3 A4	1.039	13	10%	81	80	78.05	29	1.95	69.19	8.87

(This table is a continuation of Table 2.2.)

a default by the US government on its loan guarantee, a SLABS-Treasury near-arbitrage is nearly certain to be profitable.

If the interest rate ordering condition is relaxed and $r_t > r_t^{LIBOR}$ is allowed, then sufficiently large and persistent occurrences of $r_t > r_t^{LIBOR}$ would lead to a loss on the SLABS-Treasury near-arbitrage. However, an arbitrageur could insure against $r_t > r_t^{LIBOR}$. An arbitrageur could enter a swap and pay r_t^{LIBOR} in return for $r_t + s_1$, where $s_1 > 0$. However, the swapping of r_t^{LIBOR} for r_t introduces a new source of risk. A near-arbitrage lower bound of $100 indicates that the *arbitrageur's debt*, which accrues interest at r_t, with a principal of $100 is always *repaid in full* from the cash flow on the *SLABS*; it does *not* imply that the *SLABS*, which accrues interest at r_t^{LIBOR}, with a principal of $100 is always *repaid in full* from the same cash flow. In order for the full repayment of the SLABS to co-occur with the full repayment of the arbitrageur's debt, one must place an upper bound on the difference between r_t^{LIBOR} and $\bar{r}_{t,t+1}^{FCP}$.

Therefore, after relaxing the interest rate ordering condition, a SLABS-Treasury trade with swapping of r_t^{LIBOR} for r_t exposes the arbitrageur to losses caused by an extreme widening of the r_t^{LIBOR} and $\bar{r}_{t,t+1}^{FCP}$ spread. This is in contrast with the SLABS-Treasury trade under normal interest rate ordering condition, where the spread between $\bar{r}_{t,t+1}^{FCP}$ and r_t matters for the profitability of the SLABS-Treasury near-arbitrage, but the spread between r_t^{LIBOR} and $\bar{r}_{t,t+1}^{FCP}$ can be of any sign and any size and never lead to a loss on the SLABS-Treasury near-arbitrage.[6]

During the crisis, SLM 2003-3 A4 had initial overcollateralization ratio of 1.30 or more, and two kinds of widening of the r_t^{LIBOR}-$\bar{r}_{t,t+1}^{FCP}$ spread could have caused a loss on the SLABS-Treasury trade, with swapping of r_t^{LIBOR} for r_t, initiated at $P_t = \$100$. First, a moderate scenario of default of 2% of pool balance in every quarter, leading to a full amortization of the pool over an 8 year period, combined with a constant spread of 2% over that period would break the near-arbitrage at $P_t = \$100$. Second, for extreme scenarios of default that lead to amortization of the pool balance within a one to 2 year period, a spread of 12% would break the near-arbitrage at $P_t = \$100$. As shown by Figure 2.1, 12% over a 2-year period and 2% over an 8-year period are extremely unlikely relative to the pre-crisis and in-crisis data. The mean pre-crisis spread between r_t^{LIBOR}-r_t^{FCP} was 0.14% and the 99th percentile of the distribution was 0.28%; the mean in-crisis spread

[6]Under normal interest rate ordering condition, the r_t^{LIBOR} and r_t would matter in the following way: the larger the spread between r_t^{LIBOR} and r_t, the larger the proportion of the cash flow from the pool distributed to SLABS holder (the lower the proportion of the cash flow from the pool distributed to excess distributions certificate holder), thus a larger $r_t^{LIBOR} - r_t$ is beneficial to SLABS holder. It is in this sense that setting $r_t^{LIBOR} = r_t$ to establish near-arbitrage lower bounds in Chapter 1 corresponds to the worst spread that does not violate the normal interest rate ordering condition.

was 0.13% and the 99th percentile of the distribution was 1.01%.[7]

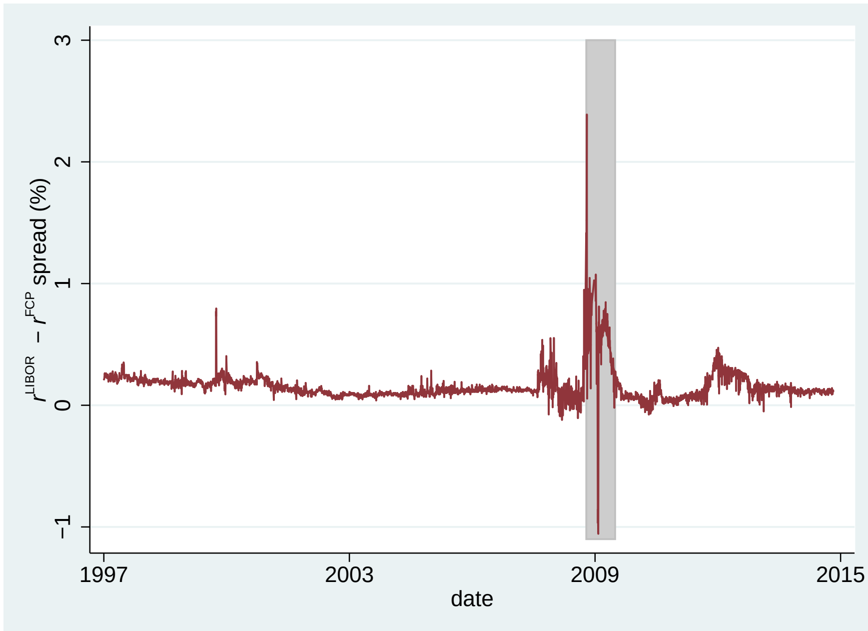


Figure 2.1: LIBOR-FCP spread. This figure plots the difference between the 3-month LIBOR rate, r_t^{LIBOR}, and the 3-month financial commercial paper rate r_t^{FCP}, expressed in percentage points. The shaded area represents an abnormal period because of the offering of guarantees covering the commercial paper issued by participating banks under the FDIC's *Temporary Liquidity Guarantee Program*. The program depressed the r_t^{FCP} rate relative to the r_t^{LIBOR}.

Furthermore, these loss-causing spreads between r_t^{LIBOR} and $\bar{r}_{t,t+1}^{FCP}$ on a SLABS-Treasury near-arbitrage after swapping of r_t^{LIBOR} for r_t are derived under the extremely conservative assumption (as used in Chapter 1) that the margin over $\bar{r}_{t,t+1}^{FCP}$ paid on the government guaranteed student loans is always 1.74%. Truly, as shown in Table 2.4, the margin is at least 0.60% higher on the loans of borrowers that have entered repayment status and who are no longer in-school, grace or deferment status. Thus, for scenarios with lower default rates and slower amortization of the pool over time, it would already be extremely conservative to use a "representative pool" of loans that accrue interest at $\bar{r}_{t,t+1}^{FCP} + 1.74\%$ during the first 3 years that follow the initiation of a SLABS-Treasury trade, and accrue interest at $\bar{r}_{t,t+1}^{FCP} + 2.34\%$ afterward. Revisiting the example of SLM 2003-3 A4, but using this better approximation yet still extremely conservative assumption relative to the empirical performance of pools, the loss-causing constant spread between r_t^{LIBOR} and $\bar{r}_{t,t+1}^{FCP}$ becomes 2.5% (instead of 2.0%) for a pool with a constant default rate of 2% that amortizes over 8 years.

The fact that the r_t^{LIBOR}-r_t^{FCP} spreads tends to be narrow is not surprising: r^{LIBOR}

[7]Includes data from 1997 to 2007.

Table 2.4: Margin over $\bar{r}^{FCP}_{t,t+1}$ on government guaranteed loans

Loan type	Status	Margin
All	In-School, Grace or Deferment	1.74%
Non-consolidation	In Repayment	2.34%
PLUS and consolidation	In Repayment	2.64%

This table reports the annualized interest rate margin over the $\bar{r}^{FCP}_{t,t+1}$ index on government guaranteed loans.

reflects the interest rate paid by very large banks to obtain an uncollateralized loan from other banks and the r^{FCP} reflects the interest rate demanded by investors to hold the uncollateralized debt of banks. LIBOR is based on the inter-bank borrowing rate of 18 of the largest banks in the world.[8] The financial commercial paper rate is based on the primary sale of commercial paper to investors, by financial institutions with "1" or "1+" ratings from Moody's and Standard & Poor's.[9] Sales are weighted by their face value, thus large issuers have a disproportionate impact on the r^{FCP} rate. Therefore, we are essentially looking at the same type of liability from groups of banks that have a lot of overlap and similar outside sources of funding.

If concerned about an extreme widening of the r^{LIBOR}_t-r^{FCP}_t spread, an arbitrageur that already swaps r^{LIBOR}_t for r_t could enter a second interest rate derivative contract. The arbitrageur receives $r_t + s_1$ for r^{LIBOR}_t, with $s_1 > 0$, and the arbitrageur could enter a basis cap where he receives $max(0, r^{LIBOR}_t - r^{FCP}_t - K\%)$ in exchange for paying s_2. K corresponds to the strike of the basis cap. With $s_1 \geq s_2$, the basis cap is financed by the spread on the swap and the cap insures against a widening of the r^{LIBOR}_t-r^{FCP}_t spread that could prevent the cash flow from the pool from repaying the SLABS in full. Figure 2.2 illustrates the strategy.

Figure 2.3 demonstrates why a good case can be made that an arbitrageur who sought a single swap-and-cap counter-party, acting as both swap-counter-party 1 and swap counter-party 2 in Figure 2.2 would have a high likelihood of finding a counter-party willing to accept a strike of 30 basis points in exchange for $s_1 - s_2 = 0$ when interest rate are low ($r_0 < 2\%$). Figure 2.3 shows the net payoff (pay-in, minus payout) for the combined swap-and-cap counter-party. Net payoffs are computed daily, using the difference between the pay-in rate r^{LIBOR}_t and the payout rate $r_t + (s_1 - s_2) + max(0, r^{LIBOR}_t - \bar{r}^{FCP}_{t,t+1} - K\%)$.

Historically, the net payoff of the swap-and-cap counter-party has been positive 98.4% of the time for $s_1 - s_2 = 0$ and $K = .30\%$. For a strike of $K >= 1.03\%$, the net payoff of

[8] Three banks are US based. All foreign banks have significant US activities, borrow in USD and use US market to raise capital.

[9] "1" or "1+" represent the highest ratings for commercial paper issuance.

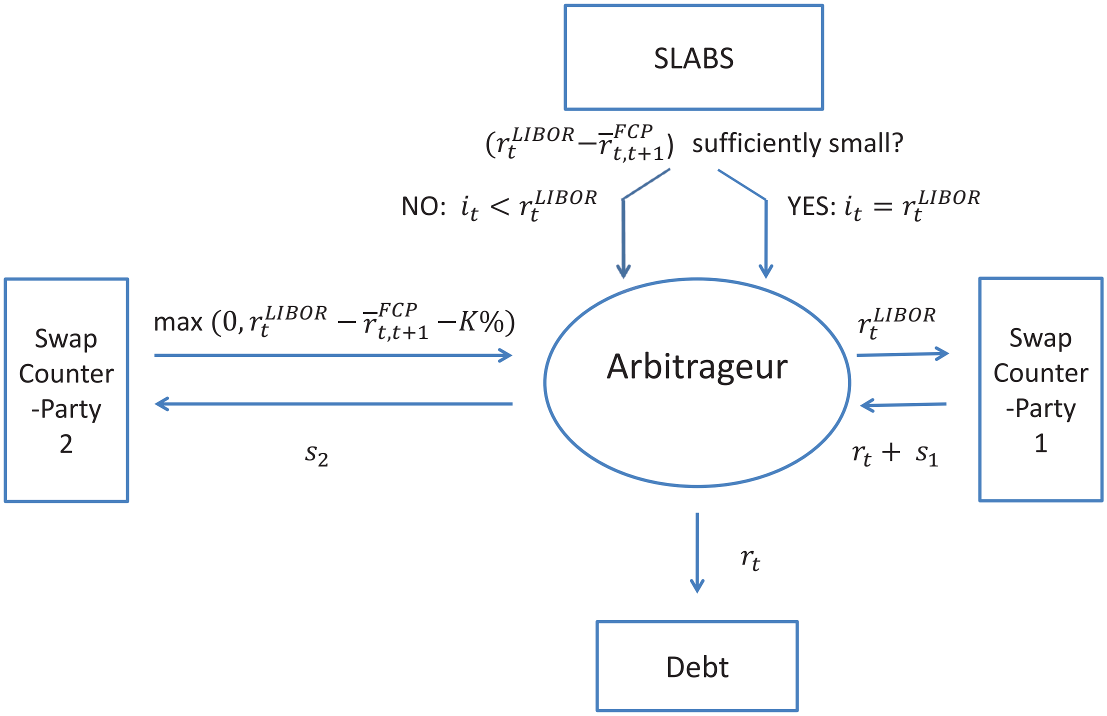


Figure 2.2: Hedging basis risk. This figure depicts a hedging strategies that insures against the risk that $r_t > r_t^{LIBOR}$. The purchase of a cap, via quarterly payments s_2, in exchange for the variable leg that pays $max(0, r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP} - K\%)$ is complementary to the swapping of r_t^{LIBOR} for r_t because of the risk that an extremely wide and positive r_t^{LIBOR}-$\bar{r}_{t,t+1}^{FCP}$ spread might prevent the full repayment of a SLABS.

the swap-and-cap counter-party has been positive 100% of the time. The rare instances of negative net payoff with $K = .30\%$ correspond to periods of declining interest rate, where the $r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP}$ spread widens, while the $r_t^{LIBOR} - r_t$ stays relatively constant.

The fact that interest rates were low during the crisis, during the period of underpricing of SLABS, helps make the case that an arbitrageur seeking a swap-and-cap counter-party and offering that this counter-party keeps the difference between $r_t^{LIBOR} - r_t$ (if positive), in exchange for insurance against a widening of the $r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP}$ beyond 30 basis points would successfully find such a counter-party. Given the high correlation between a widening of the $r_t^{LIBOR} - r_t^{FCP}$ and a widening of the $r_t^{LIBOR} - r_t$ a deterioration in the relative credit worthiness of the banks contributing to r_t^{LIBOR} versus those contributing to r_t^{FCP} is not a scenario that would lead to a negative net payoff for the swap-and-cap counter-party. As supported by Figure 2.3, historically, the only way a non-trivial widening $r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP}$ occurs without the increment in spread between $r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP}$ being matched one-to-one by increments in the $r_t^{LIBOR} - r_t$ is when interest rate falls.

Starting from a low interest, before interest rate can decline, they will have to increase. As long as interest rate are flat or increasing the net payoff of the swap-and-cap counter-party are positive. Therefore, before a drop in interest that is sufficiently large to lead to a negative net payoff on one settlement date could occur for the swap-and-cap counter-party, it would be preceded by many quarters of positive net payoff. Table 2.5 shows that conditioning on low interest rate ($r_t < 2\%$), the net payoff of the swap-and-cap counter-party have historically been positive, 100% of the time, for $s_1 - s_2 = 0$ and $K = .30\%$.

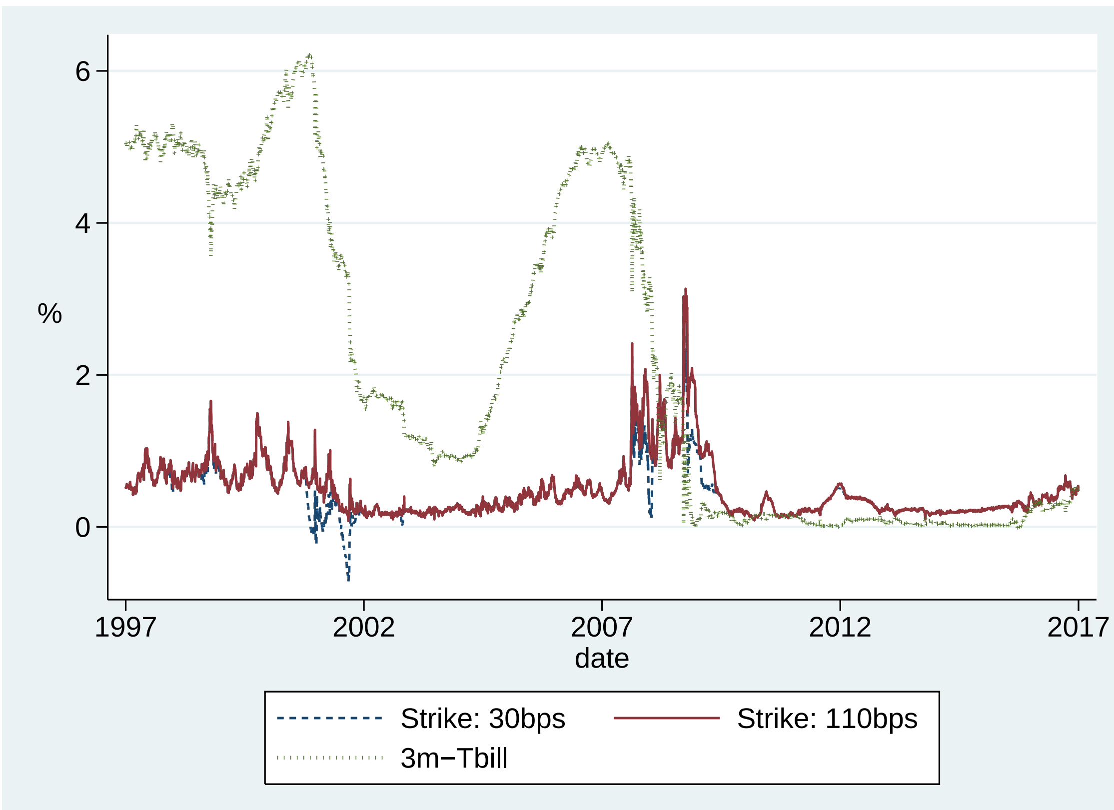


Figure 2.3: Net payoff of the swap-and-cap counter-party. This figure shows the daily net payoff (pay-in, minus payout) of the swap-and-cap counter-party for two strike values (30 basis points and 110 basis points). Payoff are for $s_1 - s_2 = 0$. The figure also shows the contemporaneous (3-month) T-bill rate.

Based on the above evidence, it appears likely that an arbitrageur would be able to find a swap-and-cap counter-party willing to enter a r_t^{LIBOR}-for-r_t swap and offer a cap on the $r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP}$ basis at a strike of 0.30% for $s_1 - s_2 = 0$ when $r_t < 2\%$ at the time of execution of the contract. The majority of SLABS that presented near-arbitrage opportunities are easily repaid in full, meaning paid the r_t^{LIBOR} rate they were promised and the full repayment of their balance, when $r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP} < 0.70\%$ holds. When

Table 2.5: Net payoff of the swap-and-cap counter-party

Panel A: No restriction on r_t

	Net payoff (pay-in, minus payout)						
Strike	Mean	Std. dev.	Min	Max	1 pctile	5 pctile	Freq(Payoff>0)
0.3	0.43	0.33	-0.72	3.03	-0.08	0.14	0.98
1.1	0.48	0.38	0.08	3.13	0.13	0.16	1.00
1.9	0.49	0.41	0.09	3.84	0.13	0.16	1.00

Panel B: $r_t < 2\%$

	Net payoff (pay-in, minus payout)						
Strike	Mean	Std. dev.	Min	Max	1 pctile	5 pctile	Freq(Payoff>0)
0.3	0.34	0.30	0.01	3.03	0.12	0.15	1.00
1.1	0.37	0.38	0.09	3.13	0.12	0.15	1.00
1.9	0.38	0.42	0.09	3.84	0.12	0.15	1.00

$r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP} \geq 0.70\%$, if the arbitrageur had simply entered a r_t^{LIBOR}-for-r_t, he would be at risk of not receiving r_t^{LIBOR} from the SLABS and not be able to deliver on his contractual promise to that swap counter-party. If failing to deliver on his contractual promise to the swap counter-party, this would be a form of realized loss on the SLABS-Treasury trade. However, thanks to having jointly entered a cap and a swap, whenever $r_t^{LIBOR} - \bar{r}_{t,t+1}^{FCP} \geq 0.70\%$ occurs, any shortfall between r_t^{LIBOR} and the interest rate received would be fine because thanks to the cap with a strike of 0.30% the shortfall in interest rate received would be offset by a one-to-one decrease in the interest rate that needs to be delivered to the swap-and-cap counter-party.

Therefore, in switching from the extremely conservative assumption that $r_t^{LIBOR} = r_t$ and $r_t^{FCP} = r_t$ that was used to derive near-arbitrage lower bound by simulations in Chapter 1, to an environment where the risk of $r_t > r_t^{LIBOR}$ is considered and hedged, and this is done based on interest rate swap and cap "pricing" that reflect the probabilistic relationship between r_t^{LIBOR}, r_t^{FCP} and r_t, there is no need to revise near-arbitrage lower bounds downward.

Therefore, revisiting our example with SLM 2003-3 A4, following the textbook narrative of an arbitrage, which implicitly assumes the frictionless shorting of Treasuries, an arbitrageur would borrow $100 at the 3-month Treasury rate, purchase SLM 2003-3 A4 bonds with $100 notional for $92, spend $0.21 to insure against inflation and enter the swap-and-cap with a single counter-party that combines the swap and cap depicted with separate counter-parties in Figure 2.2. The arbitrageur would pocket $7.79 at the initiation of the trade, then let the cash flow from the SLABS, the inflation cap, the interest rate swap and cap repay the $100 of debt with near certainty. Only one potential source of loss on the SLABS-Treasury near-arbitrage remains: default by the US government on its guarantees.

2.4 DEFAULT ON GOVERNMENT GUARANTEES

This section explores two kinds of default by the US government that could lead to a loss on the SLABS-Treasury near-arbitrage. I show that the pricing of the risk of default on government obligations and guarantees cannot explain the large underpricing of SLABS.

First, one can imagine a scenario in which the government defaults on supplemental interest payments on guaranteed student loans, without a simultaneous default on short-term Treasuries. Borrowers pay an interest rate on FFEL loans that is either i) fixed or ii) variable and subject to a cap. Holders of FFEL loans receive an interest that is variable without a cap. Therefore, the size of the supplemental interest payments that the government must make to fill the gap between interest payments by the borrowers and the interest paid to holders of the loans increases with the interest rate level. As inflation pushes interest rate higher, the nominal cost of the FFEL loans program becomes larger. One can image a scenario in which the U.S. might be able to honor its nominal debt by simply "printing more money", but then not be able to pay off its inflation-linked and variable interest rate liabilities, a group of liabilities that includes supplemental interest payments on FFEL loans and payments on TIPS. The comparison of the supplemental interest rate payments with payments on TIPS is appropriate: the promise to make supplemental interest rate payments on FFEL loans is backed by the full faith and credit of the US government.[10]

Second, one can imagine a scenario in which the government defaults on the explicit guarantee against borrower's default offered by the US Department of Education, without a simultaneous default on short-term Treasuries.[11] I concede that Treasuries play a central role in the worldwide financial system, but the prioritization of payments on Treasuries over payments on guarantees may be slightly challenged by the following consideration. Default on the guarantee on student loans could undermine the credibility of the insurance on bank deposits, offered by the FDIC, which is another form of government guarantee. If depositors no longer have confidence in deposit insurance, bank runs and an insolvent

[10] See Federal law under 20 U.S.C. §1075 and 20 U.S.C. §1087-1.

[11] Federal law (20 U.S.C. §1082) states that should the guarantor for a FFEL loan become insolvent, the holder of a FFEL loan can obtain guarantee payment directly from the Department of Education.

banking system could result. The government may succeed in convincing depositors that the guarantee on student loans is different, but it is an operation that is not without risks.

To explore what fraction, if any, of the near-arbitrage discounts could be explained by an increased risk of default on the government's obligation and guarantees with respect to FFEL loans, I turn to data on another type of variable-rate bond that benefits from US federal guarantees. The Small Business Administration guarantees a portion of the small business loans originated by banks under various programs. The guaranteed portion of SBA-guaranteed loans is often securitized and the SBA, in addition to the guarantee on the collaterals, guarantees the payment of interest and principal on the small-business loans Pool Certificates (PCs). By attributing the entire price dynamic among SBA PCs to changes in the risk of default on government guarantees and obligations, an upper bound on the price change in SLABS that can be explained by such changes in risk is obtained.

More specifically, I form price indexes from SBA PCs with similar maturities. I then construct conservatively extended amortization schedule for SLABS and based on their conservatively extended paydown date, SLABS are matched to an SBA PC price index. The arbitrage gap on SLABS is compared to the price change on their matched SBA PC price index. Table 2.6 reports the result of this comparison. Figure 2.4 shows the difference in price change for a group of SLABS with conservatively extended paydown dates ranging between 2016Q1-2016Q4 and their matched SBA price index formed from SBA PCs with a 2016-2017 maturity. The average through-to-peak price change among the SLABS is 8%.[12] The average through-to-peak price change among the matched group of SBA PCs is less than 1.7%. The computations are based on data from Bloomberg[13] for SLABS, and quoted prices obtained by the Federal Reserve System from Interactive Data Corporation for SBA PCs.[14]

Table 2.7 in the Appendix reports the price changes on the SBA PC price indexes. The price changes are computed by taking the difference between i) the maximum price index observed between 01/01/2010–10/11/2011, and ii) the minimum price observed between 11/10/2008–03/30/2009. The difference is converted into a percent price change by dividing this difference by the minimum price.

Table 2.6 reports the near-arbitrage gaps on SLABS, computed from the difference between their minimum near-arbitrage lower bound value and their minimum price

[12]Includes SLM 2003-3 A4, SLM 2003-6 A4 and SLM 2003-8 A4.

[13]I obtained quoted price data from a Bloomberg terminal and validated it with transaction data involving insurance companies retrieved from a Bloomberg terminal.

[14]The Federal Reserve System started to acquire data on the price of SBA PCs on November 10th, 2008. Hence, the comparison of through-to-peak price change between SLABS and SBA PCs over that period, instead of some other measure.

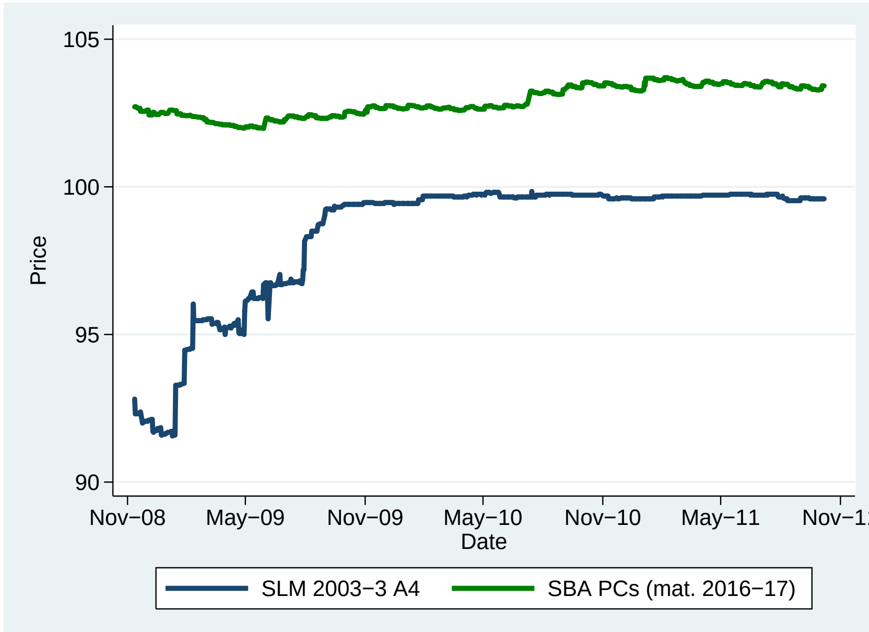


Figure 2.4: SLABS versus SBA PCs. This figure plots the price of SLM 2003-3 A4 and a price index constructed with SBA PCs maturing in 2016-2017. Ex-post, SLM 2003-3 A4 amortized roughly linearly between the crisis and 2016Q1. SBA PCs maturing in 2016-2017 have a similar or slower rate of amortization.

during the crisis of 08-09. The near-arbitrage gaps are then compared to the price changes on a matched SBA price index. The last column of Table 2.6 shows the proportion of the original near-arbitrage gap that remains after accounting for the risks associated with historically unprecedented macroeconomic events. For 90% of SLABS, 75% or more of the near-arbitrage gaps is unexplained by the aforementioned risks.

2.5 CONCLUSION

Using data on inflation caps, interest rate swaps and interest rate basis caps, and comparing the price dynamic on SLABS to other securities benefiting from a similar government guarantee, I find that for 90% of SLABS the risks associated with historically unprecedented macroeconomic events explain at most 25% of the near-arbitrage gaps. These results support the findings reported in Chapter 1 and confirm the presence of large and persistent underpricing of SLABS relative to Treasuries during the financial crisis of 2007–2009.

On the normative side, my findings support the claim from Chapter 1 that SLABS presented near-arbitrage opportunities for the US government during the crisis. These results also support the other normative implications discussed in Chapter 1. On the

Table 2.6: Near-arbitrage among SLABS versus SBA PC price changes

Tranche	Sr. O/C ratio	Near-Arb. gap		Conserv. paydown	Matched SBA index		Remaining gap	
	(min.)	(orig.)	(infl. robust)	(date)	(mat.)	(% ch.)	(% pt.)	(%)
03-3 A4	1.230	8.4%	8.3%	2016q1	2016-17	1.60%	6.7%	79%
03-6 A4	1.276	8.7%	8.5%	2016q2	2016-17	1.60%	6.9%	80%
03-8 A4	1.227	9.0%	8.8%	2016q4	2016-17	1.60%	7.2%	80%
03-9 A4	1.221	13.2%	13.0%	2016q3	2016-17	1.60%	11.4%	87%
04-4 A4	1.175	9.7%	9.5%	2016q3	2016-17	1.60%	8%	82%
04-6 A4	1.175	2.2%	2.2%	2012q2	2012-13	0.17%	2%	89%
04-6 A5	1.175	13.1%	12.9%	2017q2	2016-17	1.60%	11%	86%
04-7 A4	1.153	3.5%	3.4%	2013q2	2012-13	0.17%	3%	93%
04-7 A5	1.153	14.6%	14.3%	2017q2	2016-17	1.60%	13%	87%
04-9 A4	1.134	4.8%	4.7%	2014q3	2014-15	0.99%	4%	78%
04-9 A5	1.134	16.5%	16.2%	2017q2	2016-17	1.60%	15%	88%
05-2 A4	1.126	5.3%	5.2%	2014q3	2014-15	0.99%	4%	80%
05-2 A5	1.126	18.0%	17.7%	2017q3	2016-17	1.60%	16%	89%
05-10 A3	1.076	3.3%	3.3%	2013q2	2012-13	0.17%	3.1%	93%
05-10 A4	1.076	13.7%	13.3%	2017q3	2016-17	1.60%	11.7%	85%
05-10 A5	1.076	12.3%	10.8%	2018q4	2018-19	1.96%	8.8%	72%
06-1 A3	1.072	2.3%	2.1%	2013q4	2012-13	0.17%	2.0%	86%
06-1 A4	1.072	14.5%	14.1%	2017q3	2016-17	1.60%	12.5%	87%
06-1 A5	1.072	19.0%	17.8%	2018q4	2018-19	1.96%	15.8%	83%
06-3 A3	1.072	3.4%	3.3%	2013q4	2012-13	0.17%	3.1%	93%
06-3 A4	1.072	14.1%	13.7%	2017q3	2016-17	1.60%	12.1%	86%
06-3 A5	1.072	25.1%	23.7%	2018q4	2018-19	1.96%	21.8%	87%
07-2 A1	1.041	2.1%	2.0%	2012q4	2012-13	0.17%	1.9%	89%
07-2 A2	1.041	11.6%	11.4%	2017q4	2016-17	1.60%	9.8%	84%
07-2 A3	1.041	22.6%	21.8%	2019q1	2018-19	1.96%	19.9%	88%
07-2 A4	1.041	15.2%	13.3%	2020q1	2020-21	2.53%	10.7%	70%
07-3 A1	1.039	2.5%	2.5%	2013q2	2012-13	0.17%	2.3%	91%
07-3 A2	1.039	11.8%	11.6%	2017q4	2016-17	1.60%	10.0%	84%
07-3 A3	1.039	22.5%	21.8%	2019q1	2018-19	1.96%	19.8%	88%
07-3 A4	1.039	14.6%	11.4%	2020q1	2020-21	2.53%	8.8%	61%

This table reports the near-arbitrage price gaps on SLABS, original and inflation-risk-robust, and compares them to price changes on a matched SBA price index. Near-arbitrage price gaps are computed from the difference between the minimum near-arbitrage lower bound value on a SLABS and its minimum market price during the crisis of 07-09, expressed as a percentage of the former.

positive side, my findings also support the claim that market participants faced important arbitraging frictions during the crisis and that the underpricing of SLABS is evidence in favor of the slow-moving capital explanation of arbitrage persistence.

2.6 APPENDIX

2.6.1 SBA PC PRICE INDEXES

Table 2.7:
SBA PC price indexes

Maturity	Time to maturity (as of 12/31/2008)	Price change
2012–2013	4	0.17%
2014–2015	6	0.99%
2016–2017	8	1.60%
2018–2019	10	1.96%
2020–2021	12	2.53%
2022–2023	14	2.50%
2024–2025	16	2.61%
2026–2027	18	2.79%
2028–2029	20	2.96%
2030–2031	22	3.34%
2032–2033	24	2.73%

This table presents price changes for SBA PC price indexes constructed with SBA PCs of various maturities. The first column reports the range of maturity years of the constituent SBA PCs that contribute to an index. The second column reports the time to maturity of the constituent SBA PCs, as of 12/31/2008. The third column reports the difference in price between two points i) the maximum price observed between 01/01/2010–10/11/2011, and ii) the minimum price observed between 11/10/2008–03/30/2009. The difference is expressed as a percentage of the minimum price.

Chapter 3

Securitization with Asymmetric Information: The Case of PSL-ABS

joint with Adam Ashcraft[1]

3.1 INTRODUCTION

Getting securitization right — retaining its potential benefits while correcting flaws that contributed to the financial crisis of 2007–2009, or flaws that could potentially contribute to a future crisis — is a common objective of market participants, policy-makers, and regulators. At its best, securitization is a financial intermediation process that can create low-risk, highly-rated, information-insensitive and liquid securities via pooling and tranching (DeMarzo (2005); Gorton and Metrick (2012)). It has been well documented that the underperformance of private-label mortgage-backed securities (MBS)[2] was a contributor to the crisis of 2007–2009. For example, it has been estimated that among "all mortgage-backed securities Moody's had rated triple-A in 2006, it downgraded 73% to junk" by April 2010.[3] This is suggestive of design flaws in the structuring of MBS.

This paper analyzes a subset of private student loan asset-backed securities (PSL-ABS) that performed relatively well in comparison to MBS.[4] To our knowledge, our paper is

[1] The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System. Any errors or omissions are the responsibility of the authors.

[2] Securities collateralized by mortgages without a guarantee against default from a government sponsored enterprise.

[3] According to estimates in "The Financial Crisis Inquiry Report" of the National Commission on the Causes of the Financial and Economic Crisis in the United States, published in 2011 (p. 122).

[4] In contrast to the massive downgrades from triple-A to junk among MBS, among the PSL-ABS that we analyze, no downgrades have pushed a triple-A rated ABS below the investment grade cutoff to date. If

the first to analyze informational frictions in the securitization of PSL and we focus on the adverse selection friction between a PSL-ABS issuer and investors. We focus on the adverse selection friction because some loan characteristics that strongly predict defaults are only coarsely disclosed (e.g. borrower credit score at time of securitization) and at least one strong predictor of default (share of college drop-outs) is not disclosed at all. In contrast to private-label MBS, PSL-ABS have experienced a relatively sustained issuance during the post-crisis period.[5] This provides additional motivation for taking a closer look at PSL-ABS.

When disclosures of pool characteristics to investors are sufficiently coarse a broad range of performance outcomes is possible. In such a scenario, the past performance of observationally–similar pools risks being a poor predictor of a pool's future performance. We quantify the extent to which a PSL-ABS issuer can engage in adverse selection, given its disclosure at issuance of PSL-ABS deals; this can help PSL-ABS buyers better assess the potential for adverse selection and encourage vigilance among investors for conditions that could encourage an issuer to begin engaging in adverse selection. Financial distress, a reduced reliance on the ABS market for loan funding, the intention to exit the ABS market, or some form of increase in managerial short-termism could each contribute to greater discounting of future reputational costs. When discounted reputational costs are sufficiently low relative to the benefits of adverse selection (raising funds from investors that exceed the expected value of the cash flows from a securitized pool), an issuer is likely to engage in adverse selection.

In contrast with MBS (Piskorski et al. (2015); Elul (2016); Agarwal et al. (2011); Berndt et al. (2010)), there are fewer agents involved in the supply chain of credit among the PSL-ABS that we analyze. The loan origination, ABS issuance and loan servicing activities are all integrated within the same firm. Furthermore, the vertically-integrated originator-issuer-servicer retains ownership of the residual cash flows from the securitized assets (another feature that differs from MBS). Residual cash flows refer to the cash flows generated by a securitized pool of loans after all ABS holders have been repaid (if any such cash flows remain). This kind of ownership was not generally present among MBS

one excludes PSL-ABS deals containing senior auction rate notes, the worst downgrade to date has been to an Aa3 rating (equivalent to an AA- rating). Including PSL-ABS deals containing senior auction rate notes, the worst downgrade to date has been to Baa2 (equivalent to a BBB rating).

[5]PSL-ABS, after experiencing a hiatus in issuance in 2008 and using the support of the Term Asset-Backed Securities Loan Asset Facility (TALF) for issuances in 2009–2010, have been issued multiple times per year during the post-crisis period. The volume of issuance between 2011–2013 was about 80% of its 2005–2007 level. In contrast, the issuances of private-label MBS between 2011–2013 represented less than 5% of its 2005–2007 level.

originated before the crisis.[6] These features may have contributed to mitigating the frictions resulting from the asymmetric information between the issuer of PSL-ABS and investors as well as the superior historical performance of PSL-ABS relative to MBS. An emerging literature provides empirical evidence that credit risk retention contributes to mitigating adverse selection (Begley and Purnanandam (2017)).

Slightly more broadly, our findings inform the understanding of securitization cases involving a firm that a) vertically integrates the loan origination, ABS issuance, and loan servicing activities, b) retains credit risk, and c) coarsely discloses or does not disclose to investors important predictors of loan losses. The majority of PSL-ABS issued before the crisis possessed all those features, as do nearly all PSL-ABS issued after the crisis. Insights from our findings could also become useful if securitization cases that have historically been different (e.g. subprime mortgages (Ashcraft and Schuermann (2008)) came to converge toward the securitization case that we analyze.

We make a methodological contribution by developing and employing a novel methodology to analyze the adverse selection friction between an ABS issuer and investors. To our knowledge, all prior empirical studies on the adverse selection friction between an issuer/broker/originator and investors/buyers have used data sets that either identify securitized loans and unsecuritized loans or allow for the construction of a measure of the probability of securitization (Berndt and Gupta (2009); Agarwal et al. (2012); Benmelech et al. (2012); Krainer and Laderman (2014); Adelino et al. (2016)). This is true of the papers on the topic of adverse selection listed here and everywhere else in this paper. The loan-level data set that we use does not identify securitized loans and unsecuritized loans, thus we develop a new methodology: we put ourselves in the shoes of an issuer of PSL-ABS at the time of the selection of the loans into a securitized pool. We use historical deals to parameterize our exercise and the selection of loans is constrained by the pool characteristics disclosed to investors on these deals. For each historical deal, we form one loss-maximizing pool and multiple "random" pools that each match the disclosed characteristics but differ in their level of cumulative pool losses. Our deal-specific estimated shifts are simply the percentage difference between the cumulative gross losses on the

[6] Pre-crisis, originators did not typically hold an equity position in MBS that contained the loans that they originated, and while issuers would usually hold an equity position at issuance, they would typically sell that equity position soon after. While the lack of ownership of equity by originators is observable from MBS deal documents and covered in (Ashcraft and Schuermann (2008)), the data on the sale of issuers' equity positions is more scarce. A subset of the equity positions in MBS were sold into Net Interest Margin (NIM) securities. Park (2011) reports that about $168 billion in NIM securities on subprime MBS deals were issued between 2004 and 2007, versus $1,309 billion in other types of securities on these deals. This suggests a very high rate of sale of equity positions into NIM securities since the book value of equity was typically less than 5% of asset value in subprime MBS deals.

loss-maximizing pool and the average among random pools.

Previewing our results, we find that the PSL-ABS issuer that we analyze could increase pool losses by 6%–20% among pre-crisis deals and 16%–36% among post-crisis deals while still matching the disclosed pool characteristics. This is achieved purely by exploiting the coarseness of the disclosures — specifically, by jointly overrepresenting unseasoned loans in the low credit score region and overrepresenting seasoned loans in the high credit score region. The magnitude of these shifts is material and could result in adverse outcomes for investors (credit rating downgrades or market value loss).

The rest of this paper is organized as follows. In Section 3.2, we review the related literature, mainly on the securitization of mortgages, and use our institutional knowledge to adapt prior findings on MBS to the case of PSL-ABS. This review aids the interpretation of our estimates for potential adverse selection as lower bounds. In Section 3.3, we introduce the data that we use in greater detail. In Section 3.4, we quantify the extent of adverse selection that can be achieved via the sources of asymmetric information that we can exploit. We present concluding remarks in Section 3.5.

3.2 RELATED LITERATURE AND LOWER BOUND INTERPRETATION

At a high level, our approach assumes that all loans have a similar expected return at origination, which requires that the interest rate be set in such a way as to fully compensate for the risk of loss, as expected at origination. However, divergence in expected returns occurs over time based on the information acquired by the vertically-integrated servicer-issuer-originator. The issuer acquires this information between the origination of a loan and the time at which it is considered for securitization, but little of this information is disclosed to investors; we analyze this important source of asymmetric information.[7] Due to data limitations, we only quantify a fraction of the potential for adverse selection in PSL-ABS.

One such piece of information that is acquired after the loans are originated and which goes undisclosed to investors is enrollment status. Prior studies (Knapp and Seaks (1992); Mezza and Sommer (2015)) have estimated that a college drop-out is between 2.2

[7]The requirement that loans be fully disbursed before they become eligible for securitization and the general use of multiple disbursement on private student loans creates a minimum delay between the origination of loan and their eligibility for securitization. Empirically, delays between origination and securitization are significantly longer than the 3 quarters that generally separates the partial disbursements on a loans, with weighted average loan age at securitization *WALAS*, ranging between 4 and 11 quarters among the PSL-ABS deals we analyze.

and 9 times more likely to default on student loans than a college graduate, yet PSL-ABS issuers do not disclose the enrollment or degree-completion information they possess. Particularly relevant for our analysis is evidence presented by Mezza and Sommer (2015) that college drop-outs, even after controlling for credit score immediately before entering into repayment, are more than twice as likely to become delinquent on their loans.[8] Results that control for credit score best complement our analysis because summary statistics on credit score at securitization are disclosed to PSL-ABS investors and we match this disclosure in our exercise. However, we lack critical pieces of information (e.g. the distribution of private student loan dollars across school type and between college graduates versus college drop-outs) that would allow us to quantify how much the overrepresentation of college drop-outs would add to our shifts. Nonetheless, it is unquestionable that the effect would be more adverse to investors, and this supports to a lower bound interpretation of our estimated shifts.

Maintaining our focus on the information that is acquired after a loan is originated, a vertically-integrated originator-issuer-servicer has the opportunity to use data-mining techniques on the loans it services (both those on its books and those in the pools collateralizing the PSL-ABS it issues) to uncover predictors of default that are not disclosed to investors. While disclosed credit bureau scores (e.g. FICO credit scores) are a borrower-level measure, an originator-issuer-servicer can use the data at its disposal to develop loan-level adjustments based on observations specific to a loan. For example, Aiello (2016) finds that mortgage borrowers who make their first six payments at least a day prior to the due date are 14.8% percentage points less likely to become delinquent.[9] Payment prior to due date is thus highly correlated with non-default, observable to an issuer-servicer and never disclosed to investors; this combination makes it a clear avenue for potential adverse selection. The hypothesized presence of similar delinquency predictors for PSL further supports interpreting our estimated shifts as lower bounds on the degree of feasible adverse selection.

While we focus on default risk and the literature on MBS has mainly focused on default, selection with respect to prepayment risk can also affect the performance of a pool. In our exercise, we match both the disclosed balance-weighted average credit score at securitization and the disclosed distribution of loan balance across intervals of credit score at origination. We maximize pool losses by mixing particularly bad loans that experience decline in credit score after their origination with loans that experience

[8]Figure 4 of Mezza and Sommer (2015) summarizes their estimates of differential delinquency risk between college graduates and drop-outs controlling for credit score.

[9]Table III of Aiello (2016) shows that these results are obtained after controlling for FICO credit scores.

modest increases in credit score. This is done ignoring prepayment risk. If the good loans that are mixed with the particularly bad loans in the loss-maximizing pool were selected based on a high likelihood of prepayment, they would generate a short stream of revenues that would have a limited impact at offsetting the selection of the particularly bad loans into a securitized pool. Agarwal et al. (2012) finds evidence of adverse selection on prepayment risk in the selection of securitized mortgages. This again, intensifies the argument for the interpretation of our estimated shifts as lower bounds.

Conceptually, our exercise does not require that loans with a negative Net Present Value (NPV) be originated. Loans that undergo a downward migration in credit score between their origination and their consideration for securitization (and no observation that offsets this negative signal (e.g. staying current when positive payments are due)) provide an ample supply of loans with a negative NPV at securitization. In a nutshell, we determine which of these declining-NPV loans to select and which loans among those experiencing increases in scores to mix them with so as to maximize pool losses while matching the disclosed characteristics on credit scores.

By combining our institutional knowledge of PSL-ABS with our review of the literature, we identified multiple channels that contributed or that could contribute to the origination of PSLs with a negative NPV. The origination of such loans increases the supply of loans with a negative NPV available at the time of forming a securitized pool. Negative NPV loans that do not require downward migration in credit score, or that remain negative NPV loans despite an upward migration in score, would add to the supply of negative NPV loans that we could exploit. This is yet another reason to interpret our estimated shifts as lower bounds.

First, among contributors to the origination of private loans with negative NPV was the willingness of issuers of PSL to originate a moderate volume of private student loans that were expected to be unprofitable in order to improve their relationships with some schools and to secure the origination and servicing of larger federal (government-guaranteed) student loan volume.[10] Competition among originators of government-guaranteed loan for a top placement on the list of "Preferred Lenders" that schools provided to students was fierce.[11] For a small volume of private student loans (at most 15%) the "subsidized

[10] Senator Durbin raised awareness of "pay-to-play" or "subsidized-private-loans-for-federal-loan-volume" arrangements in 2007 (Congressional Record (2007)).

[11] Many lenders used unethical inducements. The unethical inducements were significant enough that New York state's attorney general Cuomo's "nationwide investigation into the student loan industry [...] resulted in agreements with twelve student loan companies, including the eight largest lenders in America — Citibank, Sallie Mae, Nelnet, JP Morgan Chase, Bank of America, Wells Fargo, Wachovia, and College Loan Corporation" (https://ag.ny.gov/press-release/cuomo-announces-settlement-student-loan-company). The Student Loan Sunshine Act ("H.R. 890

rate" offered on private loans was so low relative to the risk of default that lenders would negotiate a risk-sharing agreement with schools.[12] The presence of the 90–10 rule, which mandates that no more than 90% of the loan volume at a for-profit school comes from federal loans, created strong incentives for for-profit schools with exceptionally high default rates to negotiate such agreements with originators of private loans. The risk-sharing agreement ensured continued access to federal loan for their current and prospective students. The risk-sharing transfers from the schools to the lender would not be transferred to the PSL-ABS trusts. Therefore, from the point of view of a PSL-ABS investors, the overrepresentation of loans covered by a risk-sharing agreement would have had an adverse effect on investors.[13]

A second potential contributor to the origination of negative NPV private loans, or loans with relatively lower expected values, are the fair lending laws. Many studies have shown borrower race to be a statistical predictor of default, even after controlling for credit score (Board of Governors of the Federal Reserve System (2007); Jiang et al. (2014a)). But differential treatment in the approve/decline decision or the setting of the interest rate based on race is forbidden by fair lending laws. To our knowledge, however, no law prevents issuers from overrepresenting borrowers belonging to a race group associated with higher delinquencies (and for which the originator-issuer was unable to set a compensating higher interest rate because of fair lending restrictions) in a securitized pool of loans.[14]

A third potential contributor to the origination of negative NPV private loans is the competition for origination volume, especially the competition from non-bank lenders

— 110th Congress: Student Loan Sunshine Act." www.GovTrack.us. 2007. May 14, 2017 https://www.govtrack.us/congress/bills/110/hr890) was passed in Congress under expedited rules on May 9, 2007 with only three dissenting votes. It aimed to make unethical inducements practices illegal. Provisions of the Student Loan Sunshine Act were incorporated in the Higher Education Opportunity Act, which was signed into law on Aug 14, 2008.

[12]Risk-sharing agreements would take two main forms, either discount loans, which required that a school pays $0.20-$0.30 to a lender for every $1 of private student loan disbursed to its student (https://www.sec.gov/Archives/edgar/data/1286613/000114036111016427/form10k.htm, or recourse loans, which required that schools purchase back loans if default exceeded some threshold http://securities.stanford.edu/filings-documents/1050/ESI00_01/2014115_r01c_13CV01620.pdf.

[13]The federal government guaranteed-loan program ended in June 2010 and many PSL originators terminated relations with schools involved in "subsidized-private-loans-for-federal-loan-volume" arrangements by the end of 2009. For example, in SLM's 10-K for 2009, the company states "This significant level of provision expense, compared with prior and subsequent quarters, [sic] primarily related to the non-traditional portion of the Company's Private Education Loan portfolio which the Company had been expanding over the past few years. The Company has terminated these non-traditional loan programs [...]".

[14]We obtained copies of SLM's pre-crisis application forms for private loans. No information on race is collected on these forms. However, the technology to statistically categorize a borrower into a race group based on his name and ZIP Code exists; this statistical categorization could be used to overrepresent certain race groups into securitized pools.

that are subject to a different set of regulations.[15] Lenders are sometimes willing to incur losses in segments of the market where competition is the strongest in order to remain a "full-spectrum" lender that offers competitive rates to all borrowers and to potentially squeeze out competition. A good example of a fiercely competitive non-bank lender prior to the crisis is MRU Holdings. It issued private student loans under the MyRichUncle brand between 2005 and 2008 and became the fourth-largest provider of student loans in 2007. It did not accept deposits and was funded with bank loans. MRU Holdings filed for bankruptcy in February 2009. Post-crisis examples of non-bank lenders that attempt to "cream-skim" traditional lenders by focusing on students pursuing high-earning majors or programs at institutions with superior job placement include FinTech companies such as SoFi.[16]

Finally, our exercise assumes that the disclosed information properly represents the content of securitized pools. A series of recent papers have shown this assumption to be violated on a widespread scale in the MBS market (Piskorski et al. (2015); Griffin and Maturana (2016)).[17] Particularly relevant for the case of PSL-ABS is the evidence presented by Griffin and Maturana (2016) that both originator and MBS issuer fixed effects explain a particular type of misrepresentation: unreported second liens. This constitutes evidence that both false representation by originators to MBS issuers (with MBS issuers failing to perform their due diligence and passing the false information on to investors) and truthful representation by originators followed by false representation by MBS issuers were occurring. The latter form of misrepresentation is most relevant for the PSL-ABS that we analyze because of the vertical integration of the loan origination and ABS issuance activities within the same firm. Therefore, the lack of an adequate governance structure to verify that the disclosed information properly represents the content of a securitized pool is another channel that could add to the shifts that we compute and thus contribute to the lower bound interpretation of our shifts.

[15]The portion of SLM's funding that was obtained via deposits was small prior to the crisis, but the presence of Sallie Mae bank (a bank chartered in the state of Utah) within the SLM Corp. holding partially subjected SLM to the regulation that applies to banks. The portion of SLM's funding that comes from deposits has increased over time, especially after SLM was fully privatized in December 2004.

[16]Weiss, Miles. "Harvard Graduates Targeted for Loans With Alumni-Backed Funds." *Bloomberg News*. Bloomberg. Web. 12 Dec. 2012.

[17]These papers document widespread misrepresentation in the securitization of residential mortgages. These papers were preceded by others that were narrower in scope (Ben-David (2011); Jiang et al. (2014a); Garmaise (2015); Carrillo (2013).

3.3 DATA DESCRIPTION

We first describe pool-level characteristics of historical PSL-ABS deals. These characteristics are used to parameterize our computation of shifts in gross pool losses in Section 3.4. Second, we describe the loan-level performance data. Two important components of our computation of shifts in pool losses rely on this loan-level data. First, it provides the distributions of loan balance across credit scores at the time of securitization and across loan seasoning groups. Second, we use it to estimate the relationships between gross loss, credit score, and loan seasoning.

3.3.1 POOL CHARACTERISTICS DATA

We hand-collected data on pool-level characteristics of historical deals from prospectuses. All the deals that we analyze were issued by SLM Corporation[18] and have a consistent set of pool characteristics disclosed in their prospectuses. SLM is a vertically integrated originator-issuer-servicer and it held the residual claim on cash flows from the securitized pools that collateralize these deals. Table 3.1 presents the proportion of non-cosigned loans in historical pools and various statistics related to credit scores on the non-cosigned portion of pools. The table focuses on credit score statistics for the non-cosigned portion of pools because our computations of shifts in gross pool losses in Section 3.4 are obtained from samples of non-cosigned loans. Shifts computed on the non-cosigned portion of pools can confidently be treated as upper bounds on the co-signed portion of pools: in preliminary regressions using the loan-level data with credit score of the co-signer at securitization as the independent variable, we found the level of default and the sensitivity to credit score to be significantly lower among loans with a co-signer than without.

Prospectuses have only limited credit score disclosure for the time of securitization, but more detailed score information for the time of origination. At securitization, there is only the proportion of pool balance with a score below 630 and the balance-weighted mean credit score. At origination, a more detailed grid of scores is disclosed. Appendix 3.6.1 provides an example illustrating the granularity of the disclosure. We match these disclosed characteristics when computing shifts in pool losses.

Table 3.1 also presents the balance-weighted average loan age at securitization, or (*WALAS*). Loan age is simply the time elapsed since the origination of a loan, in quarters. Empirically, there is a lag between the origination of PSL and their securitization. One factor that contributes to this lag is the common practice of splitting the disbursement of

[18] SLM had a Government Sponsored Enterprise (GSE) subsidiary, Sallie Mae, until in December 2004. All the deals that we analyze were issued after the dissolution of the GSE subsidiary.

PSL across more than one semester and the requirement that loans be fully disbursed before they become eligible for securitization. Other factors contribute to lengthening the lags. The *WALAS* reported in Table 3.1 range between four and six quarters among the pre-crisis PSL-ABS deals and between seven and eleven quarters among post-crisis deals.

Let ψ^o denote credit score at origination and ψ^s denote credit score at securitization. The deals that we analyze have trivial amounts to no loans with $\psi^o < 630$, but the $\psi^s < 630$ is a region that offers greater potential for selection, as we will see in Section 3.4.3. A longer *WALAS* means more time for loans with $\psi^o \geq 630$ to transition to $\psi^s < 630$, thus deals with a longer *WALAS* generally have a greater proportion of loans with $\psi^s < 630$ and this contributes to the potential for larger shifts in gross losses.

Table 3.1:
Pool characteristics of historical deals

Deal	*WALAS* (qtr)	Non-cosigned	Credit score (Non-cosigned) At origination $\psi^o < 630$	At origination $\overline{\psi^o}$	At Securitization $\psi^s < 630$	At Securitization $\overline{\psi^s}$	*Pick*
SLM 2005-A	5	52%	0.16%	696	15%	675	3.24
SLM 2005-B	4	49%	0.20%	697	14%	681	3.21
SLM 2006-A	5	49%	0.20%	698	14%	681	3.84
SLM 2006-B	5	42%	0.08%	697	11%	681	3.68
SLM 2006-C	6	52%	0.39%	696	17%	678	6.50
SLM 2007-A	4	42%	0.16%	694	14%	681	6.16
SLM 2009-A	10	38%	0%	701	24%	675	6.53
SLM 2009-B	7	37%	0%	703	19%	680	5.86
SLM 2009-C	7	37%	0%	704	17%	684	11.60
SLM 2010-A	7	28%	0%	714	11%	691	8.87
SLM 2013-A	11	20%	0%	703	22%	683	12.73
SLM 2013-B	10	20%	0%	702	21%	684	13.26

This table presents the pool characteristics of historical deals. The first column reports deal name composed of the issuer followed by the year of issuance. The second column reports the *WALAS* for all loans in the pool that collateralizes deal. The third column reports the non-cosigned proportion of pool balance. The fourth to seventh columns report pool characteristics disclosed to investors and related to the credit scores of the non-cosigned portion of a pool. The fourth and fifth (sixth and seventh) columns report the proportion of pool balance with $\psi^o < 630$ ($\psi^s < 630$) and the balance-weighted average $\overline{\psi^o}$ ($\overline{\psi^s}$). The last column presents the *pick* parameter, which represents the inverse of the ratios of the balance of the non-cosigned portion of a pool over the balance of non-cosigned loans outstanding and not yet securitized by SLM.

The *pick* parameter, also reported in Table 3.1, is the inverse of the ratio of the non-cosigned pool balance over the balance of non-cosigned loans outstanding and not yet securitized by SLM.[19] For example, deal 2005-A has a *pick* parameter of 3.24; this means that the pool was formed by securitizing 1 out of every 3.24 outstanding loans that had not yet been securitized by SLM. Holding everything else constant, the larger *pick* is, the greater the potential to shift gross losses between a baseline pool without selection and a pool with maximum adverse selection. Appendix 3.6.2 provides additional details on the type of data used to construct the pool-level parameters *pick* and *WALAS*.

3.3.2 LOAN-LEVEL PERFORMANCE DATA

The student loan performance data is from the Consumer Credit Panel (CCP) data set of the Federal Reserve Bank of New York. The CCP data set contains quarterly longitudinal data and covers a representative sample of U.S. consumers with a credit bureau file: it contains 5% of consumers for whom Equifax, one of three major national reporting agencies, possesses a credit file.[20] The reporting on consumers in the CCP data set occurs at various levels of aggregation on different debt products. The reporting on student loans is at the loan level. Appendix 3.6.3 contains a table that lists the variables that we use from the CCP or derive from it.

The raw CCP data identifies student loans, but does not indicate whether a student loan is private or federal. Carving out a clean subsample of private student loans from the CCP is essential because the differences in the underwriting process between private and federal loans[21] lead to significantly different performance. Lucas and Moore

[19]This is a simplified description of the *pick* parameter. Truly, the *pick* parameter is the inverse of a weighted average of vintage-specific ratios of the non-cosigned pool balance over the balance of non-cosigned loans outstanding and not yet securitized. Weights are determined by the distribution of pool balance across vintages. In order to properly estimate vintage-specific time-series of balances of non-cosigned loans outstanding and not yet securitized, we had to collect data on the non-cosigned pool balance of *all* PSL-ABS deals that contain loans from vintages that are also present in the deals that we analyze.

[20]Additional information about the construction of the CCP can be found in Lee and Van der Klaauw (2010).

[21]The criteria to qualify for federal loans in the post-2000 era are minimal: an applicant must i) submit a Free Application for Federal Student Aid (FAFSA), ii) have no current delinquency on federal loans, and iii) attend an institution that is eligible (Title IV institution). There is generally greater stringency in the underwriting process for private loans. For example, a freshman who satisfies conditions i)-iii) would be approved for a federal loan; however, for 90% of postsecondary programs, the same freshman would need a co-signer in order to obtain a private loan and the credit score of the co-signer would need to be sufficiently high. As a sophomore, meeting the same criteria required of the freshman would lead to an approval for a federal loan; however, the same sophomore would need a sufficiently thick credit history in order to have a credit score and this credit score would need to be sufficiently high in order to be approved for a private loan, otherwise, the sophomore would need a co-signer, and if he is unable or unwilling to find a co-signer with a sufficiently high credit score, then, his private loan application would be declined. Hard data on schools where private loans are offered are not readily available, but anecdotal evidence suggests that 1) private loans are offered at many of the same schools that are eligible for federal loans, but not all of them, for example, private loans may not be offered at some 2-year public colleges with low cost of attendance and high dropout rates, and 2) private loans are likely offered at a small number of schools that are not

(2010) estimate that gross default rates among federal loans are 1.8% of outstanding principal per annum, versus 1% for private loans. These unconditional differences do not necessarily imply conditional differences (differences in the relationships between gross losses on loans, credit score, and loan seasoning), but they make them very likely. The quantification of these relationships is a key building block for our results. In addition to the likely differences in the estimated relationships between private and federal loans, the distribution of loan balances across credit scores and levels of seasoning would also very likely be different. Since these distributions are another building block for our results, it was critical to obtain them from samples that only contained private loans.

The best means to identify private loans that we have discovered relies on the fact that all federal loans originated after 2006Q3 have a fixed interest rate, whereas more than 95% of private loans originated between 2006Q3 and 2011Q2 have a variable rate (Consumer Financial Protection Bureau (2012)). The interest rate on loans is not directly observable in the CCP data, and imputing an interest rate based on changes in balance and payment due proved too noisy to cleanly categorize loans as having fixed or variable interest rate. However, starting in 2010Q1 a change in the reporting by a set of loan servicers, unrelated to balance and payment due, permitted us to consistently identify loans with a variable rate. A series of check on the properties of the loans that we identified as private lead us to believe that they would be representative of the loans originated by SLM.

Our computations of shifts in gross pool losses rely on the construction of deal-specific samples of private student loans. These are subsamples of the student loans we identified as private in the CCP. Summary statistics of the deal-specific samples can be found in Table 3.2 alongside historical pool characteristics. These statistics are for loans without a co-signer, as was the case for credit score in Table 3.1. This focus on non-cosigned loans will apply for all subsequent statistics reported in this paper, unless otherwise noted.

Table 3.2 shows that the origination quarters of loans in the deal-specific samples generally fall outside the range of vintages present in historical deals. This unexpected feature arises because the largest and cleanest sample of private loans we could identify from the CCP data requires that loans be originated after 2006Q2 and that their reporting continue until at least 2010Q2. For pre-crisis deals, we chose to use origination quarters 2008Q2-2009Q1. This reflected a compromise between loan volume, the length of the performance window and survival bias.[22] For post-crisis deals, we could obtain sufficiently

eligible for federal loans. The anecdotal evidence on the latter comes from schools that lost their eligibility for federal loans, because of excessively high default rates, and responded by offering institutional loans and enticing a private lender to lend to their students via a risk-sharing agreement with the lender.

[22]For the pre-crisis deals, using loans originated in 2008Q3-2009Q2 instead of 2008Q2-2009Q1 would have eliminated survival biases, mainly in the form of very small volume of missing prepaying loans, for

Table 3.2:
Loan characteristics of deal-specific samples

Deal				Deal-specific sample							
						Full sample			Subsample: $\psi^o \geq 630$		
Name	*WALAS*	Vintage	$\psi^o < 630$	Qtr orig.	Size	$\psi^o < 630$	$\psi^s < 630$	$\overline{\psi^s}$	$\psi^s < 630$	$\overline{\psi^s}$	σ_{ψ^s}
2005-A	5	03-04	0.16%	08Q2-09Q1	7500-37500	16%	21%	671	12%	687	62
2005-B	4	04-05	0.20%	08Q2-09Q1	7500-37500	16%	20%	671	11%	687	59
2006-A	5	04-05	0.20%	08Q2-09Q1	7500-37500	16%	21%	671	12%	687	62
2006-B	5	04-06	0.08%	08Q2-09Q1	7500-37500	16%	21%	671	12%	687	62
2006-C	6	04-06	0.39%	08Q2-09Q1	7500-37500	15%	22%	670	14%	686	64
2007-A	4	05-06	0.16%	08Q2-09Q1	7500-37500	16%	20%	671	11%	687	59
2009-A	10	06-07	0%	07Q3-08Q2	14500-72500	23%	34%	649	25%	668	81
2009-B	7	07-08	0%	08Q2-09Q1	7000-35000	15%	23%	669	16%	683	65
2009-C	7	07-08	0%	08Q2-09Q1	7000-35000	15%	23%	669	16%	683	65
2010-A	7	08-09	0%	08Q2-09Q1	7000-35000	15%	23%	669	16%	683	65
2013-A	11	07-12	0%	07Q2-08Q1	15000-75000	25%	35%	647	26%	666	83
2013-B	10	07-12	0%	07Q3-08Q2	14500-72500	23%	34%	649	25%	668	81

This table shows some properties of historical deals alongside summary statistics for the deal-specific samples used to form mimicking pools, loss-maximizing and random, in Section 3.4. The second and fourth columns are reproduced from Table 3.1 to facilitate understanding of the deal-specific samples. The third column reports ranges of vintages, where both endpoints of every range represent 10% or more of pool balance and the vintages within the range collectively represent more than 80% of pool balance. The fifth to twelfth columns report on the deal-specific samples. Given the low proportion of loans with $\psi^o < 630$ in historical deals, the properties of the subsamples with $\psi^o \geq 630$, which are reported in the tenth to twelfth columns are most relevant. The properties in the seventh to ninth columns apply to full samples and are provided for comparison purpose. For both the full samples and the $\psi^o \geq 630$ subsamples, the fifth column reports the range of quarters of loan origination. Deal-specific full samples include loans that were originated during the quarters reported in the fifth column and that were eligible for securitization when *WALAS* quarters old. The sixth column reports ranges of sample size for the full samples, a form of reporting that adds a layer of anonymization about the lenders that are included in the samples. The seventh to ninth columns respectively report, for the full samples, the proportion of pool balance with $\psi^o < 630$, the proportion of pool balance with $\psi^s < 630$ and the balance-weighted average ψ^s. Finally, focusing on the subsamples with $\psi^o \geq 630$, the tenth to twelfth columns respectively report the proportion of pool balance with $\psi^s < 630$, the balance-weighted average ψ^s and the balance-weighted standard deviation for ψ^s. Raw loan-level data source: FRBNY CCP/Equifax.

large loan volume and sufficiently long performance window without having to accept any survival bias, thus we chose to do so.

Finally, it should be noted that the CCP data set does not contain a variable indicating whether a loan was securitized or not. This is in contrast with much of the empirical literature on the securitization of residential mortgages. Endowed with such an indicator variable, many researchers (Elul (2015); Jiang et al. (2014b)) have analyzed the historical differences in performance between securitized and non-securitized mortgages. If we were endowed with such an indicator variable, issuer-specific private loans data and a sample covering all the vintages present in a deal, we could have conducted a similar analysis, which would have been complementary to ours. This complementary analysis would have allowed us to determine whether PSL-ABS issuer historically engaged in adverse selection or not.

However, our framework to quantify the full possible extent of adverse selection, which does not require a securitization indicator, would remain valuable. A lack of differences in historical performance would not address whether and to what extent an issuer could engage in adverse selection in the future on deals that are observationally similar to the historical ones. This is particularly relevant because incentives to engage in adverse selection can vary over time. Furthermore, finding historical evidence of adverse selection would not necessarily imply that an issuer was engaging in adverse selection to its fullest extent, whereas this could be a possibility in the future. To sum up, a securitization indicator, along with the other conditions permitting a complementary analysis, would not have fundamentally changed the approach we took to answer our research question.

3.4 SHIFTS IN POOL LOSSES VIA SELECTION

The first step in the computation of a shift in pool losses consists of forming a number of pools by randomly selecting loans while matching a set of pool characteristics disclosed to PSL-ABS investors. Additionally, we impose that pools also match a distribution of loan seasoning. The empirical derivation of the distribution of loan seasoning will be

deal 2006-C and reduced it for all other pre-crisis deals. However, using this later origination window would have shrunk the aggregate balance in the deal-specific samples by about 20%. Using loans originated in 2008Q4-2009Q3 instead of 2008Q2-2009Q1 would have eliminated survival biases for deal 2005-A, 2006-A, 2006-B and 2006-C, and reduced it for other pre-crisis deals. However, using this later origination window would have shrunk the aggregate balance by more than 60%. In addition, non-cosigned loans from 2008Q2 belong to an origination regime that is more representative of the pre-crisis period, with looser underwriting standards, whereas loans originated in 2009Q2 are more representative of the post-crisis period, with tighter underwriting standards.

explained later in this section. The second step is to form a pool that also matches the stated pool characteristics, but maximizes the expected gross pool losses. We call this pool the loss-maximizing pool. It represents the preferred securitized pool of a myopic originator-issuer who ignores or infinitely discount reputational costs.[23] By maximally leveraging the undisclosed information, the worst possible performing pool is securitized and the best performing residual pool is retained on the originator-issuer's balance sheet.

We repeat the exercise with various sets of parameters, which are chosen to proxy for 12 historical deals[24]. The key output of interest, which we call *shift*, consists of the difference in cumulative gross losses (CGL) between the loss-maximizing pool and the average CGL of the random pools with the same disclosed characteristics. Shift is expressed as a percentage of the average CGL of the random pools. For example, a loss-maximizing pool having an expected CGL of 19.3% means that the balance of the loans that enter default (at the time of entry into default) divided by the initial pool balance equals 19.3%. The average CGL among the corresponding random pools is 16.0%, so we report a shift of 20.6%.[25] This means that the CGL for the loss-maximizing pool is 20.6% higher than an investor would expect at deal issuance based on historical performance of observationally equivalent pools, assuming an historical performance without selection and no anticipation of selection on the issued deal.

3.4.1 MATCHED POOL CHARACTERISTICS AND EMPIRICAL CONSTRAINTS

Empirically, disclosure on credit score at *securitization* (ψ^s) is limited to two summary statistics: the balance-weighted average credit score and the proportion of pool balance with a credit score below 630. In our exercise, both the loss-maximizing pool and the corresponding random pools match those two summary statistics. The disclosure on credit score at *origination* (ψ^o) is more detailed. Appendix 3.6.1 shows side-by-side, for an example deal, the empirical distribution of pool balances across ψ^o intervals and the distribution over the coarsened grid that we match when computing shifts in pool losses.

Empirically, issuers also disclose the distribution of pool balance across vintages, which combined with the date of securitization, produces a distribution of pool balance across loan age at securitization. When creating the sample from which loans can be selected

[23]Myopia could be caused by managerial short-termism due to improper alignment of their incentives with those of shareholders and heavy discounting, practically equivalent to infinite discounting, could be rational for the managers of a firm that is close its bankruptcy boundary or intending to exit the ABS market.

[24]SLM had issued 6 pre-crisis deals after the dissolution of its Government Sponsored Enterprise subsidiary, Sallie Mae, in December 2004. We matched that deal count with 6 post-crisis deals that possessed pool characteristics suggesting greater potential for loan selection.

[25]((19.3% - 19.3%)/16.0%) = 20.6%.

in a deal-specific exercise, all loans must be eligible for securitization, mainly meaning no more than 30 days delinquent[26], when their loan age equals the balance-weighted average loan age of the pool under analysis.

In our exercise, all loans in a deal-specific sample have the same loan age. Despite a common loan age, loans can differ in their seasoning. Unlike other consumer loans, which have a positive required payment in the month that follows their disbursement, the required payments on student loans can be zero for months and often years after their disbursement. Following the disbursement of a student loan, as long as a student maintains sufficient enrollment (usually more than half time) then a student can choose required payments of \$0. The vast majority of students choose to do so. There is also a grace period (usually 6 months in duration) that follows the disenrollment, due to drop-out or graduation, during which the required payment on a loan can remain \$0.

If one tries to use the longest possible window of post-securitization performance so to estimate expected gross loss (EGL) on loans, there can be large differences in EGL between the loans that enter repayment soon after securitization or are already in repayment at securitization versus loans that are still years away from entering repayment when they are securitized.[27] This is why we estimate the relationship between EGL and credit score at securitization separately for different levels of seasoning.

Seasoning at the time of consideration for securitization ($seas^s$) is constructed as follows. Let $\Phi_{(p>0,\, t)}$ be an indicator function that takes value 1 if the payment due on a loan in a quarter is positive and value 0 otherwise. Let us define four time indicators: t_p for the first quarter with a positive payment due, t_o for the quarter of origination, t_s for the time of consideration for securitization and τ for the end of the post-securitization performance window. Let ℓ denote the length of the performance window. We construct

[26]Some deals treat loans 30-60 days past due as also eligible. The full disbursement of the loan is another eligibility criterion. Loans with $\psi^o < 630$ do not appear in post-crisis deals; we treat this has an eligibility criterion. For pre-crisis deals, when forming pools, loans $\psi^o < 630$ are eligible, but when estimating relations between gross loss and loan characteristics at securitization, we exclude them because this leads to a more representative sample.

[27]We use the term "at securitization" in two ways. First, the more intuitive use, which is to indicate the actual time of securitization when discussing characteristics of loans found in historical pools. Second, to indicate the point in time of potential securitization in i) our exercises of selection of pools and ii) our estimation of relationship between gross loss (over a potential post-securitization period) and loan characteristics (at the time of potential securitization). We omit the "potential" qualifier for brevity in the second case. In a deal-specific exercise, all loans from the quarters of origination selected to form the deal-specific sample (listed in Table 3.2) and that are eligible for securitization are all used to estimate the relation between gross loss and its predictors.

$seas_s$ using:

$$seas^s = \sum_{t_o}^{t_s} \Phi_{(p>0,\, t)}, \text{if } t_p \leq t_s, \tag{3.1}$$

$$seas^s = -(t_p - t_s), \text{if } t_p > t_s, \tag{3.2}$$

where we set $seas^s = -(\ell + 1)$ if $\sum_{t_o}^{\tau} \Phi_{(p>0,\, t)} = 0$.[28]

Loans belonging to relatively narrow ranges of seasoning are grouped together. We chose to group loans into four seasoning groups for each deal, with cutoffs that vary with the *WALAS* of deals. Given our sample size, this approach gives reasonable opportunities for significant differences in the relationship between EGL and ψ^s to be found between these groups. The classification of loans into seasoning groups is also a component of our approach to separate two types of shifts: i) shifts in pool losses that are likely to persist over the life of a pool, and ii) shifts in the timing of losses (especially higher losses during the in-sample window of post-securitization performance) without necessarily increasing losses over the life of a pool. To isolate the shifts in pool losses that are likely to persist over the life of a pool, we follow a three-step process.

First, for every deal-specific sample of loans, all loans are categorized across seasoning groups and we run our algorithm to form random pools that match the disclosed pool characteristics related to credit scores (both at securitization and at origination). We then compute the average proportions of pool balance in the four seasoning groups across the random pools. Appendix 3.6.4 presents these average proportions for each deal. Second, and this will be explained in greater details in Section 3.4.2, we estimate the relationship between gross loss and credit score separately for each seasoning group while compensating for the underexposure to default risk of the less seasoned loans (especially those that had not yet entered repayment at t_s). Third, we form loss-maximizing and random pools that match the proportion of pool balance across seasoning groups obtained in the first step, and use them to compute shifts in gross pool losses.

An additional set of constraints that enter our pool formation algorithm are the aggregate balance of loans at *clusters* in deal-specific samples. We form clusters of loans with identical ψ^s, common ψ^o interval (see Appendix 3.6.1 for an example of a coarsened grid of ψ^o intervals) and a common seasoning group. When forming pools, the balance from any cluster cannot exceed the aggregate balance available at that cluster in the deal-specific sample of loans eligible for securitization. Appendix 3.6.5 provides a

[28]Since we use information revealed after t_s to construct t_p for loans with $t_p > t_s$, our exercise can be interpreted as assuming perfect foresight with respect to the time it takes for loans with $\sum_{t_o}^{t_s} \Phi_{(p>0,\, t)} = 0$ to enter repayment.

comprehensive and mathematical presentation of the constraints on pool formation that the issuer faces in our exercise.

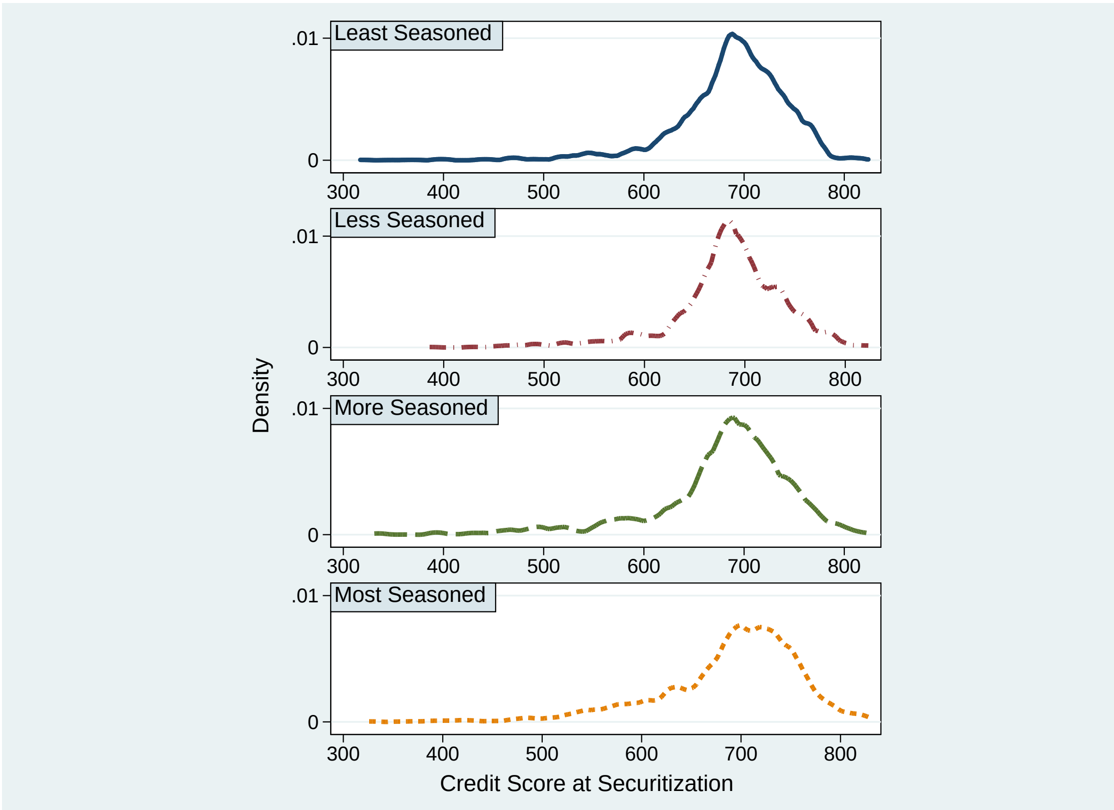


Figure 3.1:
Density of loan balance across credit scores at securitization
This figure shows the marginal densities of balance of student loan across credit scores at securitization by seasoning groups. The densities are those from the sample specifically used for deal 2006-C, which is a sample of loans that were eligible for securitization 6 quarters after their origination. The densities are for the sample obtained after loans with $\psi^o < 630$ were excluded, which is the most relevant sample, considering that loans with $\psi^o < 630$ only represent 0.39% of the balance in pool 2006-C. In contrast, without the exclusion, the aggregate balance of loans with $\psi^o < 630$ would represent 15% of the aggregate balance in the deal-specific sample and greater densities would be present in the low ψ^s regions. Raw data source: FRBNY CCP/Equifax.

Figure 3.1 presents, separately for each seasoning group, the marginal distribution of loan balance across credit score at securitization, for deal 2006-C. To give an example of how the empirical densities of a deal-specific sample could affect the computation of shifts in pool losses, consider the following. Let expected gross losses (EGL) be relatively higher for a particular seasoning group in the $\psi^s < 630$ region. Deal 2006-C has a *pick* parameter of 6.5, but the densities of the deal-specific sample are thin in $\psi^s < 630$ region. Therefore, when forming this deal's loss-maximizing pool, it might not be possible to reach 17% of pool balance with $\psi^s < 630$, as needed to match the historical proportion, by only selecting loans from the seasoning group that delivers highest EGL in the $\psi^s < 630$ region. Thus, the empirical densities might force the loss-maximizing pool to contain loans from seasoning groups other than the one with highest EGL in the $\psi^s < 630$ region. This will reduce the loss-maximizing pool's separation from the random pools. This example illustrates how the distribution of loan balance across the range of loan characteristics in the deal-specific samples is an important component of computing the shifts in pool losses.

3.4.2 ESTIMATION

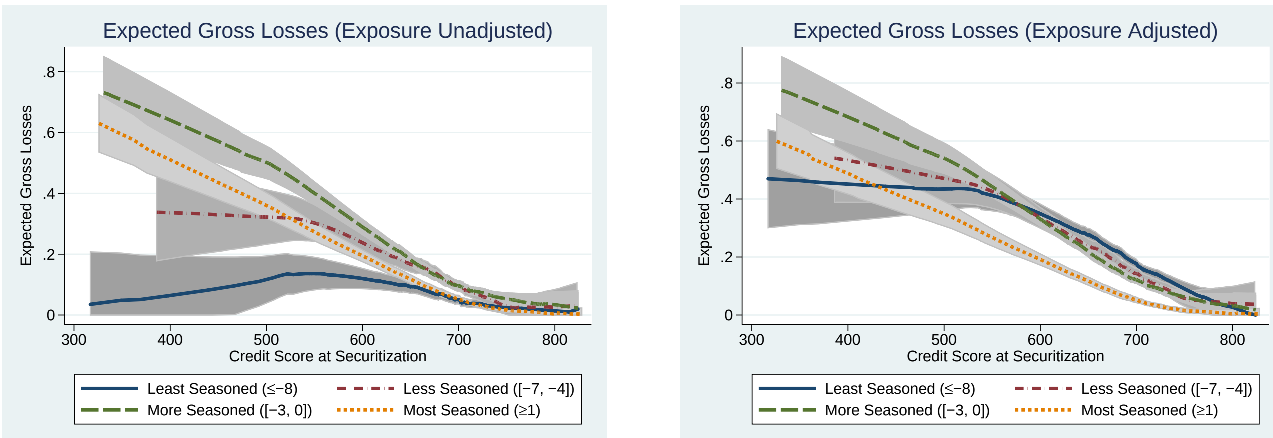
The measure of performance of interest at the pool level is cumulative gross loss (CGL) — the aggregate balance of defaulting loans at the time of default, expressed as a percentage of the initial pool balance. The dependent variable in our regressions is therefore the product of i) a binary default indicator and ii) the ratio of the balance at default over the balance at securitization (which is positive for defaulting loans and zero otherwise). We use Locally Weighted Scatterplot Smoothing (LOWESS) curves — obtained via a two-step process of weighted pointwise local linear regressions followed by weighted smoothing — to represent expected gross losses by credit score at securitization. Expected gross loss (EGL) is normalized by loan balance at securitization. We estimate a separate LOWESS curve for each seasoning group.[29] We estimate a different set of LOWESS curves for every deal-specific sample.

Our rationale for using LOWESS curves instead of alternative methods to estimate the relationship between gross loss and loan characteristics at securitization is the following. LOWESS curves lean toward linearity more than alternative methods. Given that pools, both loss-maximizing and random, must match the same $\overline{\psi^s}$, we wanted estimated relationships that only retain non-linearities that are empirically supported and avoid the risk of using estimated relationships that are non-linear as an artifact of the non-linearities that are parametrically built into many of the alternative methods.[30] Another advantage of LOWESS curves is the ease of presenting expected gross losses, along with their confidence intervals, in one plot, instead of having to show separately the results of regressions with default as the dependent variable and regressions with the ratio of

[29] In the first step, we use a bandwidth of 0.8, which means that up to 80% of each seasoning group's subsample enters the regression used to estimate EGL at a given point. Tricube weights are used: for points inside the bandwidth, less weight is placed on points that are farther away from the point at which we are trying to make a prediction, and none is placed on points outside the bandwidth. In the second step, a smoothing process turns the first-step predicted gross losses into a final EGL curve by taking a tricube weighted mean over the first-step predicted gross losses.

[30] Ex-post, the relationship estimated with LOWESS curves provides empirical support for the built-in non-linearities of other methods.

balance at default divided by balance at securitization as the dependent variable. Despite the slight biases toward linearity of LOWESS curves, we still find statistically significant shift values, an indication of the robustness of our findings.



(a) Fixed post-securitization performance window

(b) Adjusted for duration of exposure to default risk

Figure 3.2:
Expected gross losses
The figure shows the relationship between expected gross losses (EGL), credit score at securitization and seasoning at securitization. The estimates were obtained from the sample of loans used to draw mimicking pools for deal 2006-C (after exclusion of loans with $\psi^o < 630$). The lines correspond to LOWESS curves computed with a bandwidth of 0.8 and the shaded areas are 90% confidence intervals obtained from the local linear regressions performed as first step in the construction of LOWESS curves. In the left panel, the EGL estimates are over a fixed and truncated post-securitization performance window. In the right panel, the relative underexposure to default risk of all but the most seasoned loans is corrected for and the EGL estimates reflect the first e quarters of exposure to default risk after loans enter repayment (excluding post-entry into repayment quarters that occurred before a loan was considered for securitization). Raw data source: FRBNY CCP/Equifax.

The left panel of Figure 3.2 shows the estimated relationship between gross losses, credit score at securitization and seasoning over a fixed post-securitization window of 16 quarters. The relationship is deal-specific in the sense that the post-securitization window begins when loans are six quarters old, matching the *WALAS* for deal 2006-C which is six quarters.[31] Credit score at securitization is a strong predictor of gross losses. The same is true for seasoning over a wide range of ψ^s.

However, some of the predictive power of seasoning over a fixed and truncated post-securitization window is due to the mechanical relation between seasoning and duration

[31]The relationship are estimated on a subsample of loans with $\psi^o \geq 630$ because of the trivial proportion of loans with $\psi^o < 630$ in deal 2006-C. The deal-specific (sub)sample only contain loans that are eligible for securitization when six quarters old, mainly meaning outstanding and less than 30 days delinquent.

of exposure to default risk. Observing loans over a truncated post-securitization window is a challenging limitation of our data: the ideal data would show the performance of all loans until their termination, either via paydown or default. When forming a loss-maximizing pool, the originator-issuer wants to securitize the loans that perform the worst over their securitized life and retain the loans that perform the best from their time of consideration for securitization onward. Therefore, the relationships between gross losses over the securitized life of loans and loan characteristics at securitization are needed.

The right panel of Figure 3.2 also shows the relationship between EGL and loan characteristics at securitization, but these curves are adjusted to compensate for the relative underexposure to default risk of all but the most seasoned loans in the sample. We present an example to provide intuition for our adjustment procedure and define terms that would be used to present our procedure more generally along the way.

For a deal-specific sample, let the length of the post-securitization performance window that we observe be 16 quarters ($\ell = 16$). Take a loan in the deal-specific sample that is 8 quarters away from entering repayment at t_s ($seas^s = -8$). Let the mean $seas^s$ among the most-seasoned group of loans be 3 quarters ($\overline{seas^s_{most}} = 3$).[32] Let our initial loan survive until τ and cumulate 3 quarters with positive payment due by $\tau - 4$: $\sum_{t_o}^{\tau-4} \Phi_{(p>0,\, t)} = 3$. Our initial loan is only exposed to default risk "as a most seasoned loan" for 4 quarters ($h = 4$).

Because it survived until τ, our initial loans would enter the local linear regression that contributes to building the *unadjusted* LOWESS curve with a realized loss value of \$0. When LOWESS curves are adjusted for the relative underexposure to default risk of our initial loans (of all loans with $seas^s < \overline{seas^s_{most}}$ and $seas^\tau \geq \overline{seas^s_{most}}$), the loss value that enters the local linear regression corresponds to the product of i) expected gross loss between $t_s + h$ and τ among most-seasoned loans (expressed per dollar at $t_s + h$), and ii) the ratio of the balance at τ over the balance at t_s for our initial loan. The imputed value is conditional on ψ^τ. The EGL curves used for imputation are separately estimated with most-seasoned loans that survive up to $t_s + h$ for $h = 1, \ldots, \ell$.[33]

[32]For the purpose of the adjustments, we effectively use $\overline{seas^s_{most}}$ after rounding it down to the nearest integer. Effective $\overline{seas^s_{most}}$ for each deal can be recovered from Table 3.3 by taking the difference between the maximum duration of exposure to default risk, e, and ℓ.

[33]Iterating backward, we repeat the above steps for loans with $seas^\tau < \overline{seas^s_{most}}$ and $seas^\tau \geq \overline{seas^s_{more}}$. Prior to estimating gross loss between $t_s + h$ and τ among more-seasoned loans, where h is now given by $h = \tau - t(\sum_{t_o}^{t} \Phi_{(p>0,\, t)} = \overline{seas^s_{more}})$, we impute an expected gross loss over ℓ quarters forward for more-seasoned loans that survive to τ but have $seas^\tau < \overline{seas^s_{most}}$ by assuming that $seas^\tau = \overline{seas^s_{most}}$ and using the imputation method described above. Then, similar steps are followed to input non-zero realized losses into the local linear regressions for loans with $seas^\tau < \overline{seas^s_{more}}$ and $seas^\tau \geq \overline{seas^s_{least}}$.

Let the maximum exposure to default risk be denoted by e ($e = \ell + \overline{seas^s_{most}}$). After the adjustment, it is as if loans that are not yet in repayment at securitization ($seas^s \leq 0$) were exposed to e quarters of default risk, and loans that were already in repayment ($seas^s > 0$) are exposed to ($\max\{q - seas^s, \ell\}$) quarters of default risk. Across all the deal-specific samples, our adjustments ensure that loans are exposed to default risk for at least 16 quarters after their entry into repayment, i.e. $e, \ell \geq 16$. Monteverde (2000) estimates that more than half of the cumulative default on private student loans occurs within the first four years after a student's graduation.[34] Therefore, the shifts in estimated gross losses within the first e quarters after loans enter repayment, with $e > 16$, are likely to proxy well for the shifts that would be found with larger e and ℓ.

Briefly analyzing the right-panel of Figure 3.2, the significantly lower expected losses among loans with even a very modest level of seasoning (having remained current for as little as one quarter after entering repayment) can likely be attributed to the following reasons. Private loans are non-dischargeable in bankruptcy. The decrease in expected loss in response to a bit of seasoning is stronger among the loans of borrowers with a lower credit score. Borrowers with a low credit score and a positive level of seasoning, no matter how small, are likely revealing that they are prioritizing the repayment of their student loans, which are not dischargeable in bankruptcy, over the repayment of other types of debt. The dischargeability of credit cards in bankruptcy and the non-recourse nature of other types of debt, either legally or based on enforcement practices, can contribute to such a prioritization. The prioritization of mortgage and credit card payments has been analyzed (Andersson et al. (2013)), but an analysis that would incorporate student loan payments is an area for future research.

The deal-specific equivalent of the right panel of Figure 3.2 represents an important component of computing shifts in pool losses. There is some variation in the relative positioning of LOWESS curves across the deal-specific samples. However, two significant features are common across samples. First, non-linearities are present. Second, the most-seasoned group has a significantly lower EGL than all other groups. Furthermore, the distance between the EGL curve of the most-seasoned group and the other groups is always greater in the $\psi^s < 730$ region than in the $\psi^s > 730$ region. Since the deals that we analyze contain a proportion of pool balance from the most-seasoned group

[34] The sample used in Monteverde (2000) is peculiar: it consists of law school students who were scheduled to graduate between 1992 and 1994. In addition to the usual six month grace period after graduation, law students often obtain an additional period of deferment if they request it while studying for the bar exam. Thus, if trying to apply Monteverde's estimate to a more representative population of private loan borrowers, it is likely that more than half of the cumulative default occurs within the first four years after a loan enters repayment.

of at least 14%, these common features help to create separation between the EGL of loss-maximizing pools and their corresponding random pools.

3.4.3 FORMING LOSS-MAXIMIZING AND RANDOM POOLS

The loss-maximizing pool solves the issuer's problem. Let EGL denote the expected gross loss, which is a function of ψ^s and seasoning group, $seas_i^s$. EGL are deal specific (i.e. are estimated with deal-specific samples).

Using the notation of Appendix 3.6.5, K denotes the set of loans in a deal-specific sample, k indexes the loans, bal denotes balance, and δ_k denotes a scaling factor in the $[0-1]$ range. Thus, in forming a loss-maximizing pool, the objective of the issuer is to select the pool that maximizes expected gross loss:

$$\max_{\delta_k \forall k \in K} \delta_k \cdot bal_k \cdot \text{EGL}(\psi^s, seas_i^s). \tag{3.3}$$

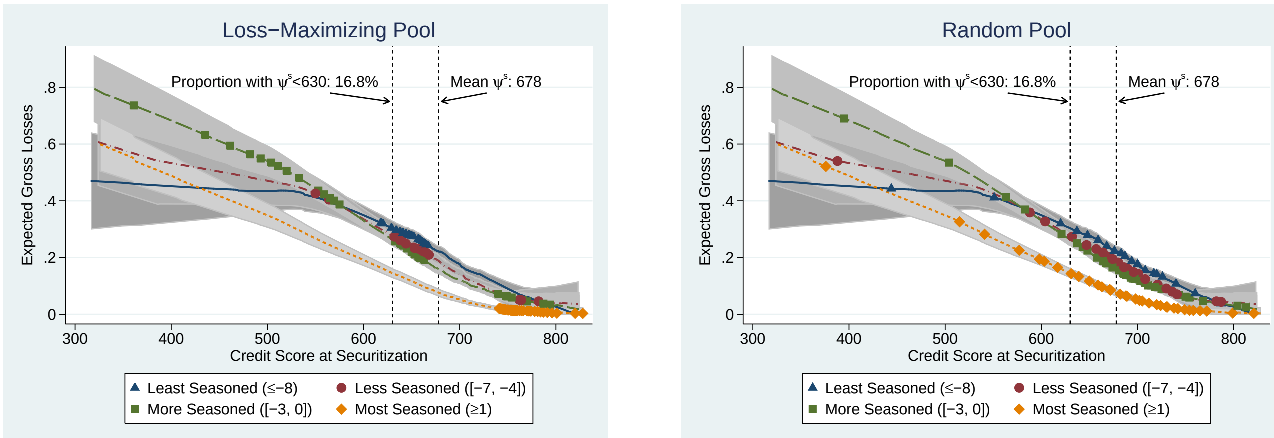
The issuer chooses the scaling factors for all $k \in K$ subject to a constraint that links the chosen scaling factors, the loan balances, the aggregate balance of loans in the deal-specific sample (ϕ^{agg}), the target pool balance (ϕ^t), and the *pick* parameter:

$$\sum_{k \in K} \delta_k \cdot bal_k = \phi^t = \phi^{agg} / pick. \tag{3.4}$$

Other constraints, related to the matching of summary statistics on ψ^s, distribution across ψ^o-intervals and distribution across $seas_i^s$ are presented in Appendix 3.6.5.

The left panel of Figure 3.3 shows our approximate solution to the issuer's problem for deal 2006-C. We use *cluster* to refer to a group of loans with identical ψ^s, a common ψ^o-interval and a common seasoning group. Given that there are more than one thousand clusters of loans (and thus more than one thousand δ_k to choose) and given that we allow for δ_k to be continuous in the $[0-1]$ range, we opted for a numerical solution method that would give us an approximate solution to the issuer's problem.

Our numerical solution method builds on the intuition obtained from an analytical solution to a simplified issuer's problem. We assume away the constraints posed by the aggregate balance of loans available at any cluster of loans (allowing $\delta_k > 1$). We only retain two constraints: the one expressed in equation (3.4) and the constraint of matching the balance-weighted mean ψ^s of the historical pool, $\overline{\psi^s}^t$. We then build on the intuition gained by solving this simplified issuer's problem, progressively add other constraints back in, develop solution methods for those increasingly more complicated problems and ultimately have our solution method for the (full) issuer's problem.



(a) Loss-maximizing pool **(b)** One of many corresponding random pools

Figure 3.3:
Loss-maximizing pool and a corresponding random pool
The left panel shows the loss-maximizing pool for the exercise parameterized to mimic historical deal 2006-C. The markers on the figure represent the location of the pool balance over the range of credit scores and the seasoning groups. For each seasoning group, one marker appears on the figure for both the minimum and maximum credit score with a positive balance. Moving from the minimum to the maximum, there are also markers whenever crossing over a ψ^s value changes the integer value of the cumulative pool balance, expressed as a percentage. The right panel shows one of the corresponding random pools. All mimicking pools have identical distribution across seasoning groups: starting from least-seasoned and ending with most-seasoned, the respective proportions of pool balance are: [17.9%, 19.5%, 30.1%, 32.5%]. Raw data source: FRBNY CCP/Equifax.

We solve the simplified issuer's problem in three steps. Much of the intuition behind our solution method can be gained by making visual reference to the $(\psi^s \times \text{EGL})$ two-dimensional plane and the EGL curves in the left panel of Figure 3.3 (and abstracting from the markers for our solution to the (full) issuer's problem). First, restricting attention to points on the upper envelope of the EGL curves, we draw lines that link every point to the left of $\overline{\psi^s}^{\,t}$ to every point to the right of $\overline{\psi^s}^{\,t}$.[35] Second, we compute the levels of EGL at the intersection of these lines with the vertical line at $\psi^s = \overline{\psi^s}^{\,t}$. These are the balance-weighted average EGL levels that can be achieved by combining those points with balances that satisfy the constraints. The (left-right) pair of points that have their line intersect the vertical line at $\psi^s = \overline{\psi^s}^{\,t}$ at the highest level are partial solutions to the simplified issuer's problem. Third, once partial solutions are found, we compute the

[35]While Figure 3.3 shows continuous EGL curves, the true EGL curves are actually seasoning-group-specific discontinuous loci of ψ^s values with a positive balance in the deal-specific sample. Points at $\psi^s = \overline{\psi^s}^{\,t}$ can be viewed as degenerate lines, with the left and right points being the same point.

corresponding balances for each point that satisfy the constraints.[36]

For the rest of this section, as the constraints that were omitted or relaxed in the simplified issuer's problem are added back to the problem, we repeatedly refer to the pair of points, among a shrinking set of points available for pair formation, that would solve the simplified issuer's problem. Let us use $(lp, rp)^*$ to denote this pair.

Given the empirical distribution of balances across clusters and the size of the target pool balance, re-adding the constraints posed by the loan balance available at clusters (i.e. re-imposing $\delta_k \leq 1$) renders the solution to the simplified issuer's problem infeasible. However, the above intuition carries over when these constraints are added back. The "one-shot" optimization problem becomes one of sequential optimization, with a shrinking set of points available for pair formation. In the context of a bottom-up approach to pool formation, in which loans are added one-by-one to the pool, this forces us to make the following adjustments.

At first, all points are available for pair formation, thus the left and right point from the $(lp, rp)^*$ pair are selected and their δ_k value is set to 1. Next, our constrained EGL-maximizing algorithm determines whether to add a left or a right point. Let ϕ denote the product of δ_k and bal_k at the points selected thus far. Our algorithm checks whether $\sum_k \frac{\delta_k \cdot bal_k}{\phi} \cdot \psi^s > \overline{\psi^s}^{\,t}$. If so, our balance-weighted mean ψ^s is too high and an additional left point is added to the pool; otherwise, an additional right point is added. The most recently selected left point remains available to form pairs with right points, as long as the algorithm is selecting right points; as soon as the algorithm attempts to add an additional left point, the most recently selected left point is removed from the set of points available for pair formation. The converse statement for right points holds as well.

Within the context of this sequential removal of points from the set of points available for pair formation, the algorithm takes three iterative steps. First, restricting itself to the set of points available for pair formation, $(lp, rp)^*$ is identified. Second, it checks whether to add a left-of-mean or right-of-mean point based on the comparison of $(\sum_k \frac{\delta_k \cdot bal_k}{\phi} \cdot \psi^s)$ with $\overline{\psi^s}^{\,t}$. Third, the left or right point that is part of $(lp, rp)^*$ is selected and the entire balance available at that point is added to the pool (i.e. δ_k is set to 1). This process continues until using $\delta_k = 1$ for a selected loan leads to $\phi \geq \phi^t$; at that point, $0 < \delta_k \leq 1$ can be used to exactly match $\phi = \phi^t$.[37]

[36]For all the sets of EGL curves estimated on deal-specific samples, there was sufficient convexity to the upper envelope of the EGL curves that the solution to the simplified issuer's problem was unique. Each had the entire pool balance split between one left and one right point. Appendix 3.6.6 presents a proof that these steps, identify solutions to the simplified (mean-ψ^s matching) issuer's problem.

[37]This simplified algorithm, given the deal parameters and given the loan densities over the range of credit score in deal-specific samples, converges to an EGL-maximizing pool with a balance-weighted mean ψ^s within one decimal point of its target.

It is relatively straightforward to modify the above algorithm to handle the additional constraint of the proportion of pool balance that can be drawn from the $\psi^s < 630$ region. All the deals that we analyze have $\overline{\psi^s}^{\,t} > 630$. Therefore, the left-of-mean points can be split into two groups, those with $\psi^s < 630$ and those with $(630 \leq \psi^s < \overline{\psi^s}^{\,t})$. Among the sets of expected gross loss curves estimated on the deal-specific samples, there is enough convexity in the upper envelope of expected gross loss curves that the sequential addition of loans from the $(lp, rp)^*$ pair among points available for pair formation tends to initially select left-of-mean points in the $\psi^s < 630$ region. Thus, the algorithm tends to achieve its target pool balance in the $\psi^s < 630$ region, explicitly given by

$$\sum_{k \in K} \Phi_{(\psi_k^s < 630)} \cdot \delta_k \cdot bal_k = \theta^t(\psi^s < 630) \cdot \phi^t, \tag{3.5}$$

before it achieves its target pool balance in the $\psi^s \geq 630$ region.

$$\sum_{k \in K} \Phi_{(\psi_k^s \geq 630)} \cdot \delta_k \cdot bal_k = (1 - \theta^t(\psi^s < 630)) \cdot \phi^t, \tag{3.6}$$

If the pool building process achieves its target pool balance in the $\psi^s < 630$ region first, the algorithm removes all points with $\psi^s < 630$ from those available for pair formation and continues to alternate between the addition of left and right points, while abiding by the restriction that left points be selected in the $(630 \leq \psi^s < \overline{\psi^s}^{\,t})$ region, until $\phi = \phi^t$ is reached.

If the pool building process achieves its target pool balance in the $\psi^s \geq 630$ region first, it is necessary to relaunch the pool-building algorithm and force-fill the $\psi^s < 630$ region before condition 3.6 occurs. This manually constrains the selection of left-of-mean points within the $\psi^s < 630$ region before condition 3.6 occurs. We experimented with changes in the timing of the force-filling, because force-filling the $\psi^s < 630$ region earlier than necessary could affect the subsequent availability of right-of-mean points for pair formation with left-of-mean points in the $(630 \leq \psi^s < \overline{\psi^s}^{\,t})$ region. Empirically, we found the timing to have little effect. Changes in timing would move the balance-weighted average EGL of the EGL-maximizing pool by less than one decimal point.

It is again relatively straightforward to modify the above algorithm to handle the addition of constraints stemming from the distribution of loans across ψ^o-intervals. Instead of having two conditional pool balance targets, those for the $\psi^s < 630$ and the $\psi^s \geq 630$ regions, there is an additional 20-22 conditional pool balance targets corresponding to the ψ^o-intervals (Appendix 3.6.1 shows the coarsened grid of ψ^o-intervals for deal 2006-C). As a conditional pool balance target is reached, all points that share this condition become excluded from the set of points available for pair formation. Otherwise, the algorithm is essentially unchanged: i) it alternates between the selection of left-of-mean and right-of-mean points, ii) picks points from the $(lp, rp)^*$ pair among points available for pair formation, and iii) $\delta_k = 1$ is used on a selected point unless doing so makes the conditional pool balance exceed its target (in which case $0 < \delta_k < 1$ such that the conditional pool balance equals its target is used).

The final algorithm solves the issuer's problem with its full set of constraints by building on the intuition gained from solving the issuer's problem with a partial set of constraints. The last set of constraints added back to the problem is related to the proportions of pool balance from the various seasoning groups. For all deals, those constraints are binding: the maximum balance-weighted average EGL achieved with these constraints in place is lower than the maximum balance-weighted average EGL that could be achieved previously. Two factors, common across deals, contribute to this reduction in maximum balance-weighted average EGL and lead us to modify our pool-building algorithm: i) as shown in Table 3.6, pools must have at least 14% of their balance from the most-seasoned group, and ii) the EGL curve for the most-seasoned group lies below the curves for other groups. An additional factor contributed to our choice of modification of our pool-building algorithm: the vertical distance between the EGL curves is much smaller in the $\psi^s > 750$ region.

Building on the intuition granted by the simplified problem, a good way to assess the marginal contribution of loans (at a point in the $(\psi^s \times \text{EGL})$ plane) toward the maximization of a balance-weighted average EGL, under mean-ψ^s constraint, is to look at the maximum balance-weighted average EGL that would be delivered by the loan if it were paired with a loan on the opposite side of $\overline{\psi^s}^{\,t}$ and if the entire pool balance came only from these two points. For a given point, it is possible to visually determine the maximum balance-weighted average EGL that could be achieved via pair formation: it is given by the highest level of EGL at the intersection of the line linking the pair of points and the vertical line at $\psi^s = \overline{\psi^s}^{\,t}$.

Anticipating the 14% or more in pool balance that needs to be selected from the most-seasoned group, and equipped with the above intuition to compare the marginal contribution of loans toward the maximization of a balance-weighted average EGL, we determined that prioritizing the selection of loans from the most-seasoned group in the $\psi^s > 750$ region would contribute toward an optimal solution. This is because the EGL curve for the most-seasoned group lies below the EGL curves for the other groups and the vertical distance between the curves is much smaller in the $\psi^s > 750$ region. The prioritization contributes to the formation of pairs that include most-seasoned loans and deviate little from those that would have been selected absent the constraints on the proportion of pool balance from the seasoning groups.

Conditional on picking right-of-mean loans, it is important to first prioritize the selection of loans from the most-seasoned group as our code does. Otherwise, by simply alternating between the selection of left-of-mean and right-of-mean points from the $(lp, rp)^*$ pair among points available for pair formation, the algorithm would end up selecting points from the most-seasoned group in the $630 < \psi^s < \overline{\psi^s}^{\,t}$ region.[38] These points are very suboptimal, based on the maximum balance-weighted average EGL they can deliver via pair formation and given the corresponding balances needed for a balance-weighted ψ^s equal to $\overline{\psi^s}^{\,t}$.

Thus, for all deals, our final algorithm first prioritizes the selection of right-of-mean loans from the most-seasoned group until the targeted pool balance from that group is reached. Appendix 3.6.7 provides the details for an additional adjustment that is used on deals 2009-B, 2009-C and 2010-A. The intuition for the additional adjustment is similar to that for the prioritized selection of right-of-mean loans from the most-seasoned group.[39] Other than for those modifications, the final algorithms select loans similarly to the algorithm that was used prior to adding back the constraints related to the proportions of pool balance across seasoning groups.

The algorithm to select random pools is similar, except that there is no attempt to sequentially select left-of-mean and right-of-mean points from the $(lp, rp)^*$ pair among points available for pair formation. The code alternates the selection of left-of-mean and right-of-mean points, but the left-of-mean and right-of-mean points are randomly sorted and the order in which conditional pool balance targets are reached has large degree of randomness.[40]

[38]An algorithm that does not anticipate the binding constraints would achieve its targeted pool balance from the least-, less- and more-seasoned groups before achieving its targeted pool balance from the most-seasoned group. When left to select exclusively among loans from the most-seasoned group, the targeted proportion in the $\psi^s < 630$ region would already have been reached. Thus, the algorithm would select points from the most-seasoned group in the $630 < \psi^s < \overline{\psi^s}^{\,t}$ region, which are very suboptimal.

[39]The additional adjustment is due the relative positioning of the EGL curve for more-seasoned loans relative to least- and less-seasoned loans. In that case, the a priori removal of points from the least- and less-seasoned groups in the $\psi^s > 730$ region from the set of points available for pair formation is used to favor the selection of loans from the more-seasoned group in the $\psi^s > 730$ region.

[40]The selection of loans cannot be perfectly random because of the requirement to achieve the same conditional pool balance on matched characteristics as the loss-maximizing pool. Our algorithm has a "look-ahead" component and automatically "force-fills the pool" with loans that will allow it to reach a conditional pool balance target when it finds that the tentative addition of another loan would make it infeasible to reach the conditional pool balance target in question. The force-filling conditionally respects the random sorting of loans. The automation of the force-filling means that force-filling is only ever used as a last resort in the formation of random pools and the initial random sorting of the loans and the joint distribution of loan characteristics strongly determine the composition of the random pools.

3.4.4 SHIFTS IN POOL LOSSES

Table 3.3 reports shifts in gross pool losses for exercises parameterized to mimic 12 historical deals. The (percentage) shifts reported in Table 3.3 are computed by taking the difference in expected CGL between the loss-maximizing pool and the average among its corresponding random pools, and expressing it as a percentage of the latter.

Table 3.3:
Shifts in gross pool losses

Deal	Window (ℓ) (qtrs)	Exposure (e) (qtrs)	Cumulative gross losses Loss-max.	Cumulative gross losses Random (av.)	Shift (Pct pts)	Shift (%)
2005-A	18	21	19.7%	18.3%	1.4%*	7.8%
2005-B	18	21	20.4%	19.2%	1.2%*	6.4%
2006-A	18	21	19.3%	17.6%	1.7%*	9.4%
2006-B	18	21	19.1%	17.6%	1.5%*	8.6%
2006-C	16	19	19.1%	15.9%	3.2%*	20.0%
2007-A	18	21	21.2%	19.3%	1.9%*	10.1%
2009-A	16	24	22.2%	18.2%	4.0%*	22.0%
2009-B	16	22	19.3%	16.6%	2.7%*	15.9%
2009-C	16	22	19.3%	16.0%	3.3%*	20.8%
2010-A	16	22	17.3%	14.5%	2.8%*	19.4%
2013-A	16	24	21.6%	15.9%	5.7%*	36.0%
2013-B	16	24	21.6%	16.8%	4.9%*	29.0%

This table shows the shifts in gross losses for 12 historical deals. The second column reports the length of the fixed performance windows, denoted by ℓ. The third column reports the maximum duration of the exposure to default risk, e, to which loans are exposed to when constructing expected gross loss curves that compensate for the underexposure to default risk of all loans but those in the most-seasoned group. Loans with $seas^s < 0$ are exposed to e quarters of default risk. Loans $seas^s \geq 0$ are exposed to $(\max\{q - seas^s, \ell\})$ quarters of default risk. The fourth column reports the expected CGL of the loss-maximizing pool. It is obtained by taking a balance-weighted average of the exposure-to-default-risk-adjusted expected gross losses. The fifth column reports the average of the expected CGL across ten corresponding random pools. The sixth column simply reports the difference in expected CGL between the loss-maximizing pool and the average among its corresponding random pools, expressed in percentage points. * indicates statistical significance at the 1% level. The seventh column reports the difference in expected CGL between the loss-maximizing pool and the average among its corresponding random pools, expressed as a percentage of the latter. Raw data source: FRBNY CCP/Equifax.

For the loss-maximizing pool for deal 2006-C, the distribution of pool balance across ψ^s and $seas_i^s$, and their corresponding EGL, is shown in the left panel of Figure 3.3. The counter-part for one random pool is shown in the right panel of Figure 3.3.

Shifts in gross losses for the non-cosigned portion of pools are generally larger among post-crisis deals than pre-crisis deals. This is despite balance-weighted average credit scores that are larger for post-crisis deals, as shown in Table 3.1. The larger proportion of pool balance with $\psi^s < 630$ and the larger *pick* parameters among post-crisis deal dominate and explain the larger feasible shifts among post-crisis deals. The differences in expected loss across seasoning groups are greater in the $\psi^s < 630$ region, thus a larger proportion of pool balance with $\psi^s < 630$ helps achieve greater separation between a loss-maximizing pool and random pools. It is intuitive that a larger *pick* parameter, a larger inverse ratio of the pool balance over the balance of loans outstanding and not yet securitized by the originator-issuer, allows for greater separation between a loss-maximizing pool and random pools.

3.4.5 MATCHED CREDIT SCORE, PERFORMANCE AND INTEREST RATE

The matching of the disclosed average ψ^s obviously limits the magnitude of the estimated shifts in gross pool losses. In addition, a combination of factors make the matching of a lower proportion of loans with $\psi^s < 630$ reduce the magnitude of the shifts. Those factors are: i) the within-seasoning-group convexity of the relations between gross losses and ψ^s, especially in the $\psi^s > 550$ region, and ii) the larger differences in expected losses between seasoning group in the $\psi^s < 630$ region.

Among the deals that we analyze, the matching of the disclosed distribution of loans across the ψ^o-intervals has a much smaller effect on the reduction of the magnitude of shifts than the matching of the disclosed summary statistics on ψ^s. There is one channel through which the matching of the distribution of loans across the ψ^o-intervals has a non-trivial effect and this channel only affects pre-crisis deals. Pre-crisis deals have a positive, but tiny proportion of loans with $\psi^o < 630$. This proportion never exceeds 0.40%, as shown in Table 3.1. In comparison, as shown in Table 3.2, 15%-16% of the aggregate balance of deal-specific samples is found in the $\psi^o < 630$ region. Furthermore, loans with $\psi^o < 630$ tend to be overrepresented in the $\psi^s < 630$ region, and our pool-building algorithm for loss-maximizing pools sequentially selects pairs of loans that deliver highest balance-weighted EGL and the left-of-mean loans tend to be selected first in the $\psi^s < 630$ region. Thus, our algorithm tends to rapidly reach its target proportion of loans from the $\psi^o < 630$ region. Once that target is reached, loans with $\psi^o < 630$ are no longer available for selection into the loss-maximizing pool and the loan densities in the $\psi^s < 630$ region becomes significantly thinner after that point. These thinner densities are one of the reasons why loans that do not lie on the upper envelope of EGL tend to be selected in the $\psi^s < 630$ region among pre-crisis deals. Other than through this effect, matching the distribution of loans across the ψ^o-intervals has little effect on the magnitude of the estimated shifts.

The matching of the disclosed distribution of pool balance across ψ^o-intervals is nonetheless important. This is because, given the information available at origination, credit score at origination is a strong predictor of default. Therefore, by matching the distribution of pool balance across ψ^o-intervals, the loss-maximizing pool and the random pools should have similar distributions of interest rates. This makes it less likely that the higher losses in the loss-maximizing pool would be offset by higher interest payments paid by the surviving loans in the loss-maximizing pool.

3.5 CONCLUDING REMARKS

A substantial literature has analyzed the moral hazard (lax origination) (Keys et al. (2010); Purnanandam (2011)) and adverse selection frictions (Elul (2015); Jiang et al. (2014b); Krainer and Laderman (2014)) in the securitization of mortgages. To our knowledge, our paper is the first to take a close look at an informational friction in the securitization chain for private student loans.

The PSL-ABS that we analyze have performed significantly better than private-label MBS. However, two features could contribute to their unexpected underperformance in the presence of time-varying incentives for an originator-issuer to engage in adverse selection. First, is the lack of a contractual commitment to a random selection of securitized loans. Second, is the coarseness of disclosures on some loan characteristics that are strong predictors of default and the non-existence of disclosures for at least one strong predictor of default.

We find it puzzling that issuers of PSL-ABS do not contractually commit to randomly select securitized loans. Theoretically, a commitment to random selection should benefit issuers of PSL-ABS — the overcollateralization required to issue a deal would be lower and/or the cost of funding the origination of loans would be lower. For clarity, we envision a flexible process of random selection that would proceed in two steps. First, the marginal distribution of loans across vintages would be pre-determined (to allow for a mix of loan seasoning that produces a sufficiently high probability that cash flows from the pool are sufficient to make periodic interest rate payments on the bonds) and a limited number of exclusion criteria could be employed (e.g. no loans subject to a risk-sharing agreement with a school). Second, after applying the exclusion criteria, loans from each vintage would be drawn randomly.

Given this flexibility, the only apparent cost of committing to random selection is foregoing the option to engage in adverse selection. Therefore, either a flexible framework of random selection has not been considered or issuers of PSL-ABS are demonstrating that they assign a positive value to the adverse selection option, and that the value of this option outweighs the benefits of a contractual commitment to random selection. Our analysis, by bringing closer scrutiny to the potential for adverse selection in PSL-ABS may reduce the value of the adverse selection option and might contribute to the adoption of a contractual commitment to the random selection of securitized loans.

3.6 APPENDIX

3.6.1 DISCLOSURE ON CREDIT SCORE AT ORIGINATION

Table 3.4:
Grids of credit score at origination

Disclosed		Coarsened	
ψ^o interval	Pool balance	ψ^o interval	Pool balance
<630	0.38%	<630	0.39%
630 - 639	3.59%	630 - 639	3.77%
640 - 649	7.90%	640 - 649	8.29%
650 - 659	8.44%	650 - 659	8.85%
660 - 669	8.95%	660 - 669	9.39%
670 - 679	8.96%	670 - 679	9.39%
680 - 689	8.47%	680 - 689	8.89%
690 - 699	7.44%	690 - 699	7.81%
...		...	
790 - 799	0.92%	790 - 799	0.96%
800 - 809	0.53%	800 - 809	0.56%
810 - 819	0.13%	810 - 819	0.14%
820 - 829	*	820+	0.01%
830 - 839	*		
840 - 849	*		
N/A	4.66%		
Total	100.00%	Total	100.00%

This table presents the disclosure on credit score at origination for a specific deal, SLM 2006-C, and is representative of the disclosure in deal prospectuses. Intervals increase by increments of 10 from 630 upward. Intervals from 700 to 789 were suppressed for brevity; the proportion of pool balance decreases monotonically in that region. * denotes a pool balance of less than 0.01% in an interval. The coarsened grid is obtained by first collapsing the top intervals, which, in isolation, are so thinly populated that when forming mimicking pools in Section 3.4, filling all of these intervals separately would be unfeasible. A second step consists of the proportional re-allocation to the other intervals of the weight placed on the "N/A" interval, which consists of loans with no credit score at origination or loans that were underwritten without relying on credit score.

3.6.2 TYPE OF DATA USED TO CONSTRUCT POOL LEVEL PARAMETERS

The balance-weighted average loan age at securitization, or (*WALAS*), is computed by combining: i) the distribution of pool balance across vintages, ii) an empirically estimated distribution of loans across quarters of origination within a year, and iii) the date of securitization.[41]

The *pick* parameter is computed by combining four pieces of data: i) the annual volume of origination of non-cosigned loans (obtained by combining various forms of disclosure by SLM)[42] ii) vintage-specific time-series of amortization of loan balances (estimated from the loan performance data), and two pieces of information disclosed in deal prospectuses, iii) the distribution of pool balance across vintages and iv) the non-cosigned pool balance.

3.6.3 LOAN LEVEL VARIABLES FROM CCP: RAW AND DERIVED

Table 3.5 lists the variables that we use and that are found in the CCP or derived from it. The construction of loan seasoning (at time of consideration for securitization) is described in Section 3.4.1 and the construction of seasoning level at any other time is similar. We construct our default indicator by combining information contained in the delinquency status variable and the narrative code variable. The narrative code contains a three digit code that can be matched to a list of descriptions that servicers use to report information that falls outside of the pre-determined fields. Some loans have delinquency status that indicate outright default and are classified as such. Other loans have delinquency status that indicate severe delinquency, which once combined with the content of the narrative code, provide sufficient evidence for the classification of a loan as a defaulting one.

[41] The disclosure on the distribution of pool balance across vintages is presented jointly for loans with a co-signer and loans without, instead of being presented separately as is the case for statistics related to credit score. Therefore, (*WALAS*) is a pool characteristic that reflects both co-signed and non-cosigned loans. When parameterizing our exercise to compute shifts in gross pool loss for the non-cosigned portion of pools, we treat (*WALAS*) as if it only represented loans without a co-signer, a simplifying assumption that we acknowledge. The assumption is likely a good reflection of investors' expectations in the absence of negative selection on deals that have a) a narrow range of vintages, and b) a relatively constant fraction of origination volume that is co-signed within that range of vintages. These features are approximately accurate for all the deals that we study, except for 2013-A and 2013-B. For deals 2013-A and 2013-B, with a range of vintages that extends from 2007 to 2012, the assumption is probably not be the best reflection of investors' expectations in the absence of negative selection, since investors would probably expect non-cosigned loans to have more of their weights on the earlier years of the range of vintages, because of a large decrease in the non-cosigned fraction of origination volume from 2007 to 2009. We nonetheless proceed with the simplifying assumption for the time being.

[42] Data on annual origination volume of private loans, combining co-signed and non-cosigned loans, can be found in SLM's 10-K filings, which are retrievable from the SEC EDGAR website. The fraction of the annual origination volume that is co-signed is not disclosed in the 10-K filings. We obtained those fractions from investor presentations by SLM, which were filed with the SEC and retrieved on the SEC EDGAR website, and earnings call presentations, which were retrieved on SLM's website.

Table 3.5:
Observable characteristics in the CCP data set

Variable	Type
Loan characteristics	
Anonymized loan identifier	Raw
Origination date	Raw
Balance	Raw
Delinquency status	Raw
Co-signed indicator	Raw
Payment due	Raw
Narrative code	Raw
Private/Federal indicator	Derived
Seasoning	Derived
Default indicator	Derived
Anonymized servicer identifier	Raw
Borrower characteristics	
Anonymized consumer identifier	Raw
Credit score (Equifax RiskScore (ERS) 3.0)	Raw

The list is not comprehensive with respect to borrower characteristics. The birthyear of borrowers and geographical information on their residence is available (e.g. ZIP Code), but we do not use these variables in our analysis.

3.6.4 DISTRIBUTION ACROSS SEASONING GROUPS

Table 3.6:
Distribution of pool balance across seasoning groups

Deal	*WALAS*	Seasoning			
		Least	Less	More	Most
		≤ -8	[-7,-4]	[-3,0]	≥ 1
2005-A	5	24.3%	24.5%	31.0%	20.3%
2005-B	4	28.2%	26.6%	29.2%	16.1%
2006-A	5	24.5%	23.9%	30.6%	21.0%
2006-B	5	25.1%	24.4%	30.7%	19.8%
2006-C	6	17.9%	19.5%	30.1%	32.5%
2007-A	4	27.7%	26.2%	30.2%	15.9%
		≤ -4	[-3,0]	[1,3]	≥ 4
2009-B	7	31.7%	27.8%	25.9%	14.6%
2009-C	7	31.6%	28.4%	25.2%	14.8%
2010-A	7	31.2%	28.1%	24.2%	16.5%
		≤ -4	[-3,0]	[1,4]	≥ 5
2009-A	10	18.1%	19.9%	30.0%	31.9%
2013-A	11	16.4%	17.4%	27.5%	38.6%
2013-B	10	17.8%	20.3%	29.3%	32.6%

This table presents the matched distribution of pool balance across seasoning groups when loss-maximizing and random pools are formed to compute shifts in gross pool losses. Across deals, 4 seasoning groups are used. The cutoffs of the seasoning groups vary in response to the balance-weighted average loan age of deals. Raw data source: FRBNY CCP/Equifax.

3.6.5 MATCHED POOL CHARACTERISTICS

In our exercise, the pools formed by an issuer must match a set of pool characteristics. Let K denote the set of loans in the deal-specific sample of loans eligible for securitization, let bal_k denote the balance at securitization of loan k and let δ_k be a scaling factor in the $[0-1]$ range. This section of the appendix will end by expanding on this point, but let us state that the δ_k scaling factors, due to the algorithm that we use to form pools, generally take either a value of 0 or 1 (this approximates the real world practice of placing whole loans into pools), but $0 < \delta_k < 1$ is allowed and is used on a small number of loans to facilitate exact matching of as many pool characteristics as possible. With the presence of scaling factors, the issuer's problem of forming a pool consist in choosing the set of δ_k, and a formed pool can be thought of as the set of loans with $\delta_k > 0$, which generally consists of loans with $\delta_k = 1$.

Letting ϕ^{agg} denote the aggregate balance of loans in the deal-specific sample, a formed pool must match the target pool balance ϕ^t, giving us the following constraint:

$$\sum_{k \in K} \delta_k \cdot bal_k = \phi = \phi^t = \phi^{agg} / pick, \tag{3.7}$$

where ϕ denotes the balance of the formed pool.

Second, the formed pool must match the target balance-weighted average credit score at securitization, i.e. the empirical $\overline{\psi^s}$ of the historical pool, giving us:

$$\sum_{k \in K} \frac{\delta_k \cdot bal_k}{\phi} \cdot \psi^s = \overline{\psi^s}^{\,t}, \tag{3.8}$$

where the ^{s} superscript denotes the time of securitization and is only used on credit scores to distinguish credit score at securitization, ψ^s, from credit score at origination, ψ^o. It is implicit that for all other loan characteristics without a time superscript, namely balance and seasoning, they are measured at securitization (i.e. at pool formation).

Third, let $\Phi_{(\psi_k^s < 630)}$ be an indicator function that takes a value of 1 if loan k has $\psi^s < 630$ and value 0 otherwise, and let $\theta^t(\psi^s < 630)$ denote the targeted proportion. The formed pool must match the empirical proportion of pool balance with credit score at securitization less than 630, giving us:

$$\sum_{k \in K} \Phi_{(\psi_k^s < 630)} \cdot \delta_k \cdot bal_k = \theta^t(\psi^s < 630) \cdot \phi^t. \tag{3.9}$$

Fourth, let j index the intervals of the coarsened grid of credit score at origination in the empirical pool, let J denote the set of intervals, and let the target proportion of pool balance in each interval be denoted by $\theta^t(\psi^o \in j)$. The formed pool must match the distribution of pool balance across intervals of the coarsened grid of ψ^o, giving us:

$$\sum_{k \in K} \Phi_{(\psi_k^o \in j)} \cdot \delta_k \cdot bal_k = \theta^t(\psi^o \in j) \cdot \phi^t \quad \forall j \in J. \tag{3.10}$$

Fifth, let i index the loan seasoning groups, and let the targeted proportion of pool balance in each seasoning group be denoted by $\theta^t(seas \in i)$. The formed pool must match the distribution of pool balance across the seasoning groups, giving us:

$$\sum_{k \in K} \Phi_{(seas_k \in i)} \cdot \delta_k \cdot bal_k = \theta^t(seas \in i) \cdot \phi^t \quad \text{for } i = 1, 2, 3, 4. \tag{3.11}$$

Sixth, the deal-specific sample places restriction on the balance at securitization available at any $(\psi^s, \psi_j^o, seas_i)$ combination, any cluster of loans with identical credit score at securitization, a common interval of credit score at origination and common seasoning group. Let $\mathcal{K}$ denote the set of clusters with positive balance. Putting the restriction in mathematical terms gives:

$$\sum_{k \in (\psi^s,\, \psi_j^o,\, seas_i)} \delta_k \cdot bal_k \leq \phi^{agg}_{(\psi^s,\, \psi_j^o,\, seas_i)} \quad \forall (\psi^s, \psi_j^o, seas_i) \in \mathcal{K}. \tag{3.12}$$

Lastly, to explicitly complete the set of constraints, we have:

$$\delta_k \geq 0 \quad \forall k \in K, \tag{3.13}$$

and

$$\delta_k \leq 1 \quad \forall k \in K. \tag{3.14}$$

Finally, we expand on the reason for allowing $0 < \delta_k < 1$ to facilitate exact matching of as many equality constraints as possible. The deal-specific samples that we use often have a loan count that is 15 times smaller or more than the aggregate sample of loans empirically available to SLM, which consists of the loans originated within the range of vintages disclosed for a securitized pool, eligible for securitization and not yet securitized. The much smaller loan count in our deal-specific samples is because the CCP data set contains a 5% sample of the U.S. population with credit reports and because we form pools with loans originated in quarters that had smaller volume of origination than the quarters of origination of the loans in historical pools. With this smaller sample size, had we restricted δ_k to only take value 0 or 1, but allowed for approximate matching with greater tolerance levels, there would have been two sources of variation behind the differences in expected gross loss across the random pools and between the random pools and the loss-maximizing pools: i) different levels of deviation from the targeted pool characteristics and ii) different pool compositions. By allowing $0 < \delta_k < 1$, the deviations from the targeted pool characteristics are tiny and differences in expected gross loss across pools are almost entirely due to differences in pool composition while matching the targeted pool characteristics.

Previewing the method that we use to form pools, it should be noted that the number of loans with $\delta_k < 1$ is very small. Our algorithm builds up pools, random and loss-maximizing, with a bottom up approach of selection of loans and only relies on $\delta_k < 1$ whenever after trying $\delta_k = 1$ on a loan, the algorithm finds out that this would push the left hand-side of equations (3.9), (3.10), or (3.11) above their right-hand side. When that occurs, the left-hand side of either equations (3.9), (3.10), or (3.11) that just exceeded its right-hand side by the largest amount is identified and δ_k is scaled back to a value such that its left-hand side equals its right-hand side. It should also be noted that the deviations from equation (3.8) are very small because our algorithm of formation of pools alternatively adds loan with $\psi_s < \overline{\psi^s}^{\,t}$ when $\sum_k \frac{\delta_k \cdot bal_k}{\phi} \cdot \psi^s > \overline{\psi^s}^{\,t}$ and vice-versa.

3.6.6 GEOMETRIC SOLUTION TO A SIMPLIFIED ISSUER'S PROBLEM

The simplified, mean-matching, issuer's problem has the following objective:

$$\max_{\delta_k \forall k \in K} \sum_{k \in K} \delta_k \cdot bal_k \cdot \text{EGL}(\psi_k^s, seas_k^s). \tag{3.15}$$

The following constraint on the pool balance target is maintained:

$$\sum_{k \in K} \delta_k \cdot bal_k = \phi = \phi^t = \phi^{agg} / pick. \tag{3.16}$$

So are the constraints on the balance-weighted mean credit score at securitization,

$$\sum_{k \in K} \delta_k \cdot \frac{bal_k}{\phi^t} \cdot \psi_k^s = \overline{\psi^s}^{\,t}, \tag{3.17}$$

and $\delta_k \geq 0$, but all other constraints are abandoned, including $\delta_k \leq 1$.

Since the simplified problem allows for $\delta_k > 1$ the objective can be re-written as the maximization of the balance-weighted average EGL, with the set of $\{bal_k\}_{k=1}^{K}$ being the choice variables:

$$\max_{bal_k \forall k \in K} \sum_{k \in K} \frac{bal_k}{\phi^t} \cdot \text{EGL}(\psi_k^s, seas_k^s). \tag{3.18}$$

The pool balance constraint can be re-written as shares of pool balance summing to 1:

$$\sum_{k \in K} \frac{bal_k}{\phi^t} = 1, \tag{3.19}$$

and the δ_k terms disappear from the mean-ψ^s constraint, which becomes:

$$\sum_{k \in K} \frac{bal_k}{\phi^t} \cdot \psi^s_k = \overline{\psi^s}^{\,t}. \tag{3.20}$$

The above makes it immediately apparent that the objective is to maximize a linear combination of $\text{EGL}(\psi^s, seas^s)$. We use the terms *point* and *line* in reference to the $(\psi^s \times \text{EGL})$ 2-dimensional plane. Given a level of ψ_s, there are up to 4 levels of feasible EGL associated with that level of ψ_s, one for each seasoning group. The 4 curves linking the various (ψ_s, EGL) points associated with each seasoning group give an overview of the feasible points, although underneath there is truly a discontinuous locus of points, with ψ_s only taking integer values and gaps in ψ_s value can be present. We do not assume continuity of the expected gross loss curves at any point in this appendix. We assume continuity on the balances, however, which allows us to match constraints exactly.

Solving the simplified issuer's problem can be viewed as a stepwise process of selecting points or combination of points, truly meaning selecting on which points to place a positive balance, and then computing/selecting what level of balance to place on each one of those points to satisfy the constraints. This is the context behind the use of terms such as "combination of points", or "selecting points and their corresponding balances".

Note that given the mean-ψ^s constraint, any combination that contains a point with $\psi^s < \overline{\psi^s}^{\,t}$, a point to the left of $\overline{\psi^s}^{\,t}$ for short, or a *left point* for additional brevity, must contain at least one *right point*, a point with $\psi^s > \overline{\psi^s}^{\,t}$, and vice-versa.

This appendix proves three lemmas, which together support the following proposition:

Proposition 1 *When searching for a solution to the simplified, mean-ψ^s matching, issuer's problem, over a given set of K points, it is sufficient to compare combinations formed from points on the upper envelope of expected gross loss curves. Within those points, it is sufficient to compare i) the levels of balance-weighted average* EGL *achieved by all the left-right 2-point combinations, along with the unique corresponding pair of balances that satisfy the mean-ψ^s constraint, and ii) the level of* EGL *achieved by the single point at $\psi^s = \overline{\psi^s}^{\,t}$, with the entire pool balance placed at that point. The maximum level of balance-weighted* EGL *achieved in that set is the maximum level that can be achieved by any combinations on the given set of K points. All point-balance* *combinations that match that maximum level of* EGL*, are solutions to the simplified issuer's problem. The level of balance-weighted* EGL *achieved by a left-right 2-point combination, and corresponding balances that satisfy the balance-weighted mean-*ψ^s *constraint, is given by the level of* EGL *at the intersection of the line linking the two points and the vertical line drawn at* $\psi^s = \overline{\psi^s}^{\,t}$.

Proposition 1 implies that in order to find a solution to the simplified issuer's problem, it is sufficient to follow the following steps. First, compare a) the levels of EGL at the intersection of the line linking all the left-right combinations of points with the vertical line drawn at $\psi^s = \overline{\psi^s}^{\,t}$, and b) the highest level of EGL among single points at $\psi^s = \overline{\psi^s}^{\,t}$, if any. Second, select one, of possibly many, combination of points that achieves the highest level of EGL among a) and b). Third, if the selected combination of points is from a pair of points, compute the balances that satisfy constraints on balance.

A formal definition of the upper envelope of the expected gross loss curves will be given in Lemma 2, but the term is sufficiently intuitive that Proposition 1 can be momentarily understood without a formal definition. Note that Proposition 1 does not rule out the possibility that a combination involving 3 or more points could be a solution to the simplified issuer's problem. The proposition is about sufficient steps to find *a* solution to the simplified issuer's problem. The proposition states that should a combination of 3 or more points be a solution to the simplified issuer's problem, it would be possible to match the balance-weighted EGL delivered by the combination of 3 or more points with *at least one* combination of 2 points or less.

Lemma 1 *Given a combination with one point with* $\psi^s < \overline{\psi^s}^{\,t}$ *and one point with* $\psi^s > \overline{\psi^s}^{\,t}$*, the balance-weighted level of* EGL *that is delivered by selecting shares of pool balance that satisfy the mean-*ψ^s *constraint corresponds to the level of* EGL *at the intersection of the line linking the two points and the vertical line drawn at* $\psi^s = \overline{\psi^s}^{\,t}$.

Proof of Lemma 1: Let the $_{lp}$ subscript denote the left point, with $\psi^s < \overline{\psi^s}^{\,t}$, and the $_{rp}$ subscript denote the right point, with $\psi^s > \overline{\psi^s}^{\,t}$. When only taking a linear combination between a left point and a right point, the mean-ψ^s constraint can be re-written as:

$$\frac{bal_{lp}}{\phi^t} \cdot \psi^s_{lp} + (1 - \frac{bal_{lp}}{\phi^t}) \cdot \psi^s_{rp} = \overline{\psi^s}^{\,t}. \tag{3.21}$$

The level of EGL at the point of intersection between the line linking the left and right points and the vertical line drawn at $\psi^s = \overline{\psi^s}^{\,t}$, denoted EGL* is given by:

$$\mathrm{EGL}^* = \mathrm{EGL}_{lp} + (\overline{\psi^s}^{\,t} - \psi^s_{lp}) \cdot \frac{\mathrm{EGL}_{rp} - \mathrm{EGL}_{lp}}{\psi^s_{rp} - \psi^s_{lp}}. \tag{3.22}$$

The right-hand side can be re-arranged in way that more closely resembles a weighted average between EGL$_{lp}$ and EGL$_{rp}$:

$$\mathrm{EGL}^* = (1 - \frac{\overline{\psi^s}^{\,t} - \psi^s_{lp}}{\psi^s_{rp} - \psi^s_{lp}}) \cdot \mathrm{EGL}_{lp} + \frac{\overline{\psi^s}^{\,t} - \psi^s_{lp}}{\psi^s_{rp} - \psi^s_{lp}} \cdot \mathrm{EGL}_{rp}\,. \tag{3.23}$$

With the above equation re-arranged this way, the question becomes whether setting

$$\frac{bal_{lp}}{\phi^t} = (1 - \frac{\overline{\psi^s}^{\,t} - \psi^s_{lp}}{\psi^s_{rp} - \psi^s_{lp}}) \tag{3.24}$$

is indeed a proportion of pool balance on the left point, along with proportion $(1 - bal_{lp}/\phi^t)$ on the right point, that satisfies the mean-ψ^s constraint.

With a couple steps of algebra, equation (3.21) can be re-written to isolate bal_{lp}/ϕ^t:

$$\frac{bal_{lp}}{\phi^t} \cdot (\psi^s_{lp} - \psi^s_{rp}) = \overline{\psi^s}^{\,t} - \psi^s_{rp}\,, \tag{3.25}$$

$$\frac{bal_{lp}}{\phi^t} = \frac{\overline{\psi^s}^{\,t} - \psi^s_{rp}}{\psi^s_{lp} - \psi^s_{rp}}, \tag{3.26}$$

$$= \frac{\psi^s_{rp} - \overline{\psi^s}^{\,t}}{\psi^s_{rp} - \psi^s_{lp}}. \tag{3.27}$$

Finally, note how the right-hand side of equation (3.24) can be re-arranged as:

$$(1 - \frac{\overline{\psi^s}^{\,t} - \psi^s_{lp}}{\psi^s_{rp} - \psi^s_{lp}}) = (\frac{\psi^s_{rp} - \psi^s_{lp}}{\psi^s_{rp} - \psi^s_{lp}} - \frac{\overline{\psi^s}^{\,t} - \psi^s_{lp}}{\psi^s_{rp} - \psi^s_{lp}}) = \frac{\psi^s_{rp} - \overline{\psi^s}^{\,t}}{\psi^s_{rp} - \psi^s_{lp}}. \tag{3.28}$$

■

Lemma 2 *Let the construction of the upper envelope of the expected gross loss curves be the following: i) take the set of all ψ^s values taken by the loan in set K, ii) at each ψ^s value, retain the loan(s) with maximum* EGL$(\psi^s, seas^s_i)$ *value. All solutions to the simplified, mean-ψ^s matching, issuer's problem consist of combinations of points on the upper envelope of the expected gross loss curves.*

Proof of Lemma 2: The proof proceeds by contradiction. Suppose a solution to the issuer's problem, which maximizes the balance-weighted EGL, has shares of pool balance that satisfy the mean-ψ^s constraint and contains a point that is not on the upper envelope of the expected gross loss curves. Then, it is feasible to substitute the loan that is not on the upper envelope by a point that is on the upper envelope, maintain shares of pool balance and continue to satisfy the mean-ψ^s constraint since the substituting loan has the same ψ^s. In addition, all other terms that enter the expression for the balance-weighted EGL stay the same, but the product of the share of pool balance and EGL on the loan that was substituted in is higher than the product of the same share of pool balance and EGL on the loan that was substituted out, by definition of the loan being substituted in being on the upper envelope. Thus, we have:

$$\text{EGL}_{sub-in} > \text{EGL}_{sub-out}\,, \tag{3.29}$$

$$\frac{bal_{sub-out}}{\phi^t}\,\text{EGL}_{sub-in} > \frac{bal_{sub-out}}{\phi^t}\,\text{EGL}_{sub-out}\,, \tag{3.30}$$

$$\frac{bal_{sub-in}}{\phi^t}\,\text{EGL}_{sub-in} > \frac{bal_{sub-out}}{\phi^t}\,\text{EGL}_{sub-out}\,. \tag{3.31}$$

A contradiction with the claim that the point that was substituted out was part of combination of points that solved the issuer's problem.

■

Lemma 3 *Let a set of combinations of points consist of i) all the pairs of left and right points and corresponding shares of pool balance, given by* $\frac{bal_{lp}}{\phi^t} = \frac{\psi^s_{rp} - \overline{\psi^s}^{\,t}}{\psi^s_{rp} - \psi^s_{lp}}$ *and* $\frac{bal_{rp}}{\phi^t} = \frac{\overline{\psi^s}^{\,t} - \psi^s_{lp}}{\psi^s_{rp} - \psi^s_{lp}}$*, and ii) the points with* $\psi^s = \overline{\psi^s}^{\,t}$*, with a share of pool balance of 1 placed at* $\psi^s = \overline{\psi^s}^{\,t}$*. Furthermore, limit the set of combinations to those formed exclusively from points that are on the upper envelope formed by the expected gross loss curves. It is sufficient to consider the points and corresponding balances from this set in order to achieve the maximum balance-weighted average* EGL *under the constraints of the simplified, mean-*ψ^s *matching, issuer's problem. All combinations, possibly including combinations of more than two points, that satisfy the constraints and match the maximum balance-weighted average* EGL *achieved in the aforementioned set are solutions to the simplified issuer's problem.*

Proof of Lemma 3: Limiting the search for a solution to points on the upper envelope formed by the expected gross loss curves is without loss of generality, as shown by *Lemma 2*. This reduces notation since EGL(ψ^s) will be used, with the understanding that the maximum level of EGL among the, up to 4, levels of EGL$(\psi^s, seas^s_i)$ at a ψ^s value is being used.

The proof will proceed by contradiction. We will show that for an any combination of NL left points and NR right points, with $(NL + NR) > 2$, and corresponding balances on those points of $\{bal_k\}_{k=1}^{NL+NR}$ that satisfy the constraints, the balance-weighted average EGL delivered by the combination cannot exceed the maximum balance-weighted EGL that can be achieved from the $(NL \times NR)$ left-right pairwise combination that can be formed from the $(NL + NR)$ points. We will also show, via comments in the proof for the case with $NL + NR$ points, that any combination with NL left points, NR right points, with $NL + NR \geq 2$, and one point at $\overline{\psi^s}^{\,t}$, and corresponding balances on those points of $\{bal_k\}_{k=1}^{(NL+NR+1)}$ that satisfy the constraints, the balance-weighted average EGL delivered by the combination of $(NL + NR + 1)$ points cannot exceed both i) the maximum balance-weighted EGL that can be achieved from the $(NL \times NR)$ left-right pairwise combination that can be formed from the $(NL + NR)$ points, and ii) the EGL at $\psi^s = \overline{\psi^s}^{\,t}$. This has for implication that the balance-weighted average EGL that can be delivered by any combination of $(NL + NR)$ points, with $(NL + NR) > 2$, or any combination of $(NL + NR + 1)$ points, with $(NL + NR) \geq 2$, is bounded from above by the balance-weighted average EGL that can be delivered by either i) or ii). This also has for implications that if one computes the maximum balance-weighted average EGL that can be achieved from the set of iii) *all* possible pairwise combination of left and right points, with corresponding balances such that the constraints are met, and iv) placing the entire pool balance at $\psi^s = \overline{\psi^s}^{\,t}$, then this maximum is an upper bound on the balance-weighted average EGL that can be achieved by *any* combination of $(NL + NR)$ points, with $(NL + NR) > 2$, and any combination $(NL + NR + 1$ points, with $(NL + NR) \geq 2$. Therefore, it is sufficient to limit the search to iii) and iv) in order to find the maximum balance-weighted average EGL that can be achieved by any combinations.

The statement, which we will show to be contradictory, is the following. Let there be a combination of NL left points and NR right points, with $(NL + NR) > 2$, with corresponding balances that satisfy the constraints, and assume that this combination delivers a balance-weighted average EGL that exceeds the balance-weighted average EGL that can be delivered by any 2-point or single-point combinations, with corresponding balances that satisfy the constraints. We will show this statement to be contradictory without even having to compare the balance-weighted average EGL of the combination of $(NL + NR)$ points to all pairwise combination of left and right points in the set of K points: we will show this statement to be contradictory even when one limits the comparison to the balance-weighted average EGL that can be achieved with the $(NL \times NR)$ left-right pairwise combination that can be formed from the $(NL + NR)$ points.

We explicitly conduct our proof for the case with $(NL + NR)$ points that can be separated into NL left point(s), with $\psi^s < \overline{\psi^s}^{\,t}$, and NR right point(s), with $\psi^s > \overline{\psi^s}^{\,t}$, but we will comment at one point in the proof on how it would be modified to handle the addition of one point with $\psi^s = \overline{\psi^s}^{\,t}$. It is sufficient to consider the case with one point at $\psi^s_c = \overline{\psi^s}^{\,t}$ and corresponding balance bal_c since, by Lemma 2, limiting the search for combination of points that maximize balance-weighted EGL means only considering points on the upper envelope of the expected gross loss curves. Therefore, our proof will comprehensively cover all cases.

Let the balance-weighted average EGL achieved with the $(NL + NR)$ points be denoted with $\overline{\text{EGL}}$. We have:

$$\sum_{k=1}^{NL+NR} \frac{bal_k}{\phi^t} \cdot \text{EGL}(\psi_k) = \overline{\text{EGL}}. \tag{3.32}$$

Let a ($NL \times NR$)-by-2 vector contains all the pairwise combinations that can be formed from the NL left points and the NR right points. For ease of exposition, we illustrate segments of the proof with an example, an example with 3 left points and 2 right points. Denoting the location of the points by $\psi_{lp1}, \psi_{lp2}, \psi_{lp3}, \psi_{rp1}, \psi_{rp2}$ the vector of pairwise combination would be:

$$\begin{aligned}
&\psi_{lp1}, \psi_{rp1},\\
&\psi_{lp1}, \psi_{rp2},\\
&\psi_{lp2}, \psi_{rp1},\\
&\psi_{lp2}, \psi_{rp2},\\
&\psi_{lp3}, \psi_{rp1},\\
&\psi_{lp3}, \psi_{rp2}.
\end{aligned}$$

Let there be a set of scalars $0 \leq \alpha_{k,j} \leq 1$ with $j = 1, ..., \max\{1, NR - 1\}$ on the left point(s) and $j = 1, ..., \max\{1, NL - 1\}$ on the right point(s). Create a new ($NL \times NR$)-by-1 vector that multiplies the element on each row of the ($NL \times NR$)-by-2 vector by their corresponding shares of pool balance and then sums them. Insert the α_{kj} scalar sequentially, from top to bottom, as pre-multipliers on each term in the sums , letting the indices increase up to their maximum, and using $(1 - \sum_j \alpha_{k,j})$ after that. Returning to our example, this gives:

$$
\begin{aligned}
\alpha_{lp1,1} \cdot \frac{bal_{lp1}}{\phi^t} \cdot \psi_{lp1} + \alpha_{rp1,1} \cdot \frac{bal_{rp1}}{\phi^t} \cdot \psi_{rp1} \ , \\
(1-\alpha_{lp1,1}) \cdot \frac{bal_{lp1}}{\phi^t} \cdot \psi_{lp1} + \alpha_{rp2,1} \cdot \frac{bal_{rp2}}{\phi^t} \cdot \psi_{rp2} \ , \\
\alpha_{lp2,1} \cdot \frac{bal_{lp2}}{\phi^t} \cdot \psi_{lp2} + \alpha_{rp1,2} \cdot \frac{bal_{rp1}}{\phi^t} \cdot \psi_{rp1} \ , \\
(1-\alpha_{lp2,1}) \cdot \frac{bal_{lp2}}{\phi^t} \cdot \psi_{lp2} + \alpha_{rp2,2} \cdot \frac{bal_{rp2}}{\phi^t} \cdot \psi_{rp2} \ , \\
\alpha_{lp3,1} \cdot \frac{bal_{lp3}}{\phi^t} \cdot \psi_{lp3} + (1-\alpha_{rp1,1}-\alpha_{rp1,2}) \cdot \frac{bal_{rp1}}{\phi^t} \cdot \psi_{rp1} \ , \\
(1-\alpha_{lp3,1}) \cdot \frac{bal_{lp3}}{\phi^t} \cdot \psi_{lp3} + (1-\alpha_{rp2,1}-\alpha_{rp2,2}) \cdot \frac{bal_{rp2}}{\phi^t} \cdot \psi_{rp2}.
\end{aligned}
$$

Note that summing down the rows of the above ($NL \times NR$)-by-1 vector, one simply recovers the left-hand side of the mean-ψ^s constraint, thus the sum of the rows is equal to $\overline{\psi^s}^{\,t}$. This is true in our example and in the general case as well. Furthermore, note that this holds whatever the set of scalars chosen. Let's arbitrarily pick a set of $\{\alpha_{k,j}\}$. Given those $\{\alpha_{k,j}\}$ and looking one-by-one at the rows of the ($NL \times NR$)-by-1 vector, only two terms, out of the ($NL \times NR$) terms that guarantee that $\overline{\psi^s}^{\,t}$ is reached, are being summed. Note, for completeness, that $bal_k \geq 0$ and $\psi^s > 0$. Therefore, one-by-one, the rows of the ($NL \times NR$)-by-1 vector are smaller or equal to $\overline{\psi^s}^{\,t}$ and thanks to all the terms in each row being non-negative, this means that each row can be expressed as the product of a scalar θ_i, with $0 \leq \theta_i \leq 1$, and $\overline{\psi^s}^{\,t}$. Returning to our example, this means:

$$\alpha_{lp1,1} \cdot \frac{bal_{lp1}}{\phi^t} \cdot \psi_{lp1} + \alpha_{rp1,1} \cdot \frac{bal_{rp1}}{\phi^t} \cdot \psi_{rp1} = \theta_1 \cdot \overline{\psi^s}^{\,t}, \tag{3.33}$$

$$(1-\alpha_{lp1,1}) \cdot \frac{bal_{lp1}}{\phi^t} \cdot \psi_{lp1} + \alpha_{rp2,1} \cdot \frac{bal_{rp2}}{\phi^t} \cdot \psi_{rp2} = \theta_2 \cdot \overline{\psi^s}^{\,t}, \tag{3.34}$$

$$\alpha_{lp2,1} \cdot \frac{bal_{lp2}}{\phi^t} \cdot \psi_{lp2} + \alpha_{rp1,2} \cdot \frac{bal_{rp1}}{\phi^t} \cdot \psi_{rp1} = \theta_3 \cdot \overline{\psi^s}^{\,t}, \tag{3.35}$$

$$(1-\alpha_{lp2,1}) \cdot \frac{bal_{lp2}}{\phi^t} \cdot \psi_{lp2} + \alpha_{rp2,2} \cdot \frac{bal_{rp2}}{\phi^t} \cdot \psi_{rp2} = \theta_4 \cdot \overline{\psi^s}^{\,t}, \tag{3.36}$$

$$\alpha_{lp3,1} \cdot \frac{bal_{lp3}}{\phi^t} \cdot \psi_{lp3} + (1-\alpha_{rp1,1}-\alpha_{rp1,2}) \cdot \frac{bal_{rp1}}{\phi^t} \cdot \psi_{rp1} = \theta_5 \cdot \overline{\psi^s}^{\,t}, \tag{3.37}$$

$$(1-\alpha_{lp3,1}) \cdot \frac{bal_{lp3}}{\phi^t} \cdot \psi_{lp3} + (1-\alpha_{rp2,1}-\alpha_{rp2,2}) \cdot \frac{bal_{rp2}}{\phi^t} \cdot \psi_{rp2} = \theta_6 \cdot \overline{\psi^s}^{\,t}. \tag{3.38}$$

Let us briefly comment on how the proof would be modified if one point with $\psi^s = \overline{\psi^s}^{\,t}$ was added to a combination, giving us a combination with $(NL + NM + 1)$ points. We have yet to explain how λ_i is constructed on each left-right pair of points, but let us state that for the point at $\psi^s = \overline{\psi^s}^{\,t}$, it is straightforward: $\lambda_c = \overline{\text{EGL}}/(bal_c \cdot EGL(\psi^s_c))$, where $\overline{\text{EGL}}$ would of course be the balance-weighted average EGL over the $(NL + NM + 1)$ points and their corresponding balances. In terms of modifying the $(NL \times NM)$ equivalent of equations (3.33) to (3.38), one would simply add an $(NL \times NM + 1)^{th}$ equation, with $(bal_c/\phi^t) \cdot \psi^s_c = \theta_{NL\times NM+1} \cdot \overline{\psi^s}^{\,t}$. The rest of the proof for a combination of $(NL \times NM + 1)$ points would essentially proceed as done below for combinations of $NL + NM$ points.

The case with one left point, one right point and one point at $\psi^s = \overline{\psi^s}^{\,t}$ is trivial, in the sense that the θ_1 achieved by the pair of left and right points can immediately be computed and so can the λ_1 on the pair, just like the θ_2 on the point at $\psi^s = \overline{\psi^s}^{\,t}$ and the λ_2. Given those θ_i and λ_i, one could immediately jump to the paragraph that contains equation (3.56) and use the argumentation from that paragraph onward to complete the proof for the special case with one left point, one right point and one point at $\psi^s = \overline{\psi^s}^{\,t}$.

Returning to our proof for combinations of left and right points only, note that one can express $\overline{\text{EGL}}$ as the sum of the rows of the $(NL \times NR)$-by-1 vector after the ψ^s terms have been replaced by $\text{EGL}(\psi^s)$ terms. Returning to our example, this means:

$$\begin{aligned}\overline{\text{EGL}} = &\left[\alpha_{lp1,1} \cdot \frac{bal_{lp1}}{\phi^t} \cdot \text{EGL}(\psi_{lp1}) + \alpha_{rp1,1} \cdot \frac{bal_{rp1}}{\phi^t} \cdot \text{EGL}(\psi_{rp1})\right] + \ldots \\ &+ \left[(1 - \alpha_{lp3,1}) \cdot \frac{bal_{lp3}}{\phi^t} \cdot \text{EGL}(\psi_{lp3}) + (1 - \alpha_{rp2,1} - \alpha_{rp2,2}) \cdot \frac{bal_{rp2}}{\phi^t} \cdot \text{EGL}(\psi_{rp2})\right].\end{aligned} \tag{3.39}$$

Since all of the above holds for any set of $\{\alpha_{k,j}\}$, with the understanding that different $\{\alpha_{k,j}\}$ lead to different $\{\theta_i\}$, let us choose the $\{\alpha_{k,j}\}$ in a way that will be useful. Starting with the specifics of our example and generalizing it afterward, consider the first row of the $(NL \times NR)$-by-1 vector. $\alpha_{lp1,1}$ and $\alpha_{rp1,1}$ can be chosen in such a way that if one took the $\alpha_{lp1,1} \cdot \frac{bal_{lp1}}{\phi^t}$ weight on the $lp1$ point and the $\alpha_{rp1,1} \cdot \frac{bal_{rp1}}{\phi^t}$ weight on the $rp1$ point, and scaled it up by $1/\theta_1$ then the pair, in isolation, would satisfy the share of pool balance constraint and the mean-ψ^s constraint. How does one do that? If we want $\alpha_{lp1,1}$ and $\alpha_{rp1,1}$ to allow for a scale up by $1/\theta_1$ to deliver a pair of points and weights that would constitute shares of pool balance that satisfy the mean-ψ^s constraint, according to the work done to prove Lemma 1, in particular equation (3.27) we need to set:

$$\frac{\alpha_{lp1,1} \cdot bal_{lp1}/\phi^t}{\theta_1} = w_{lp}(lp1, rp1) = \frac{\psi_{rp1} - \overline{\psi^s}^t}{\psi_{rp1} - \psi_{lp1}} \Rightarrow \alpha_{lp1,1} = \frac{\phi^t}{bal_{lp1}} \cdot \frac{\psi_{rp1} - \overline{\psi^s}^t}{\psi_{rp1} - \psi_{lp1}} \cdot \theta_1, \tag{3.40}$$

$$\frac{\alpha_{rp1,1} \cdot bal_{rp1}/\phi^t}{\theta_1} = (1 - w_{lp}(lp1, rp1)) = \frac{\overline{\psi^s}^t - \psi_{lp1}}{\psi_{rp1} - \psi_{lp1}} \Rightarrow \alpha_{rp1,1} = \frac{\phi^t}{bal_{rp1}} \cdot \frac{\overline{\psi^s}^t - \psi_{lp1}}{\psi_{rp1} - \psi_{lp1}} \cdot \theta_1. \tag{3.41}$$

Note that picking $\alpha_{lp1,1}$ and $\alpha_{rp1,1}$ to allow for a scale up by $1/\theta_1$ of the weights on the (ψ_{lp1}, ψ_{rp1}) pair that would deliver a pair of points and weights that would constitute shares of pool balance that satisfy the mean-ψ^s constraint implies:

$$\frac{\alpha_{lp1,1}}{\alpha_{rp1,1}} = \frac{bal_{rp1}}{bal_{lp1}} \cdot \frac{\psi_{rp1} - \overline{\psi^s}^t}{\overline{\psi^s}^t - \psi_{lp1}}. \tag{3.42}$$

Using a similar logic line-by-line gives:

$$\frac{(1 - \alpha_{lp1,1})}{\alpha_{rp2,1}} = \frac{bal_{rp2}}{bal_{lp1}} \cdot \frac{\psi_{rp2} - \overline{\psi^s}^t}{\overline{\psi^s}^t - \psi_{lp1}}, \tag{3.43}$$

$$\frac{\alpha_{lp2,1}}{\alpha_{rp1,2}} = \frac{bal_{rp1}}{bal_{lp2}} \cdot \frac{\psi_{rp1} - \overline{\psi^s}^t}{\overline{\psi^s}^t - \psi_{lp2}}, \tag{3.44}$$

$$\frac{(1 - \alpha_{lp2,1})}{\alpha_{rp2,2}} = \frac{bal_{rp2}}{bal_{lp2}} \cdot \frac{\psi_{rp2} - \overline{\psi^s}^t}{\overline{\psi^s}^t - \psi_{lp2}}, \tag{3.45}$$

$$\frac{\alpha_{lp3,1}}{(1 - \alpha_{rp1,1} - \alpha_{rp1,2})} = \frac{bal_{rp1}}{bal_{lp3}} \cdot \frac{\psi_{rp1} - \overline{\psi^s}^t}{\overline{\psi^s}^t - \psi_{lp3}}, \tag{3.46}$$

$$\frac{(1 - \alpha_{lp3,1})}{(1 - \alpha_{rp2,1} - \alpha_{rp2,2})} = \frac{bal_{rp2}}{bal_{lp3}} \cdot \frac{\psi_{rp2} - \overline{\psi^s}^t}{\overline{\psi^s}^t - \psi_{lp3}}. \tag{3.47}$$

Equations (3.42) to (3.47) is a system of 7 variables, with 6 restrictions imposed on the value that the ratios can take. The system can be re-arranged in such a way that all $\alpha_{k,j}$ other than $\alpha_{lp1,1}$ are functions of $\alpha_{lp1,1}$. Using β_i to represent the terms on the right hand-side of equations (3.42) to (3.47), we have:

$$\text{Eq. 39} \Rightarrow \alpha_{rp1,1} = 1/\beta_1 \cdot \alpha_{lp1,1} \ , \tag{3.48}$$
$$\text{Eq. 40} \Rightarrow \alpha_{rp2,1} = 1/\beta_2 - 1/\beta_2 \cdot \alpha_{lp1,1} \ , \tag{3.49}$$
$$\text{Eq. 41} \Rightarrow \alpha_{rp1,2} = 1/\beta_3 \cdot \alpha_{lp2,1} \ , \tag{3.50}$$
$$\text{Eq. 42} \Rightarrow \alpha_{rp2,2} = 1/\beta_4 - 1/\beta_4 \cdot \alpha_{lp2,1} \ , \tag{3.51}$$
$$\text{Eq. 43} \Rightarrow \alpha_{lp3,1} = \beta_5 \cdot (1 - 1/\beta_1 \cdot \alpha_{lp1,1} - 1/\beta_3 \cdot \alpha_{lp2,1}) \ , \tag{3.52}$$
$$\text{Eq. 44} \Rightarrow (1/\beta_6 - 1/\beta_6 \cdot \alpha_{lp3,1}) = (1 - (1/\beta_2 - 1/\beta_2 \cdot \alpha_{lp1,1}) - (1/\beta_4 - 1/\beta_4 \cdot \alpha_{lp2,1})). \tag{3.53}$$

Therefore, plugging the right-hand side of equation (3.52) to substitute for $\alpha_{lp3,1}$ into equation (3.53), gives an expression with constant terms, $\alpha_{lp2,1}$ terms and $\alpha_{lp1,1}$ terms, thus this can be used to obtain $\alpha_{lp2,1}$ as a function of $\alpha_{lp1,1}$. Thus, arbitrarily picking an $\alpha_{lp1,1}$ value, with $0 < \alpha_{lp1,1} < 1$, and using equations (3.48) to (3.53), this would immediately set the $\alpha_{rp1,1}$, $\alpha_{rp2,1}$ and $\alpha_{lp2,1}$ value. With the $\alpha_{lp2,1}$ value set, equations (3.50) and (3.51) indicate how to set the $\alpha_{rp1,2}$ and $\alpha_{rp2,2}$ values. Finally, using the set values for $\alpha_{lp1,1}$ and $\alpha_{lp2,1}$, equation (3.52) indicates how to set the $\alpha_{lp3,1}$ value.

Thus, picking an $\alpha_{lp1,1}$ value, with $0 < \alpha_{lp1,1} < 1$, after imposing restrictions on how the relative weights placed on each element of the pairs of left and right points found in the $(NL \times NR)$-by-2 vector, restrictions such that the ratios of balances on the pairs are identical to the ratio of balances needed to match the mean-ψ^s constraint when the pairs are viewed in isolation, sets all other $\alpha_{k,j}$ values. With all $\alpha_{k,j}$ values set, the θ_i values appearing on the right-hand side of equations (3.33) to (3.38) are set.

All of the above, which we showed to be feasible for the $NL = 3$ and $NM = 2$ cases could be used for an arbitrary number of left and right points in any initial combination with $NL \geq 2$ & $NM \geq 2$ & $(NL + NM) \geq 5$. There would be $(NL \times NM)$ linear restrictions on the $\alpha_{k,j}$ ratios, or $(NL \times NM)$ linear relations for the $(NM \times (NL-1) + NL \times (NM-1))$ $\{\alpha_{k,j}\}$ parameters, with $(NL \times NM) < (NM \times (NL-1) + NL \times (NM-1))$ allowing for the arbitrary choice of $((NM \times (NL-1) + NL \times (NM-1)) - (NL \times NM))$ free parameters. Setting the value on $((NM \times (NL-1) + NL \times (NM-1)) - (NL \times NM))$ parameters located on the same side (either left or right) and setting values sequentially would prevent the setting of values that would become contradictory with the linear restrictions. It should be noted that $((NM \times (NL-1) + NL \times (NM-1)) - (NL \times NM))$ is the minimum number of free parameters, the number of free parameters when none of the lines linking the left-right pairwise combination have both a common intercept and a common slope.

Cases with $N \geq 2$ points on one side and 1 point on the other, are special cases with

N linear restrictions on $(N-1)$ parameters. Letting the N points be on the left side, it is straightforward to show that in those cases, setting $\{\alpha_{k,j}\}$ such that $\frac{1}{\alpha_{rp,i}} = \frac{bal_{rp}}{bal_{lpi}} \cdot \frac{\psi_{rp} - \overline{\psi^s}^{\,t}}{\overline{\psi^s}^{\,t} - \psi_{lpi}}$ on the first $N-1$ parameters, as required by the first $N-1$ restrictions, delivers a $(1 - \sum_{i=1}^{N-1} \alpha_i)$ term that agrees with the $(1 - \sum_{i=1}^{N-1} \alpha_i)$ term required by the N^{th} restriction. The N^{th} restriction requires $\frac{1}{(1-\sum_{i=1}^{N-1} \alpha_i)} = \frac{bal_{rp}}{bal_{lpN}} \cdot \frac{\psi_{rp} - \overline{\psi^s}^{\,t}}{\overline{\psi^s}^{\,t} - \psi_{lpN}}$. After the first $N-1$ restrictions have been imposed, verifying whether the N^{th} restriction simultaneously holds amounts to checking, after a couple steps of algebra, that
$bal_{rp} \cdot (\psi_{rp} - \overline{\psi^s}^{\,t}) = \sum_i bal_{lpi} \cdot (\overline{\psi^s}^{\,t} - \psi^s_{lpi})$, which clearly holds due to the fact that the initial (ψ^s_k, bal_k) combination satisfied the balance-weighed mean-ψ^s constraint. Assuming that the N points were on the left side was arbitrary, a similar logic could easily be used on a case with N right points and one left point.

The case with, with $NL = 2$ and $NM = 2$, is similar to the above: there would generally be a unique solution to the set of $\alpha_{k,j}$ ratios that satisfy the four restrictions on the pairwise ratios, unless the lines linking the $(lp1, rp1)$ and $(lp2, rp2)$ points had a common slope and a common intercept.

After linear restrictions are imposed and free parameters, $0 < \alpha_{k,j} < 1$, if any, are picked, the θ_i values are set. Given values of $\{\alpha_{k,j}\}$ translate into a given decomposition of $\overline{\text{EGL}}$, the balance-weighted average EGL achieved with the combination of $(NL + NM)$ points. Returning to our example with $NL = 3$ and $NM = 2$, we have:

$$\begin{aligned}\overline{\text{EGL}} = &\left[\alpha_{lp1,1} \cdot \frac{bal_{lp1}}{\phi^t} \cdot \text{EGL}(\psi_{lp1}) + \alpha_{rp1,1} \cdot \frac{bal_{rp1}}{\phi^t} \cdot \text{EGL}(\psi_{rp1})\right] + \ldots \\ &+ \left[(1 - \alpha_{lp3,1}) \cdot \frac{bal_{lp3}}{\phi^t} \cdot \text{EGL}(\psi_{lp3}) + (1 - \alpha_{rp2,1} - \alpha_{rp2,2}) \cdot \frac{bal_{rp2}}{\phi^t} \cdot \text{EGL}(\psi_{rp2})\right],\end{aligned} \tag{3.54}$$

$$\overline{\text{EGL}} = \lambda_1 \cdot \overline{\text{EGL}} + \ldots + \lambda_6 \cdot \overline{\text{EGL}}, \tag{3.55}$$

where $\lambda_1 = \left[\alpha_{lp1,1} \cdot \frac{bal_{lp1}}{\phi^t} \cdot \text{EGL}(\psi_{lp1}) + \alpha_{rp1,1} \cdot \frac{bal_{rp1}}{\phi^t} \cdot \text{EGL}(\psi_{rp1})\right] / \ \overline{\text{EGL}}$. Thus, for a chosen set of $\{\alpha_{k,j}\}$, whether it was fully determined by the linear restrictions or it required the choice of some free parameters, we have $(NL \times NM)$ pairwise combinations that each account for a θ_i fraction of the sum of the products of shares of pool balance and ψ^s, which together sum to $\overline{\psi^s}^{\,t}$, and a λ_i fraction of the sum of the products of shares of pool balance and $\text{EGL}(\psi^s)$, which together sum to $\overline{\text{EGL}}$.

Note, that given the construction of the pairs and the restrictions on the $\alpha_{k,j}$, the following balance-weighted average EGL are feasible:

$$\text{EGL}_1 = \frac{1}{\theta_1} \cdot \lambda_1 \overline{\text{EGL}}, \tag{3.56}$$

$$\ldots, \tag{3.57}$$

$$\text{EGL}_{(NL \times NM)} = \frac{1}{\theta_{(NL \times NM)}} \cdot \lambda_{(NL \times NM)} \overline{\text{EGL}}. \tag{3.58}$$

Achieving those levels of EGL is simply the result of using the left and right points of the i^{th} pairwise combination of the $(NL \times NM)$-by-2 vector and using balances corresponding to the terms in front of the ψ^s term in the $(NL \times NM)$ equivalent of equations (3.33) to (3.38), and scaling them up by $1/\theta_i$.

We have finally reached the point of the contradiction. If $\lambda_i/\theta_i > 1$ for any pairwise combinations, then $\text{EGL}_i > \overline{\text{EGL}}$ and it is possible to exceed the balance-weighted average EGL achieved with the $(NL + NM)$ combination of points with a 2-point combination, which is a contradiction with the initial claim. If $\lambda_i/\theta_i = 1, \forall i$, then any left-right 2-point combination formed from the $(NL + NM)$ points matches the balance-weighted average EGL achieved with the combination of $(NL + NM)$ points, which is another type of contradiction with the initial claim.

■

3.6.7 Forming Loss-Maximizing Pool (Post-Crisis Deals)

To identify the loss-maximizing pool for deals 2009-B, 2009-C and 2010-A, which have identical deal-specific samples thus identically shaped expected gross loss curves (and similar $\overline{\psi^s}^t$ parameters and similar proportions of pool balance in the $\psi^s < 630$ region) we make an additional adjustment to the EGL maximizing algorithm we use to solve the (full) issuer's problem. For those deals, the curve for the more-seasoned loans lies sufficiently below the curves for the least- and less-seasoned loans in the $(620 < \psi^s < 730)$ range that excluding least- and less-seasoned loans from being selected in the $\psi^s > 730$, favors the selection of more-seasoned loans in $\psi^s > 730$ region and leads to higher levels of balance-weighted average EGL. Figure 3.4 shows the loss-maximizing pool for deal 2009-B.

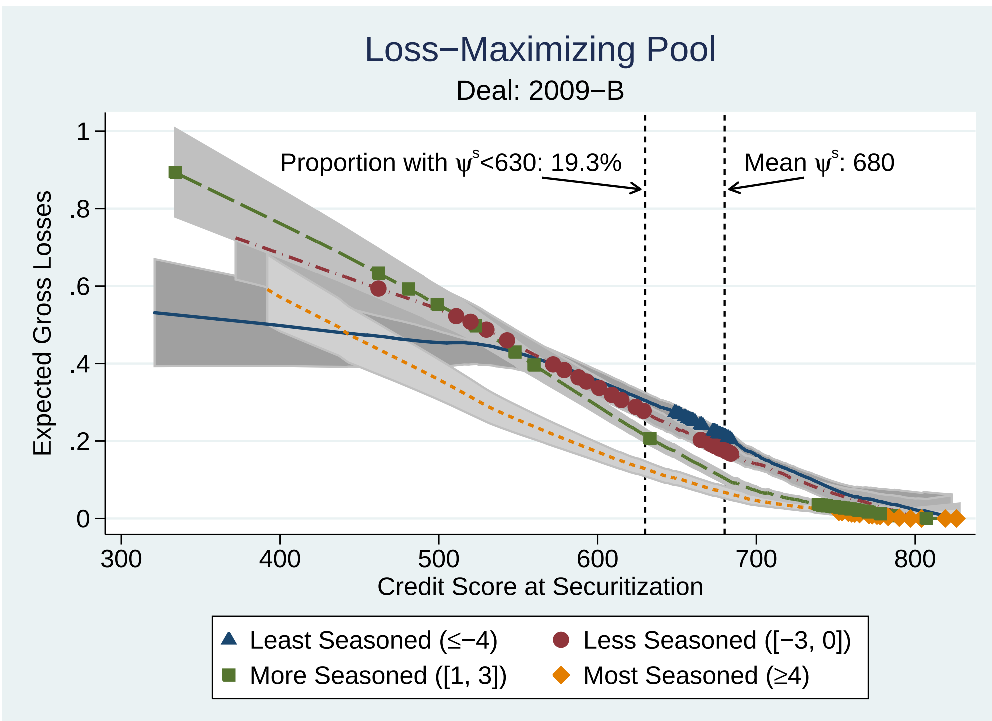


Figure 3.4:
Loss-maximizing pool for a post-crisis deal
This figure shows the loss-maximizing pool for deal 2009-B. For each seasoning group, one marker appears on the figure for both the minimum and maximum credit score with a positive balance. Moving from the minimum to the maximum, there are also markers whenever crossing over a credit score value changes the integer value of the cumulative pool balance, expressed as a percentage. Raw data source: FRBNY CCP/Equifax.

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