Bank liabilities channel

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October 14, 2016

Abstract

The financial intermediation sector is important not only for channeling resources from agents in excess of funds to agents in need of funds (lending channel). By issuing liabilities it also creates financial assets held by other sectors of the economy for insurance (or liquidity) purpose. When the intermediation sector creates less liabilities or their value falls, agents are less willing to engage in activities that are individually risky but desirable in aggregate (bank liabilities channel). The paper studies how financial crises driven by self-fulfilling expectations about the liquidity of the banking sector are transmitted to the real sector of the economy. Since the government could also create financial assets by borrowing, the paper also studies how public debt affects the liabilities issued by the financial intermediation sector and the impact on real allocations.

*I would like to thank Satyajit Chatterjee for an insightful discussion and seminar participants at Bank of Mexico, Bank of Portugal, Boston College, Carlos III University, Cemfi, Cheung Kong Graduate School of Business, EEA meeting in Geneve, Federal Reserve Board, Istanbul school of Central Banking, ITAM Mexico, NOVA University in Lisbon, Purdue University, Shanghai University of Finance and Economics, University of Maryland, University of Melbourne, University of Notre Dame, University of Pittsburgh, UQAM in Montreal. Financial support from NSF Grant 1460013 is gratefully acknowledged.

1 Introduction

There is a well established branch of macroeconomics that added financial market frictions to general equilibrium models. The seminal work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) are the classic references for most of the work done in this area during the last two decades. Although these contributions differ in many details ranging from the micro-foundation of market incompleteness to the scope of the application, they typically share two common features. The first is that the role played by financial frictions in the propagation of shocks to the real sector of the economy is based on the 'credit channel'. The idea is that various shocks can affect the financing capability of borrowers—either in the availability of credit or in its cost—which in turn affects their economic decisions (consumption, investment, employment, etc.).

The second common feature of these models is that they assign a limited role to the financial intermediation sector. This is not to say that there are not studies that emphasize the role of banks for the aggregate economy. Holmstrom and Tirole (1997) provided a theoretical foundation for the central roles of banks in general equilibrium, inspiring subsequent contributions such as Van den Heuvel (2008) and Meh and Moran (2010). However, it is only after the recent crisis that the role of financial intermediaries became central to the research agenda in macroeconomics.

With the renewed interest in financial intermediation, several studies have proposed new models to understand the role of financial intermediaries for the dynamics of the macro-economy.[1] In many of these studies the primary role of the intermediation sector is to channel funds to borrowers. Because of frictions, the funds intermediated depend on the financial conditions of banks. When these conditions deteriorate, the volume of intermediated funds declines, which in turn forces borrowers to cut investments and other economic activities. Therefore, the primary channel through which financial intermediation affects real economic activity remains the typical 'credit or lending channel'. The goal of this paper is to emphasize an additional, possibly complementary, channel which I call 'bank liabilities channel'.

The importance of the financial intermediation sector is not limited to

[1]See for example Adrian, Colla, and Shin (2013), Boissay, Collard, and Smets (2010), Brunnermeier and Sannikov (2014), Corbae and D'Erasmo (2012), De Fiore and Uhlig (2011), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Mendoza and Quadrini (2010), Rampini and Viswanathan (2012).

channeling resources from agents in excess of funds to agents in need of funds (credit channel). By issuing liabilities, it also creates financial assets that can be held by other sectors of the economy for insurance purposes. When the stock or value of bank liabilities decline, the holders of these liabilities (being them households or firms) are less willing to engage in activities that are individually risky because they hold a lower insurance buffer. This has negative consequences for the macroeconomy.

The difference between the 'bank credit channel' and the 'bank liability channel' can be illustrated with an example. Suppose that a bank issues 1 dollar liability and sells it to agent A. The dollar is then used by the bank to make a loan to agent B. By doing so the bank facilitates a more efficient allocation of resources because, typically, agent B is in a condition to create more value than agent A (because of higher productivity or higher marginal utility of consumption). However, if the bank is unable or unwilling to issue the dollar liability, it cannot make the loan and, as a consequence, agent B is forced to cut investment and/or consumption. This illustrates the standard 'credit or lending channel' of financial intermediation.

In addition to the credit channel just described, when the bank issues the dollar liability, it creates a financial asset held by agent A. For this agent, the bank liability represents a financial asset that can be used to insure the uncertain outcome of various economic activities including investment, hiring, consumption. Then, when the holdings of bank liabilities decline, agent A is discouraged from engaging in economic activities that are individually risky but desirable in aggregate. Therefore, it is through the supply of bank liabilities that the financial intermediation sector also plays an important role for the real sector of the economy.

The example illustrates the insurance role played by financial intermediaries in a simple fashion: issuance of traditional bank deposits. However, the complexity of assets and liabilities issued by the intermediation sector has grown over time and many of these activities are important for providing insurance. In some cases, the assets and liabilities issued by the financial sector do not involve significant intermediation of funds in the current period but create the conditions for future payments as in the case of derivatives. In other cases, intermediaries simply facilitate the direct issuance of liabilities by non-financial sectors as in the case of public offering of corporate bonds and shares or the issuance of mortgage-backed securities. Even though these securities do not remain in the portfolio of financial firms, banks still play an important role in facilitating the creation of these securities and, later on, in affecting their value in the secondary market. Corporate mergers and acquisitions can also be seen in this logic since, in addition to promote operational efficiency, they also allow for corporate diversification (*i.e.*, insurance). Still, the direct involvement of banks is crucial for the success of these operations. Therefore, even if many financial assets held by the nonfinancial sector are directly created in the nonfinancial sector (this is the case, for example, for government and corporate bonds), financial intermediaries still play a central role for the initial issuance and later for the functioning of the secondary market. This motivates the focus of the paper on the creation of financial assets by the overall financial intermediation sector which is much broader than commercial banks. Therefore, even if I often use the generic term 'bank', it should be clear that with this term I refer, possibly, to any type of financial intermediary, not just depository institutions.

Another goal of this paper is to explore a possible mechanism that affects the value of bank liabilities. The mechanism is based on self-fulfilling expectations about the liquidity in the financial intermediation sector: when the market expects the intermediation sector to be liquid, banks have the capability of issuing additional liabilities and, therefore, they are liquid. On the other hand, when the market expects the intermediation sector to be illiquid, banks are unable to issue additional liabilities and, as a result, they end up being illiquid. Through this mechanism the model could generate multiple equilibria: a 'good' equilibrium characterized by expanded financial intermediation, sustained economic activity and high asset prices, and a 'bad' equilibrium characterized by reduced financial intermediation, lower economic activity and depressed asset prices. A financial crisis takes place when the economy switches from a good equilibrium to a bad equilibrium.

The existence of multiple equilibria and, therefore, the emergence of a crisis is possible only when banks are highly leveraged. This implies that structural changes that increase the incentives of banks to take more leverage create the conditions for greater financial and macroeconomic instability. In the application of the model I will consider the role of financial innovations.

Although the primary goal of this paper is to study the role of financial intermediaries in creating financial assets, government debt is also a financial instrument that can be held for insurance purpose. In the second part of the paper I will discuss the role of governments in creating financial assets and how this interacts with the assets created by the financial intermediation sector. As we will see, the impact of government debt on the real sector of the economy depends on the type of taxes that the government uses to finance the debt burden (interests and repayments). Although it is possible to design a tax scheme under which public debt improves real allocations, the required tax structure may not be political feasible.

The organization of the paper is as follows. Section 2 describes the theoretical framework and characterizes the equilibrium. Section 3 applies the model to study how financial innovations could affect the stability of the macro-economy. Section 4 discusses the role of government debt. Section 5 concludes.

2 Model

There are three sectors: the entrepreneurial sector, the household sector and the financial intermediation sector. The role of financial intermediaries is to facilitate the transfer of resources between entrepreneurs and households. In the process of intermediating funds, however, financial intermediaries might have an incentive to leverage which could create the conditions for financial and macroeconomic instability.

I describe first the entrepreneurial and household sectors. After characterizing the equilibrium with direct borrowing and lending between these two sectors, I introduce the financial intermediation sector under the assumption that direct borrowing and lending is not possible or efficient.

2.1 Entrepreneurial sector

In the entrepreneurial sector there is a unit mass of entrepreneurs, indexed by i, with lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t^i)$. Entrepreneurs are individual owners of firms, each operating the production function $y_t^i = z_t^i h_t^i$, where h_t^i is the input of labor supplied by households at the market wage w_t, and z_t^i is an idiosyncratic productivity shock. The productivity shock is independently and identically distributed among firms and over time, with probability distribution $\Gamma(z)$. As in Arellano, Bai, and Kehoe (2011), the input of labor h_t^i is chosen before observing z_t^i, and therefore, labor is risky.

Since the productivity of labor is observed after the hiring decision and entrepreneurs are risk-averse, labor is risky. It becomes then important to define what is available for entrepreneurs to insure this risk. I assume that entrepreneurs have access only to a market for bonds that cannot be contingent on the realization of the idiosyncratic productivity. Therefore, markets are incomplete. As we will see, the bonds held by entrepreneurs are liabilities issued by banks with gross interest rate R_t^b.

An entrepreneur i enters period t with bonds b_t^i and chooses the labor input h_t^i. After the realization of the idiosyncratic shock z_t^i, he/she chooses consumption c_t^i and next period bonds b_{t+1}^i. The budget constraint is

$$c_t^i + \frac{b_{t+1}^i}{R_t^b} = (z_t^i - w_t)h_t^i + b_t^i. \tag{1}$$

Because labor h_t^i is chosen before the realization of z_t^i, while the saving decision is made after the observation of z_t^i, it will be convenient to define $a_t^i = b_t^i + (z_t^i - w_t)h_t^i$ the entrepreneur's wealth after production. Given the timing structure, the input of labor h_t^i depends on b_t^i while the saving choice b_{t+1}^i depends on a_t^i. The optimal entrepreneur's policies are characterized by the following lemma:

Lemma 2.1 *Let ϕ_t satisfy the condition $\mathbb{E}_z\left\{\frac{z-w_t}{1+(z-w_t)\phi_t}\right\} = 0$. The optimal entrepreneur's policies are*

$$\begin{aligned} h_t^i &= \phi_t b_t^i, \\ c_t^i &= (1-\beta)a_t^i, \\ \frac{b_{t+1}^i}{R_t^b} &= \beta a_t^i. \end{aligned}$$

Proof 2.1 *See Appendix A.*

The demand for labor is linear in the initial wealth of the entrepreneur b_t^i. The term of proportionality ϕ_t is defined by condition $\mathbb{E}_z\left\{\frac{z-w_t}{1+(z-w_t)\phi_t}\right\} = 0$, where the expectation is over the idiosyncratic shock z with probability distribution $\Gamma(z)$. Since the only endogenous variable that affects ϕ_t is the wage rate, I will denote this term by the function $\phi(w_t)$. It can be verified that this function is strictly decreasing in w_t.

Because $\phi(w_t)$ is the same for all entrepreneurs, I can derive the aggregate demand for labor as

$$H_t = \phi(w_t)\int_i b_t^i = \phi(w_t)B_t,$$

where capital letters denote average (per-capita) variables. The aggregate demand depends negatively on the wage rate—which is a standard property—and positively on the financial wealth of entrepreneurs—which is a special property of this model. This derives from the fact that labor is risky and entrepreneurs are willing to hire labor only if they hold financial wealth that allows for consumption smoothing in the eventuality of a low realization of the productivity shock.

Also linear is the consumption policy which follows from the logarithmic specification of the utility function. This property allows for linear aggregation. Another property worth emphasizing is that in a stationary equilibrium with constant B_t, the interest rate must be lower than the intertemporal discount rate,[2] that is, $R^b < 1/\beta - 1$.

2.2 Household sector

There is a unit mass of households with lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_t - \alpha \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right)$, where c_t is consumption and h_t is the supply of labor. Households do not face idiosyncratic risks and the assumption of risk neutrality is not important for the key results of the paper as I will discuss later.

Each household holds a non-reproducible asset available in fixed supply $\overline{K}$, with each unit producing χ units of consumption goods. The asset is divisible and can be traded at the market price p_t. We can think of the asset as housing and χ as the services produced by one unit of housing. Households can borrow at the gross interest rate R_t^l and face the budget constraint

$$c_t + l_t + (k_{t+1} - k_t)p_t = \frac{l_{t+1}}{R_t^l} + w_t h_t + \chi k_t,$$

where l_t is the loan contracted in period $t-1$ and due in the current period t, and l_{t+1} is the new debt that will be repaid in the next period $t+1$.

Debt is constrained by the following borrowing limit

$$l_{t+1} \leq \kappa + \eta \mathbb{E}_t p_{t+1} k_{t+1}, \tag{2}$$

[2]To see this, consider the first order condition of an individual entrepreneur for the choice of b_{t+1}^i. This is the typical euler equation that, with log preferences, takes the form $1/c_t^i = \beta R^b \mathbb{E}_t(1/c_{t+1}^i)$. Because individual consumption c_{t+1}^i is stochastic, $\mathbb{E}_t(1/c_{t+1}^i) > 1/\mathbb{E}_t c_{t+1}^i$. Therefore, if $\beta R^b = 1$, we would have that $\mathbb{E}_t c_{t+1}^i > c_t^i$, implying that individual consumption would growth on average. But then aggregate consumption would not be bounded, which violates the hypothesis of a stationary equilibrium. I will come back to this property later.

where κ and η are constant parameters.

Later I will consider two special cases. In the first case I set $\eta = 0$ so that the borrowing limit is just a constant. This special case allows me to characterize the equilibrium analytically but the asset price p_t will be constant. In the second case I set $\kappa = 0$ so that the borrowing limit depends on the collateral value of the asset. With this specification the model also generates interesting predictions about the asset price p_t but the full characterization of the equilibrium can be done only numerically.

Appendix C writes down the households' problem and derives the first order conditions. They take the form

$$\alpha h_t^{\frac{1}{\nu}} = w_t, \tag{3}$$

$$1 = \beta R_t^l (1 + \mu_t), \tag{4}$$

$$p_t = \beta \mathbb{E}_t \Big[\chi + (1 + \eta \mu_t) p_{t+1} \Big], \tag{5}$$

where $\beta \mu_t$ is the Lagrange multiplier associated with the borrowing constraint. From the third equation we can see that, if $\eta = 0$, the asset price p_t must be constant.

2.3 Equilibrium with direct borrowing and lending

Before introducing the financial intermediation sector it would be instructive to characterize the equilibrium with direct borrowing and lending. In this case the bonds held by entrepreneurs are equal to the loans taken by households and market clearing implies $R_t^b = R_t^l = R_t$.

Proposition 2.1 *In absence of aggregate shocks, the economy converges to a steady state in which households borrow from entrepreneurs and $\beta R < 1$.*

Proof 2.1 *See Appendix B*

The fact that the steady state interest rate is lower than the intertemporal discount rate is a consequence of the uninsurable risk faced by entrepreneurs. If $\beta R = 1$, entrepreneurs would continue to accumulate bonds without limit in order to insure the idiosyncratic risk. The supply of bonds from households, however, is limited by the borrowing constraint of households. To insure that entrepreneurs do not accumulate an infinite amount of bonds, the interest rate has to fall below the intertemporal discount rate.

The equilibrium in the labor market can be characterized as the simple intersection of aggregate demand and supply as depicted in Figure 1. The aggregate demand was derived in the previous subsection and takes the form $H_t^D = \phi(w_t)B_t$. It depends negatively on the wage rate w_t and positively on the aggregate wealth (bonds) of entrepreneurs, B_t. The supply is derived from the households' first order condition (3) and takes the form $H_t^S = \left(\frac{w_t}{\alpha}\right)^{\nu}$.

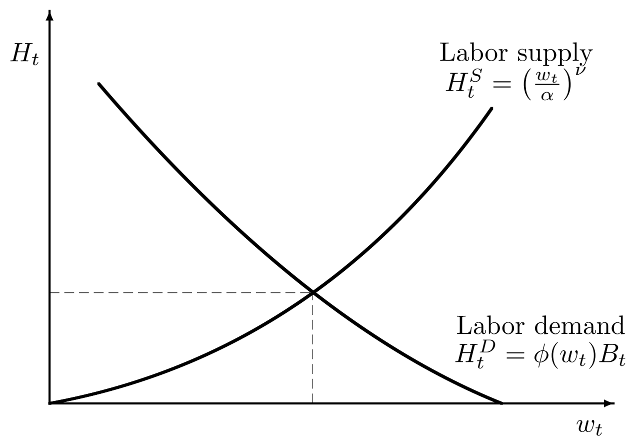

Figure 1: Labor market equilibrium.

The dependence of the demand of labor from the financial wealth of entrepreneurs is a key property of this model. When entrepreneurs hold a lower value of B_t, the demand for labor declines and in equilibrium there is lower employment and production. Importantly, the reason lower values of B_t decreases the demand for labor is not because employers do not have funds to finance hiring or because they face a higher financing cost. In fact, employers do not need any financing to hire and produce. Instead, the transmission mechanism is based on the lower financial wealth of entrepreneurs which is held as an insurance buffer against the idiosyncratic risk. This mechanism is clearly distinct from the traditional 'credit channel' where firms are in need of funds to finance employment (for example, because wages are paid in advance) or to finance investment.

The next step is to introduce financial intermediaries and show that a fall in B_t could be the result of a crisis that originates in the financial sector.

2.4 Some remarks on equilibrium properties

In the equilibrium described above, producers (entrepreneurs) are net savers while households are net borrowers. Since it is customary to work with models in which firms are net borrowers (for example in the studies referenced in the introduction), this property may seem counterfactual. This financial structure, however, is not inconsistent with the recent changes observed in the United States.

It is well known that during the last two and half decades, US corporations have increased their holdings of financial assets. As shown in Figure 2, the *net* financial assets—that is, the difference between financial assets and liabilities—have become positive in the 2000s for the nonfinancial corporate sector. The only exception is at the pick of the 2008 crisis when the financial assets held by corporations declined in value. Therefore, recent evidence shows that US corporations are no longer net borrowers in aggregate.

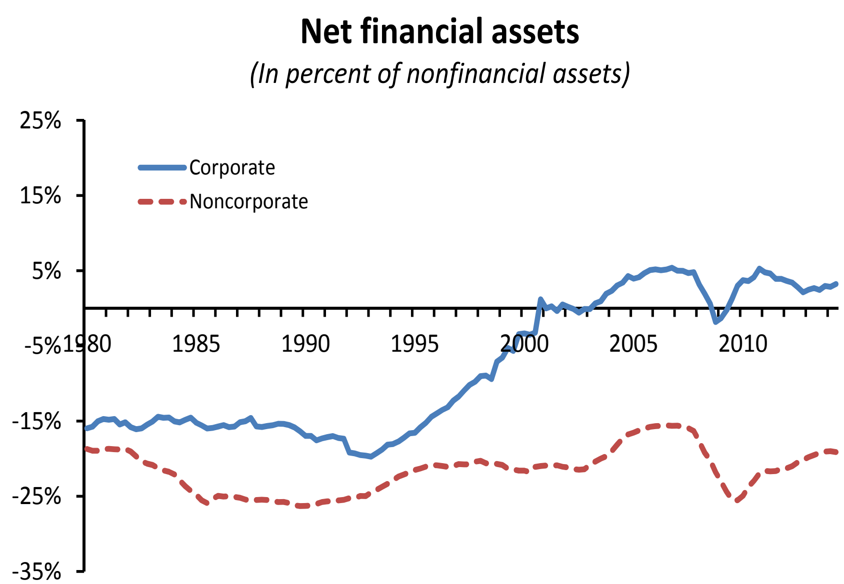

Figure 2: Net financial assets (assets minus liabilities) in the nonfinancial business sector as a percentage of nonfinancial assets. Source: Flows of Funds Accounts.

The reversal from net borrower to net lender did not arise in the noncorporate sector. The net financial assets of the noncorporate sector remained negative, without any particular trend. Still, the change experienced by the corporate sector shows that a large segment of the business sector is no longer dependent on external financing.[3] Even if there is significant heterogeneity hidden in the aggregate figures, these numbers suggest that the proportion of financially dependent firms has declined significantly over time. This pattern is shown in more details in Shourideh and Zetlin-Jones (2012) using data from the Flows of Funds and firm level data from Compustat. Also related is Eisfeldt and Muir (2012) showing that there is a strong correlation between the funds raised externally by corporations and their accumulation of liquid financial assets (suggesting that raising external funds does not necessarily increase the net financial liabilities of firms). The model developed here is meant to capture the growing importance of firms that are no longer dependent on external financing.

The second remark relates to the view that firms are not dependent on external financing if they hold positive net financial assets. This is a 'static' definition of external dependence and captures the idea that a firm is capable of increasing spending in the current period only if it can borrow more. Similarly, a firm is financially independent if it can increase its current spending without the need of borrowing. This definition of financial independence, however, does not guarantee that a firm is financially independent in the future. Negative shocks could reduce the financial wealth of entrepreneurs and force them to cut future consumption (or dividends). This introduces a 'dynamic' concept of financial dependence which is different from the most common definition formalized in traditional models with financial constraints. In these models, the financial mechanism affects the production and investment decisions in important ways only when firms are financially constrained in the period in which these decisions are made. More specifically, the financial mechanism becomes important only when the multiplier associated with the borrowing constraint turns positive.

The third remark is that the primary reason for which the entrepreneurial sector is a net lender in equilibrium is not because entrepreneurs are more risk-averse than households. Instead, it follows from the assumption that only entrepreneurs are exposed to uninsurable risks. As long as producers face more risk than households, the former would continue to lend to the latter even if households were risk averse.

[3] If we aggregate the corporate sector with the noncorporate sector, the overall net borrowing remains positive but has declined dramatically from about 20 percent in the early nineties to about 5 percent.

The final remark relates to the assumption that the idiosyncratic risk faced by entrepreneurs cannot be insured away (market incompleteness). Given that households are risk neutral, it would be optimal for entrepreneurs to offer a wage that is contingent on the output of the firm. Although this is excluded by assumption, it is not difficult to extend the model so that the lack of insurance from households is an endogenous outcome of information asymmetries. The idea is that, when the wage is state-contingent, firms could use their information advantage to gain opportunistically from workers. The same argument can be used to justify more generally the absence of a market for claims that are contingent on the realization of the idiosyncratic shock.

2.5 Financial intermediation sector

If direct borrowing is not feasible or efficient, financial intermediaries become important for transferring funds from lenders (entrepreneurs) to borrowers (households) and to create financial assets that could be held for insurance purposes. It is under this assumption that I introduce the financial intermediation sector.

There is a continuum of infinitely lived banks. Banks are profit maximizing firms owned by households. Even if I use the term 'banks' as a reference to financial intermediaries, it should be clear that the financial sector in the model is representative of all financial firms, not only commercial banks or depositary institutions. The assumption that banks are held by households, as opposed to entrepreneurs, is an important assumption as will become clear later. It also makes the analysis simpler because households are risk neutral while entrepreneurs are risk averse.

Banks start the period with loans made to households, l_t, and liabilities held by entrepreneurs, b_t. The difference between loans and liabilities is the bank equity $e_t = l_t - b_t$.

Given the beginning of period balance sheet position, the bank could default on its liabilities. In case of default creditors have the right to liquidate the bank assets l_t. However, they may not recover the full value of the assets. In particular, with probability λ creditors recover only a fraction $\underline{\xi} < 1$.

Denoting by $\xi_t \in \{\underline{\xi}, 1\}$ the fraction of the bank assets recovered by creditors, the recovery value can be written more generally as $\xi_t l_t$. Therefore, with probability λ creditors recover $\underline{\xi} l_t$ and with probability $1-\lambda$ they recover the full value l_t. The variable ξ_t is the same for all banks (aggregate stochastic variable) and its value was unknown when the bank issued the liabilities b_t

and made the loans l_t in period $t-1$.

The recovery fraction ξ_t will be derived endogenously in the model. For the moment, however, it will be convenient to think of ξ_t as an exogenous stochastic variable.

Once $\xi_t \in \{\underline{\xi}, 1\}$ becomes known at the beginning of period t, the bank could use the threat of default to renegotiate the outstanding liabilities. Assuming that the bank has the whole bargaining power, the liabilities can be renegotiated to $\xi_t l_t$. Therefore, after renegotiation, the residual liabilities of the bank are

$$\tilde{b}_t(b_t, l_t) = \begin{cases} b_t, & \text{if } b_t \leq \xi_t l_t \\ \\ \xi_t l_t & \text{if } b_t > \xi_t l_t \end{cases} \tag{6}$$

Financial intermediation implies an operation cost that depends on the leverage chosen by the bank. Denoting the leverage by $\omega_{t+1} = b_{t+1}/l_{t+1}$, the cost takes the form

$$\varphi\left(\omega_{t+1}\right) q_t b_{t+1}.$$

The cost is proportional to the funds raised by the bank, $q_t b_{t+1}$, and the unit cost $\varphi(\omega_{t+1})$ is a function of the leverage.

Assumption 1 *The function $\varphi(\omega_{t+1})$ is positive and twice continuously differentiable with $\varphi'(\omega_{t+1}), \varphi''(\omega_{t+1}) = 0$ if $\omega_{t+1} \leq \underline{\xi}$ and $\varphi'(\omega_{t+1}), \varphi''(\omega_{t+1}) > 0$ if $\omega_{t+1} > \underline{\xi}$.*

The assumption that the derivative of the cost function becomes positive when the leverage exceeds the threshold $\underline{\xi}$ captures, in reduced form, the potential agency frictions that become more severe when the leverage increases.

Denote by $\overline{R}_t^b$ the expected gross return on the market portfolio of bank liabilities issued in period t and repaid in period $t+1$ (expected return on liabilities issued by the whole banking sector). Since banks are atomistic and the sector is competitive, the expected return on the liabilities issued by an individual bank must be equal to the aggregate expected return $\overline{R}_t^b$. Therefore, the price of liabilities $q_t(b_{t+1}, l_{t+1})$ issued by an individual bank at t must satisfy

$$q_t(b_{t+1}, l_{t+1}) b_{t+1} = \frac{1}{\overline{R}_t^b} \mathbb{E}_t \tilde{b}_{t+1}(b_{t+1}, l_{t+1}). \tag{7}$$

The left-hand-side is the payment made by investors (entrepreneurs) to purchase b_{t+1}. The term on the right-hand-side is the expected repayment in the next period, discounted by $\overline{R}_t^b$ (the expected market return).

The budget constraint of the bank, after the renegotiation of the liabilities at the beginning of the period, can be written as

$$\tilde{b}_t(b_t, l_t) + \frac{l_{t+1}}{R_t^l} + d_t = l_t + q_t(b_{t+1}, l_{t+1})b_{t+1}\left[1 - \varphi\left(\frac{b_{t+1}}{l_{t+1}}\right)\right], \tag{8}$$

The left-hand-side of the budget contains the residual liabilities after renegotiation, the cost of issuing new loans, and the dividends paid to shareholders (households). The right-hand-side contains the initial loans and the funds raised by issuing new liabilities net of the operation cost. Using the arbitrage condition (7), the funds raised with new debt are equal to $\mathbb{E}_t\tilde{b}_{t+1}(b_{t+1}, l_{t+1})/\overline{R}_t^b$.

The problem solved by the bank can be written recursively as

$$V_t(b_t, l_t) = \max_{d_t, b_{t+1}, l_{t+1}} \left\{ d_t + \beta \mathbb{E}_t V_{t+1}(b_{t+1}, l_{t+1}) \right\} \tag{9}$$

subject to (6), (7), (8).

The decision to renegotiate existing liabilities is implicitly accounted by the function $\tilde{b}_t(b_t, l_t)$. The leverage cannot exceed 1 since in this case the bank would renegotiate with certainty. Once the probability of renegotiation is 1, a further increase in b_{t+1} does not increase the borrowed funds $\mathbb{E}_t\tilde{b}_{t+1}(b_{t+1}, l_{t+1})/\overline{R}_t^b$ but raises the operation cost. Therefore, Problem (9) is also subject to the constraint $b_{t+1} \leq l_{t+1}$.

The optimal policies of the bank are characterized by the first order conditions with respect to b_{t+1} and l_{t+1}. Denote by $\omega_{t+1} = b_{t+1}/l_{t+1}$ the bank leverage. The first order conditions, derived in Appendix D, take the form

$$\frac{1}{\overline{R}_t^b} \geq \beta\Big[1 + \Phi(\omega_{t+1})\Big] \tag{10}$$

$$\frac{1}{R_t^l} \geq \beta\Big[1 + \Psi(\omega_{t+1})\Big], \tag{11}$$

where $\Phi(\omega_{t+1})$ and $\Psi(\omega_{t+1})$ are increasing functions of the leverage ω_{t+1}. These conditions are satisfied with equality if $\omega_{t+1} < 1$ and with inequality if $\omega_{t+1} = 1$ (given the constraint $\omega_{t+1} \leq 1$).

Condition (10) makes clear that it is the leverage of the bank $\omega_{t+1} = b_{t+1}/l_{t+1}$ that matters, not the scale of operation b_{t+1} or l_{t+1}. This follows from the linearity of the intermediation technology and the risk neutrality of banks. The leverage matters because the renegotiation cost is convex in the leverage. These properties imply that in equilibrium all banks choose the same leverage (although they could chose different scales of operation).

Because the first order conditions (10) and (11) depend only on one individual variable—the leverage ω_{t+1}—there is no guarantee that these conditions are both satisfied for arbitrary values of $\overline{R}_t^b$ and R_t^l. In the general equilibrium, however, these rates adjust to clear the markets for bank liabilities and loans. Thus, both conditions will be satisfied in equilibrium.

Further exploration of the first order conditions (10) and (11) reveals that the funding cost $\overline{R}_t^b$ is smaller than the interest rate on loans R_t^l, which is necessary to cover the operation cost of the bank. This property is stated formally in the following lemma.

Lemma 2.2 *If $\omega_{t+1} > \underline{\xi}$, then $\overline{R}_t^b < R_t^l < \frac{1}{\beta}$ and the return spread $R_t^l/\overline{R}_t^b$ increases with ω_{t+1}.*

Proof 2.2 *See Appendix E*

Therefore, there is a spread between the funding rate and the lending rate. Intuitively, the choice of a positive leverage increases the operation cost. The bank will choose to do so only if there is a differential between the cost of funds and the return on the investment. As the spread increases so does the leverage chosen by banks. When the leverage exceeds $\underline{\xi}$, banks could default with positive probability. This generates a loss of financial wealth for entrepreneurs, causing a macroeconomic contraction through the 'bank liabilities channel' as described earlier.

2.6 Banking liquidity and endogenous ξ_t

To make ξ_t endogenous, I now interpret this variable as the liquidation price of bank assets which will be determined in equilibrium. The liquidity of the whole banking sector plays a central role in determining this price. The structure of the market for liquidated assets is based on two assumptions which are similar to Perri and Quadrini (2011).

Assumption 2 *If a bank is liquidated, the assets l_t are divisible and can be sold either to other banks or to other sectors (households and entrepreneurs). However, other sectors can recover only a fraction $\underline{\xi} < 1$.*

Therefore, in the event of liquidation, it is more efficient to sell the liquidated assets to other banks since they have the ability to recover the full value l_t while other sectors can recover only $\underline{\xi} l_t$. This is a natural assumption since banks have, supposedly, a comparative advantage in the management of financial investments. However, even if it is more efficient to sell the liquidated assets to banks, for this to happen they need to have the liquidity to purchase the assets.

Assumption 3 *Banks can purchase the assets of a liquidated bank only if $b_t < \xi_t l_t$.*

A bank is liquid if it can issue new liabilities at the beginning of the period without renegotiating. Obviously, if the bank starts with $b_t > \xi_t l_t$—that is, the liabilities are greater that the liquidation value of its assets—the bank will be unable to raise additional funds: potential investors know that the new liabilities (as well as the outstanding liabilities) are not collateralized and the bank will renegotiate immediately after receiving the funds.

To better understand these assumptions, consider the condition for not renegotiating, $b_t \leq \xi_t l_t$, where now $\xi_t \in \{\underline{\xi}, 1\}$ is the liquidation price of bank assets at the beginning of the period. If this condition is satisfied, banks have the option to raise additional funds at the beginning of the period to purchase the assets of a defaulting bank. This insures that the market price of the liquidated assets is $\xi_t = 1$. However, if $b_t > \xi_t l_t$ for all banks, there will not be any bank with unused credit. As a result, the liquidated assets can only be sold to non-banks and the price will be $\xi_t = \underline{\xi}$. Therefore, the value of liquidated assets depends on the financial decision of banks, which in turn depends on the expected liquidation value of their assets. This interdependence creates the conditions for multiple self-fulfilling equilibria.

Proposition 2.2 *There exists multiple equilibria if and only if the leverage of the bank is within the two liquidation prices, that is,* $\underline{\xi} \leq \omega_t \leq 1$.

Proof 2.2 *See appendix F.*

Given the multiplicity, the equilibrium will be selected stochastically by sunspot shocks. Denote by ε a variable that takes the value of zero with probability λ and 1 with probability $1-\lambda$. The probability of a low liquidation price, denoted by $\theta(\omega_t)$, is equal to

$$\theta(\omega_t) = \begin{cases} 0, & \text{if} \quad \omega_t < \underline{\xi} \\ \lambda, & \text{if} \quad \underline{\xi} \leq \omega_t \leq 1 \\ 1, & \text{if} \quad \omega_t > 1 \end{cases}$$

If the leverage is sufficiently small ($\omega_t < \underline{\xi}$), banks do not renegotiate even if the liquidation price is low. But then the price cannot be low since banks remain liquid for any expectation of the liquidation price ξ_t and, therefore, for any draw of the sunspot variable ε. Instead, when the leverage is between the two liquidation prices ($\underline{\xi} \leq \omega_t \leq 1$), the liquidity of banks depends on the expectation of this price. Therefore, the equilibrium outcome depends on the realization of the sunspot variable ε. When $\varepsilon = 0$—which happens with probability λ—the market expects the low liquidation price $\xi_t = \underline{\xi}$, making the banking sector illiquid. On the other hand, when $\varepsilon = 1$—which happens with probability $1 - \lambda$—the market expects the high liquidation price $\xi_t = 1$ so that the banking sector remains liquid. The dependence of the probability $\theta(\omega_t)$ on the leverage of the banking sector plays an important role for the results of this paper.

2.7 General equilibrium

To characterize the general equilibrium I first derive the aggregate demand for bank liabilities from the optimal saving of entrepreneurs. I then derive the supply by consolidating the demand of loans from households with the optimal policy of banks. In this section I assume that $\eta = 0$ so that the borrowing limit specified in equation (2) reduces to $l_{t+1} \leq \kappa$. This allows me to characterize the equilibrium analytically.

Demand for bank liabilities As shown in Lemma 2.1, the optimal saving of entrepreneurs takes the form $b^i_{t+1}/R^b_t = \beta a^i_t$, where a^i_t is the end-of-period wealth $a^i_t = \tilde{b}^i_t + (z^i_t - w_t)h^i_t$. The lemma was derived under the assumption that the bonds purchased by the entrepreneurs were not risky, that is, entrepreneurs receive b_{t+1} units of consumption goods with certainty in the next period $t+1$. In the extension with financial intermediation, bank liabilities are risky since banks can renegotiate their debt. Thanks to the log specification of the utility function, however, Lemma 2.1 continue to hold once we replace b^i_t with its renegotiated value $\tilde{b}^i_t$.[4]

Since $h^i_t = \phi(w_t)\tilde{b}^i_t$ (see Lemma 2.1), the end-of-period wealth can be rewritten as $a^i_t = [1 + (z^i_t - w_t)\phi(w_t)]\tilde{b}^i_t$. Substituting into the optimal saving and aggregating over all entrepreneurs we obtain

$$B_{t+1} = \beta R^b_t \Big[1 + (\bar{z} - w_t)\phi(w_t)\Big]\tilde{B}_t. \tag{12}$$

This equation defines the aggregate demand for bank liabilities as a function of the interest rate R^b_t, the wage rate w_t, and the beginning-of-period aggregate wealth of entrepreneurs $\tilde{B}_t$. Remember that the tilde sign denotes the financial wealth of entrepreneurs after the renegotiation of banks. Also notice that R^b_t is not the 'expected' return from bank liabilities which we previously denoted by $\overline{R}^b_t$ since banks will repay B_{t+1} in full only with some probability. Instead, R^b_t is the inverse of the price at time t of bank liabilities.

Using the equilibrium condition in the labor market, we can express the wage rate as a function of $\tilde{B}_t$. In particular, equalizing the demand for labor, $H^D_t = \phi(w_t)\tilde{B}_t$, to the supply from households, $H^S_t = (w_t/\alpha)^\nu$, the wage w_t becomes a function of only $\tilde{B}_t$. We can then use this function to replace w_t in (12) and express the demand for bank liabilities as a function of only $\tilde{B}_t$ and R^b_t. This takes the form

$$B_{t+1} = s(\tilde{B}_t)\, R^b_t, \tag{13}$$

where $s(\tilde{B}_t)$ is strictly increasing in the wealth of entrepreneurs $\tilde{B}_t$.

Figure 3 plots this function for a given value of $\tilde{B}_t$. As we change $\tilde{B}_t$, the slope of the demand function changes. More specifically, keeping the interest rate constant, higher initial wealth $\tilde{B}_t$ implies higher demand for B_{t+1}.

[4] The proof requires only a trivial extension of the proof of Lemma 2.1 and is omitted.

Supply of bank liabilities The supply of bank liabilities is derived from consolidating the borrowing decisions of households with the investment and funding decisions of banks.

According to Lemma 2.2, when banks are highly leveraged, that is, $\omega_{t+1} > \underline{\xi}$, the interest rate on loans must be smaller than the intertemporal discount rate ($R_t^l < 1/\beta$). From the households' first order condition (4) we can see that $\mu_t > 0$ if $R_t^l < 1/\beta$. Therefore, the borrowing constraint for households is binding, which implies $L_{t+1} = \kappa$. Since $B_{t+1} = \omega_{t+1} L_{t+1}$, the supply of bank liabilities is then $B_{t+1} = \kappa\omega_{t+1}$.

When the lending rate is equal to the intertemporal discount rate, instead, the demand of loans from households is undetermined, which in turn implies indeterminacy in the supply of bank liabilities. In this case the liabilities of banks are demand determined. In summary, the supply of bank liabilities is

$$B^s(\omega_{t+1}) = \begin{cases} \text{Undetermined}, & \text{if} \quad \omega_{t+1} < \underline{\xi} \\ \\ \kappa\omega_{t+1}, & \text{if} \quad \omega_{t+1} \geq \underline{\xi} \end{cases} \tag{14}$$

So far I have derived the supply of bank liabilities as a function of bank leverage ω_{t+1}. However, the leverage of banks also depends on the cost of borrowing $\overline{R}_t^b/(1-\tau)$ through condition (10). The expected return on bank liabilities for investors, $\overline{R}_t^b$, is in turn related to the interest rate R_t^b by the condition

$$\overline{R}_t^b = \left[1 - \theta(\omega_{t+1}) + \theta(\omega_{t+1})\left(\frac{\underline{\xi}}{\omega_{t+1}}\right)\right] R_t^b. \tag{15}$$

With probability $1 - \theta(\omega_{t+1})$ banks do not renegotiate and the ex-post return is R_t^b. With probability $\theta(\omega_{t+1})$ banks renegotiate and investors recover only a fraction $\underline{\xi}/\omega_{t+1}$ of the initial investment. Therefore, when banks renegotiate, the actual ex-post return is $(\underline{\xi}/\omega_{t+1})R_t^b$.

Using (15) to replace $\overline{R}_t^b$ in equation (10) I obtain a function that relates the interest rate R_t^b to the leverage of banks ω_{t+1}. Finally, I combine this function with $B_{t+1} = \kappa\omega_{t+1}$ to obtain the supply of bank liabilities as a function of R_t^b. This function is plotted in Figure 3. As can be seen from the figure, the demand is undetermined when the interest rate is equal to $1/\beta$ and strictly decreasing for lower values of the interest rate until it reaches κ.

Equilibrium The intersection of demand and supply of bank liabilities plotted in Figure 3 defines the general equilibrium. The supply (from banks)

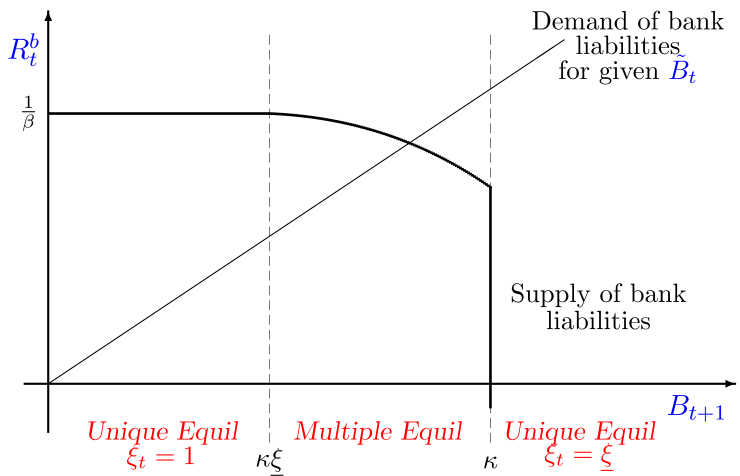

Figure 3: Demand and supply of bank liabilities.

is decreasing in the funding rate R_t^b while the demand (from entrepreneurs) is increasing in R_t^b. The demand is plotted for a particular value of outstanding post-renegotiation liabilities $\tilde{B}_t$. By changing the outstanding liabilities, the slope of the demand function changes.

The figure also indicates the regions with unique or multiple equilibria. When the interest rate is $1/\beta$, banks are indifferent in the choice of leverage $\omega_{t+1} \leq \underline{\xi}$. When the funding rate falls below this value, however, the optimal leverage starts to increase above $\underline{\xi}$ and the economy enters in the region with multiple equilibria. Once the leverage reaches $\omega_{t+1} = 1$, a further decline in the interest rate paid by banks does not lead to higher leverages since the choice of $\omega_{t+1} > 1$ would cause renegotiation with probability 1.[5]

The equilibrium illustrated in Figure 3 is for a particular value of entrepreneurial wealth $\tilde{B}_t$. Given the equilibrium value of B_{t+1} and the random draw of the sunspot shock ε, we determine the next period wealth of entrepreneurs $\tilde{B}_{t+1}$. The new $\tilde{B}_{t+1}$ will determine a new slope for the demand of bank liabilities, and therefore, new equilibrium values for B_{t+1}. Depending on the parameters, the economy may or may not reach a steady state. This would be the case if the economy does not transit to the region with multiple equilibria if it starts outside the multiplicity region. This will depend on the

[5] The dependence of the existence of multiple equilibria from the leverage of the economy is also a feature of the sovereign default model of Cole and Kehoe (2000).

operation cost $\varphi(\omega_t)$.

Let $\tau = \varphi(\omega_t)$ for $\omega_t \leq \underline{\xi}$. Remember that according to Assumption 1 the operation cost is constant for values of $\omega_t \leq \underline{\xi}$. This constant value is denoted by τ. I can then state the following proposition.

Proposition 2.3 *There exists $\hat{\tau} > 0$ such that: If $\tau \geq \hat{\tau}$, the economy converges to a steady state without renegotiation. If $\tau < \hat{\tau}$, the economy never converges to a steady state but switches stochastically between equilibria with and without renegotiation in response to the sunspot shock ε.*

Proof 2.3 *See Appendix G*

In a steady state, the interest rate paid on bank liabilities must be equal to $R_t^b = (1 - \tau)/\beta$. With this interest rate banks do not have incentive to leverage because the funding cost is equal to the return on loans. This requires that the demand for bank liabilities is sufficiently low, which cannot be the case when $\tau = 0$. With $\tau = 0$, in fact, the steady state interest rate must be equal to $1/\beta$. But then entrepreneurs continue to accumulate bank liabilities without bound for precautionary reasons. The demand for bank liabilities will eventually become bigger than the supply (which is bounded by the borrowing constraint of households), driving the interest rate below $1/\beta$. As the interest rate falls, multiple equilibria become possible.

Bank leverage and crises Figure 3 illustrates that different equilibria can emerge depending the leverage of banks. When banks increase their leverage, the economy switches from a state in which the equilibrium is unique (no crises) to a state with multiple equilibria. But even if the economy is already in a state with multiple equilibria, the increase in leverage implies that the consequences of a crisis are bigger. In fact, when the economy switches from a good equilibria to a bad equilibria, bank liabilities are renegotiated to $\kappa\underline{\xi}$. Therefore, bigger are the liabilities issued by banks and larger are the losses incurred by entrepreneurs holding these liabilities. Larger financial losses incurred by entrepreneurs then imply larger declines in the demand for labor, which in turn cause larger macroeconomic contractions (financial crisis). In the next section I will examine the role of financial innovations in determining the incentive to leverage and the economic magnitude of crises.

3 Financial innovations and macroeconomic stability

In this section I use the model to study the impact of financial innovation numerically. The period in the model is a quarter and the discount factor is set to $\beta = 0.9825$, implying an annual intertemporal discount rate of about 7%. The parameter ν in the utility function of households is the elasticity of the labor supply. I set this elasticity to 3, which is in the range of values used in macroeconomic models. The utility parameter α is chosen to have an average working time of 0.3.

The average productivity of entrepreneurs is normalized to $\bar{z} = 1$. Since the average input of labor is 0.3, the average production in the entrepreneurial sector is also 0.3. The supply of the fixed asset is normalized to $\bar{k} = 1$ and its production flow is set to $\chi = 0.05$. Total production is the sum of entrepreneurial production (0.3) plus the production from the fixed asset (0.05). Therefore, total out is 0.35 per quarter (about 1.4 per year).

The borrowing constraint (2) has two parameters: κ and η. The parameter κ is a constant borrowing limit which I set to zero. The parameter η determines the fraction of the fixed asset that can be used as a collateral. This is set to 0.6. The productivity shock follows a truncated normal distribution with standard deviation of 0.3. Given the baseline parametrization, this implies that the standard deviation of entrepreneurial wealth is about 7%. Notice that with $\eta > 0$ the price of the fixed asset, p_t, is not constant and the model cannot be solved analytically. The numerical procedure is described in Appendix H.

The last set of parameters pertain to the banking sector. The low value of ξ is set to $\underline{\xi} = 0.75$. The probability that the sunspot variable ε takes the value of zero (which could lead to a bank crisis) is set to 1 percent ($\lambda = 0.01$). Therefore, provided that the economy is in a region that admits multiple equilibria, a crisis arises on average every 25 years. The operation cost is assumed to be quadratic, that is, $\varphi(\omega) = \tau[1 + (\omega - \underline{\xi})^2]$ with $\tau = 0.0045$.

Simulation exercise The financial sector has gone through a significant process of innovations. Some innovations were allowed by institutional liberalization while others followed from product and technological innovations. One way of thinking about financial innovations in the context of the model is to reduce the bank operation cost to raise funds, which is captured by the parameter τ. The idea is that, thanks to financial liberalization and/or the introduction of new products and technologies, banks have been able to simplify their funding activity. In reduced form this is captured by a reduction in the parameter τ.

From Proposition 2.3 we know that the cost τ determines the existence of multiple equilibria. For a sufficiently high τ, the economy converges to a state with a unique equilibrium without crises. With a sufficiently low τ, instead, the economy will eventually reaches a state with multiple equilibria. As a result, the economy experiences stochastic fluctuations where gradual booms are reversed by sudden crises. Therefore, as the operation cost τ declines, the economy could move from a state where the equilibrium is unique to states with multiple equilibria. In the simulation I consider a permanent reduction in the value of τ from its baseline value of 0.0045 to 0.0035.

The only aggregate shock in the model is the sunspot shock which takes the value of zero with a 1 percent probability. To better illustrate the stochastic nature of the economy, I repeat the simulation of the model 1,000 times (with each simulation performed over 2,000 periods as described above). Each repeated simulation is based on a sequence of 2,000 random draws of the sunspot shock.

Figure 4 plots the average as well as the 5th and 95th percentiles of the 1,000 repeated simulations for each quarter over the period 1981-2020. This corresponds to periods 960 to 1,120 of each simulation. The range of variation between the 5th and 95th percentiles provides information about the volatility of the economy at any point in time.

During the simulation period, the value of τ is assumed to decrease permanently and unexpectedly in 1991 from 0.0045 to 0.0035. This reduction is interpreted as the consequence of innovations that reduced the cost to raise funds for financial institutions. The choice of 1991 for the structural break is only for illustrative purposes. Figure 4 shows the simulation statistics.

The first panel of Figure 4 plots the value of τ, which is exogenous in the model. The next five panels plot five endogenous variables: the total liabilities of banks, their leverage, the lending rate, the price of the fixed asset and the input of labor.

Following the decrease in τ, the interval delimited by the 5th and 95th percentiles of the repeated simulations widens. Therefore, financial and macroeconomic volatility increases substantially as we move to the 2000s. The probability of a bank crisis is always positive even before the structural break induced by the change in τ. However, after the reduction in the funding cost for banks, the consequence of a crisis could be much bigger (since the percentiles interval widens).

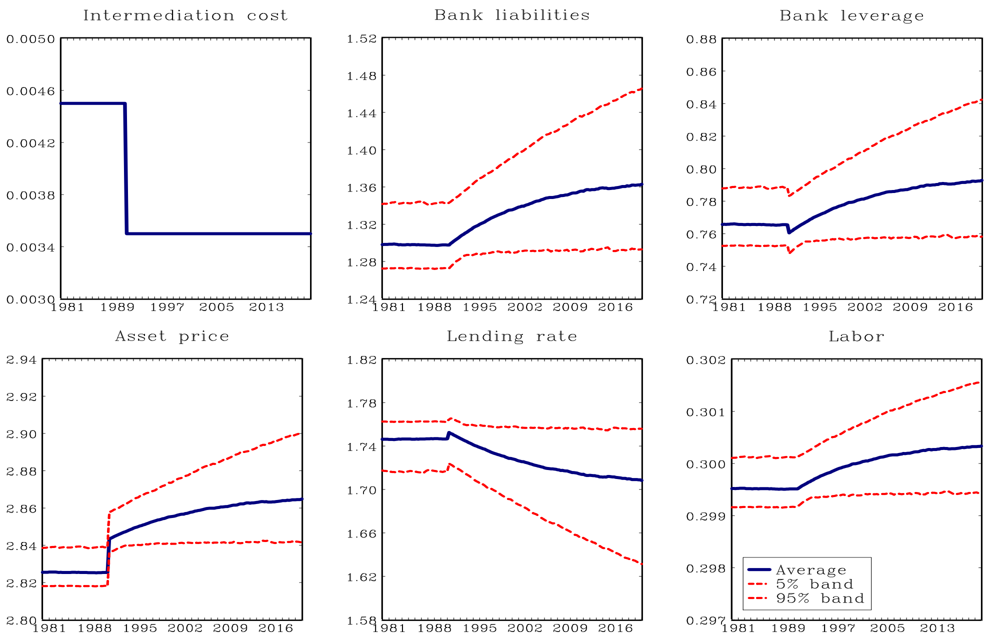

Figure 4: Change in bank operation cost τ. Responses of 1,000 simulations.

Besides the increase in financial and macroeconomic volatility, the figure reveals other interesting patterns. First, as a consequence of the lower τ, on average banks raise their leverage while the interest rate on loans decreases. The economy also experiences, on average, an increase in asset prices (the price of the fixed asset interpreted as housing) and in labor. The latter follows from the increase in the demand of labor. As the intermediation cost falls, banks issue more liabilities and in equilibrium entrepreneurs hold more financial wealth. This increases their willingness to take risk, leading to the higher demand for labor.

It is important to point out that, although labor increases on average in all repeated simulations, the actual dynamics of labor during the 20 years that followed the 1991 break could be increasing or decreasing depending on the actual realizations of the sunspot shocks.

To show this point, I repeat the experiment shown in Figure 4 but for a particular sequence of sunspot shocks. More specifically, I simulate the model under the assumption that, starting in the first quarter of 1991, the economy experiences a sequence of draws of the sunspot variable $\varepsilon = 1$ until the second quarter of 2008. Then in the third quarter of 2008 the draw becomes $\varepsilon = 0$ but returns to $\varepsilon = 1$ in all subsequent quarters. This captures the idea that expectations may have turned pessimistic in the fourth quarter of 2008 potentially leading to a sudden financial and macroeconomic crisis. The resulting responses are plotted in Figure 5 by the continuous line.

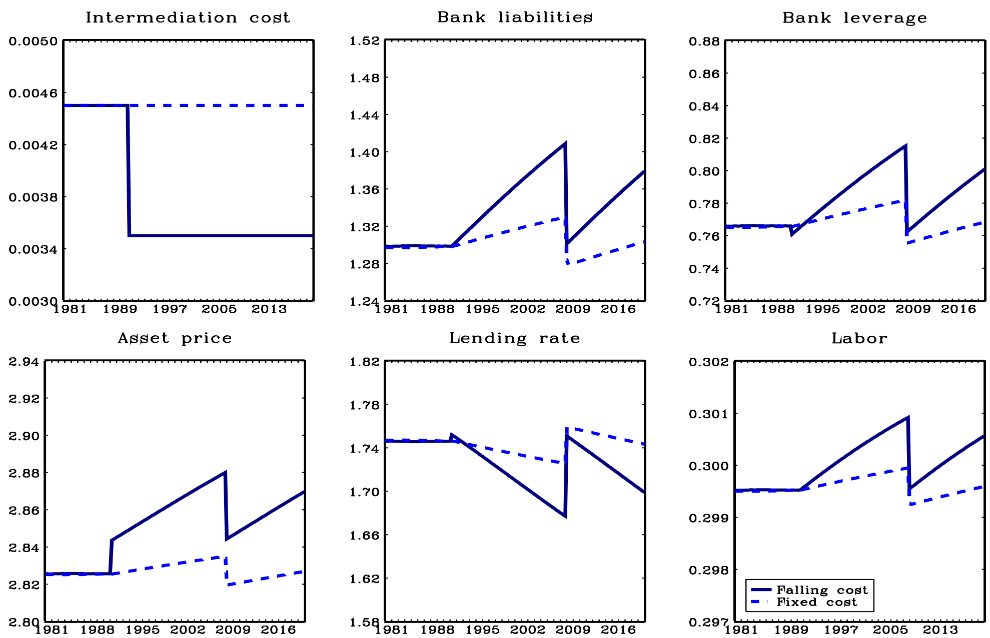

Figure 5: Responses of 1,000 simulations with same draws of the sunspot variable starting in 1991, with and without change in the operation cost τ.

The fall in the intermediation cost τ induces a large increase in the leverage of banks. This is consistent with the evidence provided in Kalemli-Ozcana, Sorensen, and Yesiltas (2012). To the extent that the banking sector in the model is interpreted broadly and including all financial institutions, this paper shows that there has been a significant increase in the leverage of the banking sector prior to the sub-prime crisis, with the increase primarily driven by investment banks. Furthermore, even if commercial banks did not show a significant increase in formal measures of leverage, they had significant off-balance sheet liabilities, suggesting that they were also highly exposed to risk.

As long as the draw of the sunspot variable is $\varepsilon = 1$, asset prices continue to increase and the input of labor expands. However, a single realization of $\varepsilon = 0$ for the sunspot shock in 2008 triggers a sizable decline in labor.

Furthermore, even if the negative shock is only for one period and there are no crises afterwards, the recovery in the labor market is very slow. This is because the crisis generates a large decline in the financial wealth of employers and it takes a long time for them to rebuilt the lost wealth with savings.

Another way to show the importance of financial innovations for macroeconomic stability, is to conduct the following exercise. I repeat the simulation of the model in response to the same sequence of sunspot draws but under the assumption that τ does not change, that is, it remains at the pre-1991 value after 1991. This counterfactual exercise illustrates how different the financial and macroeconomic dynamics in response to the same shocks would have been in absence of financial innovations. The resulting simulation is shown in Figure 5 by the dash line.

As can be seen, without the change in τ, the same sequence of sunspot shocks would have generated a much smaller financial expansion before 2008 as well as a much smaller financial and macroeconomic contraction in the third quarter of 2008. Therefore, financial innovations could have contributed to the observed expansion of the financial sector in industrialized countries but it also created the conditions for greater financial and macroeconomic fragility that became evident only after the crisis materialized.

Downward wage rigidity Although the re-adjustment in financial variables shown in Figure 5 is quite large, the response of labor is relatively small. The reason is because the decline in the demand for labor caused by the crisis is counterbalanced by a significant reduction in the wage rate. However, if wages cannot fall because of downward rigidity, the response of labor could be much bigger. To show this, I now consider downward wage rigidities.

Suppose that wages are perfectly flexible only when the demand of labor induces an increase in the wage or a moderate decline. More specifically, given w_{t-1} the equilibrium real wage at $t-1$, the current wage w_t must satisfy $w_t \geq \rho w_{t-1}$. The coefficient ρ determines the degree of downward rigidity. With $\rho = 1$ wages never decline. With $\rho = 0$ wages are perfectly flexible. Although 'nominal' wage rigidity is a more common feature of the economy, downward 'real' rigidity should be interpreted as capturing downward nominal rigidity when inflation is very close to zero.

Denote by $\bar{w}_t$ the wage rate that equalizes the demand and supply of labor, that is, $\phi_t(\bar{w}_t)\tilde{B}_t = \left(\frac{\bar{w}_t}{\alpha}\right)^{\nu}$. Equilibrium employment is always equal to the demand, $H_t = \phi_t(w_t)\tilde{B}_t$, with the wage rate given by

$$w_t = \begin{cases} \bar{w}_t, & \text{if } \bar{w}_t \geq \rho w_{t-1} \\ \\ \rho w_{t-1}, & \text{if } \bar{w}_t < \rho w_{t-1} \end{cases} \tag{16}$$

Figure 6 plots the simulation with $\rho = 0.9999$. As can be seen, the crisis induces a drop in labor that is more than 7 percent (continuous line). Without the change in τ, the drop in labor is also sizable but significantly smaller than in the case with lower τ. Therefore, the combination of higher leverages caused by the reduction in the intermediation cost together with downward wage rigidities could create the conditions for more severe macroeconomic contractions in response to a banking crisis.

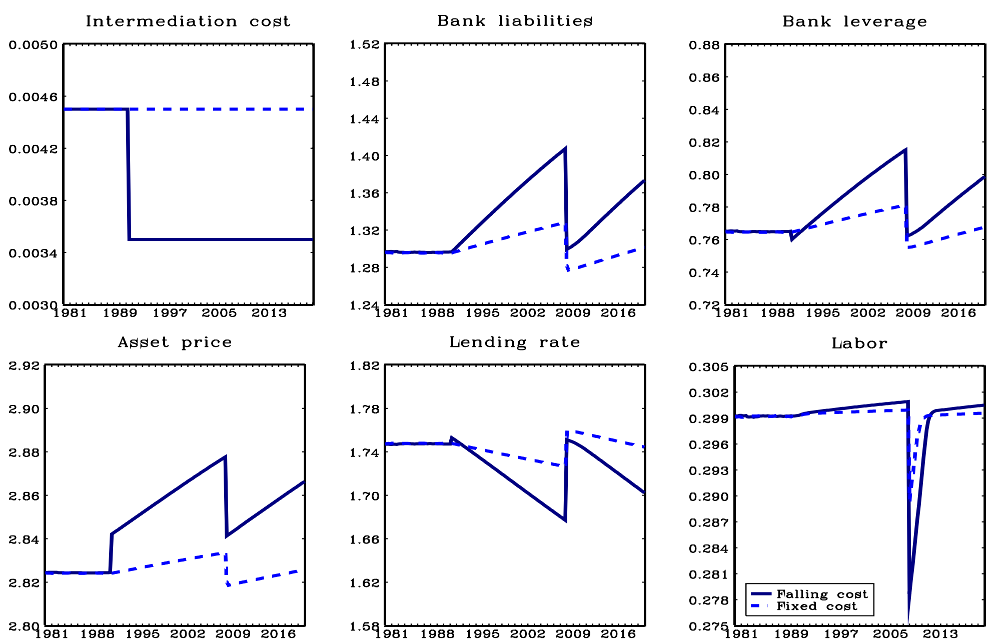

Figure 6: Model with downward wage rigidity. Responses of 1,000 simulations with same draws of the sunspot variable starting in 1991, with and without change in the operation cost τ.

4 Government debt

The analysis conducted so far shows that, due to market incompleteness and the limited creation of financial assets, the equilibrium is inefficient. More specifically, the premium over the wage that entrepreneurs require to hire labor represents a wedge that distorts the labor market. If markets were complete, producers would be able to insure the idiosyncratic risk with the purchase of state contingent claims and the wedge would be zero. But in the environment presented in this paper the only way for producers to achieve consumption smoothing (by insuring the risk) is by holding financial assets that cannot be contingent on the idiosyncratic risk. Since the supply of these assets is limited, the equilibrium is inefficient.

Given the limited ability of the financial sector to create financial assets, it is natural to ask whether the government could obviate this limitation by issuing public debt. In principle this could provide additional financial assets held by producers and could reduce the labor wedge. In this section I will show that the ability of public debt to improve the equilibrium depends on how the burden of the debt is financed.[6]

4.1 Public debt and taxation

After issuing public debt, the government has to pay the interests that mature on the debt with taxes. It becomes then important what type of taxes are used to fund the burden of debt. I will consider four types of taxes:

1. Lump-sum taxes on entrepreneurs.
2. Lump-sum taxes on workers.
3. Profit taxes on entrepreneurs.
4. Wage taxes on workers.

As we will see, public debt would unambiguously improve real allocations only if the burden of public debt is financed with lump-sum taxes on workers.

[6]There are other mechanisms that could generate financial assets held for insurance purposes. Money, for example, could also play this role as in Brunnermeier and Sannikov (2016). In this paper money is a bubble that agents are willing to hold because of the insurance it provides. More generally, bubbles like those considered in Miao and Wang (2011) could also play this role. In my framework, however, the reason entrepreneurs would hold bubbles is not because they relax the borrowing constraints (as in Miao and Wang (2011)) but because they provide insurance.

Lump-sum taxes on entrepreneurs: The following proposition establishes that with lump-sum taxes paid by entrepreneurs, public debt is irrelevant for real sector of the economy (Ricardian equivalence).

Proposition 4.1 *If the government uses lump-sum taxes charged only to entrepreneurs, then public debt does not affect real allocations.*

To illustrate this result, consider first the government budget constraint,

$$T_t = D_t - \frac{D_{t+1}}{R_t^b},$$

where T_t denotes the lump-sum taxes charged to entrepreneurs and D_t is the government debt due at time t and D_{t+1} is the new debt due at $t+1$. For simplicity I am abstracting from the possibility that banks could default so that in equilibrium the interest rate on government bonds must be equal to the interest rate on bank liabilities, R_t^b. This implies that for entrepreneurs bank liabilities are equivalent to government liabilities.

Let's consider now the budget constraint for entrepreneurs,

$$c_t^i + \frac{b_{t+1}^i}{R_t^b} + \frac{d_{t+1}^i}{R_t^b} + T_t = (z_t^i - w_t)h_t^i + b_t^i + d_t,$$

where d_t^i is the individual debt held by entrepreneur i. Eliminating T_t using the government budget constraint, the budget constraint for an individual entrepreneur i can be rewritten as

$$c_t^i + \frac{\bar{b}_{t+1}^i}{R_t^b} = (z_t^i - w_t)h_t^i + \bar{b}_t^i,$$

where $\bar{b}_t^i = b_t^i + d_t^i - D_t$.

The resulting budget constraint has the same structure as the budget constraint without government debt. The only difference is that b_t^i has been replaced with $\bar{b}_t^i$. Therefore, the solution is still given by Lemma 2.1 and takes the form

$$h_t^i = \phi(w_t)\bar{b}_t^i.$$

Again, the only difference is that now we have $\bar{b}_t^i$ instead of b_t^i. Using the individual demands for labor, we can then derive the aggregate demand

$$H_t = \phi(w_t)\int \bar{b}_t^i = \phi(w_t)\int \Big(b_t^i + d_t^i - D_t\Big) = \phi(w_t)B_t.$$

The last equality derives from the fact that the sum of the public debt held by all entrepreneurs is equal to the aggregate public debt. Thus, the aggregate demand for labor is unaffected by the issuance of public debt but depends only on the liabilities issued by the financial intermediation sector, B_t.

This result has a simple intuition. Even though the public debt will be held by entrepreneurs and it represents a financial asset for them, this is totally offset by the tax liabilities that they have to pay. As a result, since their net wealth (net of the tax liabilities) does not change, their economic decisions are unaffected (Ricardian equivalence).

Lump-sum taxes on households: If the government finances the burden of public debt with lump-sum taxes on households, then effectively the government borrows on behalf of households (Azzimonti and Quadrini (2014)). In this case public debt improves allocations as stated by the following proposition.

Proposition 4.2 *If the government uses lump-sum taxes charged only to households, then higher public debt is associated with higher average employment and output and lower macroeconomic volatility.*

The intuitive proof follows the same steps of the proof provided above starting with the budget constraint of entrepreneurs

$$c_t^i + \frac{b_{t+1}^i}{R_t^b} + \frac{d_{t+1}^i}{R_t^b} = (z_t^i - w_t)h_t^i + b_t^i + d_t.$$

The only difference is that now entrepreneurs do not pay taxes T_t. We can again rewrite the budget constraint as

$$c_t^i + \frac{\bar{b}_{t+1}^i}{R_t^b} = (z_t^i - w_t)h_t^i + \bar{b}_t^i,$$

where now $\bar{b}_t^i = b_t^i + d_t^i$ (there is not D_t).

The resulting budget constraint has the same structure as the budget constraint without government debt and the solution is still given by Lemma 2.1. In particular, the demand for labor is given by

$$h_t^i = \phi(w_t)\bar{b}_t^i.$$

Using the individual labor demands we can derive the aggregate demand

$$H_t = \phi(w_t)\int \bar{b}_t^i = \phi(w_t)\int \Big(b_t^i + d_t^i\Big) = \phi(w_t)\Big(B_t + D_t\Big).$$

Now the aggregate demand of labor depends on both bank liabilities B_t and government debt D_t. Thus the government can affect the labor demand by issuing more debt.

Essentially, by borrowing on behalf of workers, the government creates financial assets that in equilibrium are held by entrepreneurs. Differently from the previous case, the tax liabilities are paid by households, not entrepreneurs. This implies that higher public debt will be associated with higher net financial wealth of entrepreneurs. Since entrepreneurs are better insured, they will hire more workers with higher equilibrium output.

In equilibrium, an extra dollar of public debt increases entrepreneurs' wealth less than one dollar. In fact, in order to induce entrepreneurs to hold more assets, the interest rate R_t^b has to increase. But as the interest rate increases, banks will issue less liabilities. So the public debt will crowd out bank liabilities. However, the crowding out is only partial and aggregate entrepreneurial wealth increases.

The fact that banks reduce the issuance of liabilities implies that they reduce their leverage. Lower leverage will then imply that the macroeconomic consequences of crises are smaller, reducing macroeconomic volatility.

Profit taxes on entrepreneurs: In terms of efficiency, this is the worse funding scheme. In equilibrium entrepreneurs will hold more financial wealth. However, they also incur the tax liabilities to serve the public debt. Therefore, the net financial wealth (net of tax liabilities) does not change. But now taxes are proportional to profits. For the entrepreneur this is equivalent to reducing the expected productivity of labor which reduces the labor demand. Therefore, public debt will be associated with lower production and consumption.

Wage taxes on workers: The macroeconomic effects are now ambiguous. On the one hand, entrepreneurs will hold more financial assets (because they hold the public debt) without facing the tax liabilities (which are now paid by households). This increases the demand of labor. On the other, taxes on wages reduce the supply of labor, which in equilibrium leads to higher wages and lower employment. Which of the two effects dominate depends on the elasticity of the labor supply. If the supply of labor is sufficiently elastic, the second effect dominates and the equilibrium will be characterized by lower employment and output. However, aggregate volatility may be lower. This is because the higher supply of bonds increases the interest rate which in turn will be associated with lover bank leverage.

4.2 Discussion

The above analysis suggests that, provided that the government uses the 'right' taxes to fund the debt burden, public debt could be welfare improving. This conclusion, however, ignores political feasibility. First, lump-sum taxes are difficult to use in practice because households earn different incomes. So, only proportional or progressive taxes are feasible. But then, which proportional taxes, the optimality of debt is no longer guaranteed.

The second consideration is time consistency. It may be possible that issuing debt and later taxing households is ex-ante optimal. But once the debt has been issued and households have to pay it, they may have an incentive to lobby the government to default on the debt. Default can take different forms, not necessarily outright repudiation. One less direct form of default is to shift the taxation burden from households to entrepreneurs. Provided that households have sufficient political power, this is likely to happen. So we end up again in a situation in which the net financial wealth of entrepreneurs (public debt minus the tax liabilities) does not change. This is likely to be anticipated when the government issues the debt in the first place, which neutralizes the positive effects of issuing public debt.

To conclude, public debt could improve the equilibrium allocation if associated with the proper taxation scheme. In practise, the 'proper' taxation scheme may not be feasible or credible. This limits the role of public debt as a way to complete the market and improve real allocations.

5 Conclusion

The traditional role of banks is to facilitate the transfer of resources from agents in excess of funds to agents in need of funds. This paper emphasizes a second important role played by banks: the issuance of liabilities that can be held by the nonfinancial sector for insurance purposes. This is similar to the role of banks in creating liabilities that can be used for transaction as in Williamson (2012). The difference is that in the current paper bank liabilities are valued not for their use as a mean of exchange but as an insurance instrument. When the stock of bank liabilities or their value are low, agents are less willing to engage in risky economic activities and this causes a macroeconomic downturn.

The paper also shows that booms and busts in financial intermediation can be driven by self-fulfilling expectations about the liquidity of banks. When the economy expects the banking sector to be liquid, banks have an incentive to leverage and this generates a macroeconomic boom. But as the leverage increases, the banking sector becomes vulnerable to pessimistic expectations that could generate self-fulfilling liquidity crises.

The model has been used to study the impact of financial innovations on financial and macroeconomic stability. Financial innovations can generate a macroeconomic expansion but could also increase the potential instability of the macro-economy. As long as expectations remain optimistic, countries experience a macroeconomic boom. However, when expectations turn pessimistic, the economy experiences deeper macroeconomic contractions.

Can the issuance of government debt improve the allocation of resources and reduce the probability and/or the macroeconomic consequences of crises? The paper has shown that the issuance of pubic debt could play a role in this regard but only if associated to a specific taxation scheme. The 'right' taxation scheme, however, may not be politically feasible or credible.

Appendix

A Proof of Lemma 2.1

The optimization problem of an entrepreneur can be written recursively as

$$\begin{aligned} V_t(b_t) &= \max_{h_t} \mathbb{E}_t \tilde{V}_t(a_t) \qquad (17) \\ & \text{subject to} \\ & a_t = b_t + (z_t - w_t)h_t \end{aligned}$$

$$\begin{aligned} \tilde{V}_t(a_t) &= \max_{b_{t+1}} \Big\{ \ln(c_t) + \beta \mathbb{E}_t V_{t+1}(b_{t+1}) \Big\} \qquad (18) \\ & \text{subject to} \\ & c_t = a_t - \frac{b_{t+1}}{R_t} \end{aligned}$$

Since the information set changes from the beginning of the period to the end of the period, the optimization problem has been separated according to the available information. In sub-problem (17) the entrepreneur chooses the input of labor without knowing the productivity z_t. In sub-problem (18) the entrepreneur allocates the end of period wealth in consumption and savings after observing z_t.

The first order condition for sub-problem (17) is

$$\mathbb{E}_t \frac{\partial \tilde{V}_t}{\partial a_t}(z_t - w_t) = 0.$$

The envelope condition from sub-problem (18) gives

$$\frac{\partial \tilde{V}_t}{\partial a_t} = \frac{1}{c_t}.$$

Substituting in the first order condition we obtain

$$\mathbb{E}_t \left(\frac{z_t - w_t}{c_t} \right) = 0. \qquad (19)$$

At this point we proceed by guessing and verifying the optimal policies for employment and savings. The guessed policies take the form:

$$\begin{aligned} h_t &= \phi_t b_t \qquad (20) \\ c_t &= (1 - \beta) a_t \qquad (21) \end{aligned}$$

Since $a_t = b_t + (z_t - w_t)h_t$ and the employment policy is $h_t = \phi_t b_t$, the end of period wealth can be written as $a_t = [1 + (z_t - w_t)\phi_t]b_t$. Substituting in the guessed consumption policy we obtain

$$c_t = (1-\beta)\Big[1 + (z_t - w_t)\phi_t\Big]b_t. \tag{22}$$

This expression is used to replace c_t in the first order condition (19) to obtain

$$\mathbb{E}_t\left[\frac{z_t - w_t}{1 + (z_t - w_t)\phi_t}\right] = 0, \tag{23}$$

which is the condition stated in Lemma 2.1.

To complete the proof, we need to show that the guessed policies (20) and (21) satisfy the optimality condition for the choice of consumption and saving. This is characterized by the first order condition of sub-problem (18), which is equal to

$$-\frac{1}{c_t R_t} + \beta\mathbb{E}_t\frac{\partial V_{t+1}}{\partial b_{t+1}} = 0.$$

From sub-problem (17) we derive the envelope condition $\partial V_t/\partial b_t = 1/c_t$ which can be used in the first order condition to obtain

$$\frac{1}{c_t} = \beta R_t \mathbb{E}_t\frac{1}{c_{t+1}}.$$

We have to verify that the guessed policies satisfy this condition. Using the guessed policy (21) and equation (22) updated one period, the first order condition can be rewritten as

$$\frac{1}{a_t} = \beta R_t \mathbb{E}_t\frac{1}{[1 + (z_{t+1} - w_{t+1})\phi_{t+1}]b_{t+1}}.$$

Using the guessed policy (21) we have that $b_{t+1} = \beta R_t a_t$. Substituting and rearranging we obtain

$$1 = \mathbb{E}_t\left[\frac{1}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}}\right]. \tag{24}$$

The final step is to show that, if condition (23) is satisfied, then condition (24) is also satisfied. Let's start with condition (23), updated by one period. Multiplying both sides by ϕ_{t+1} and then subtracting 1 in both sides we obtain

$$\mathbb{E}_{t+1}\left[\frac{(z_{t+1} - w_{t+1})\phi_{t+1}}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}} - 1\right] = -1.$$

Multiplying both sides by -1 and taking expectations at time t we obtain (24).

B Proof of Proposition 2.1

As shown in Lemma 2.1, the optimal saving of entrepreneurs takes the form $b^i_{t+1}/R^b_t = \beta a^i_t$, where a^i_t is the end-of-period wealth $a^i_t = b^i_t + (z^i_t - w_t)h^i_t$. Since $h^i_t = \phi(w_t)b^i_t$ (see Lemma 2.1), the end-of-period wealth can be rewritten as $a^i_t = [1 + (z^i_t - w_t)\phi(w_t)]b^i_t$. Substituting into the optimal saving and aggregating over all entrepreneurs we obtain

$$B_{t+1} = \beta R^b_t \Big[1 + (\bar{z} - w_t)\phi(w_t)\Big] B_t. \tag{25}$$

This equation defines the aggregate demand for bonds as a function of the interest rate R^b_t, the wage rate w_t, and the beginning-of-period aggregate wealth of entrepreneurs B_t. Notice that the term in square brackets is bigger than 1. Therefore, in a steady state equilibrium where $B_{t+1} = B_t$, the condition $\beta R < 1$ must be satisfied.

Using the equilibrium condition in the labor market, I can express the wage rate as a function of B_t. In particular, equalizing the demand for labor, $H^D_t = \phi(w_t)\tilde{B}_t$, to the supply from households, $H^S_t = (w_t/\alpha)^\nu$, the wage w_t can be expressed as a function of only $\tilde{B}_t$. We can then use this function to replace w_t in (25) and express the demand for bank liabilities as a function of only B_t and R^b_t as follows

$$B_{t+1} = s(B_t)\, R^b_t. \tag{26}$$

The function $s(B_t)$ is strictly increasing in the wealth of entrepreneurs, B_t.

Consider now the supply of bonds from households. For simplicity I assume that $\eta = 0$ in the borrowing constraint (2). Therefore, this constraint takes the form $l_{t+1} \leq \kappa$. Using this limit together with the first order condition (4), we have that, either the interest rate satisfies $1 = \beta R^b_t$ or households are financially constrained, that is, $B_{t+1} = \kappa$. When the interest rate is equal to the inter-temporal discount rate (first case), we can see from (25) that $B_{t+1} > B_t$. So eventually, the borrowing constraint of households becomes binding, that is, $B_{t+1} = \kappa$ (second case). When the borrowing constraint is binding, the multiplier μ_t is positive and condition (4) implies that the interest rate is smaller than the inter-temporal discount rate. So the economy has reached a steady state. The steady state interest rate is determined by condition (26) after setting $B_t = B_{t+1} = \kappa$. This is the only steady state equilibrium.

When $\eta > 0$ in the borrowing constraint (2), the proof is more involved but the economy also reaches a steady state with $\beta R < 1$.

C First order conditions for households

The optimization problem of a household can be written recursively as

$$
\begin{aligned}
V_t(l_t, k_t) \quad = \quad & \max_{h_t, l_{t+1}, k_{t+1}} \left\{ c_t - \alpha \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta V_{t+1}(l_{t+1}, k_{t+1}) \right\} \\
& \text{subject to} \\
& c_t = w_t h_t + \chi k_t + \frac{l_{t+1}}{R_t^l} - l_t - (k_{t+1} - k_t) p_t \\
& \eta \geq l_{t+1}.
\end{aligned}
$$

Given $\beta \mu_t$ the lagrange multiplier associated with the borrowing constraint, the first order conditions with respect to h_t, l_{t+1}, k_{t+1} are, respectively,

$$
\begin{aligned}
& -\alpha h_t^{\frac{1}{\nu}} + w_t = 0, \\
& \frac{1}{R_t^l} + \beta \frac{\partial V_{t+1}(l_{t+1}, k_{t+1}}{\partial l_{t+1}} - \beta \mu_t = 0, \\
& -p_t + \beta \frac{\partial V_{t+1}(l_{t+1}, k_{t+1}}{\partial k_{t+1}} + \eta \beta \mu_t \mathbb{E}_t p_{t+1} = 0.
\end{aligned}
$$

The envelope conditions are

$$
\begin{aligned}
\frac{\partial V_t(l_{t+1}, k_{t+1}}{\partial l_{t+1}} \quad &= \quad -1, \\
\frac{\partial V_t(l_{t+1}, k_{t+1}}{\partial k_{t+1}} \quad &= \quad \chi + p_t.
\end{aligned}
$$

Updating by one period and substituting in the first order conditions we obtain (3), (4), (5).

D First order conditions for problem (9)

The probability of renegotiation, denoted by θ_{t+1}, is defined as

$$
\theta_{t+1} = \begin{cases} 0, & \text{if} \quad \omega_{t+1} < \underline{\xi} \\ \bar{\lambda}, & \text{if} \quad \underline{\xi} \leq \omega_{t+1} \leq 1 \\ 1, & \text{if} \quad \omega_{t+1} > 1 \end{cases}
$$

Define $\beta(1-\theta_{t+1})\gamma_t$ the Lagrange multiplier associated to the constraint $b_{t+1} \leq l_{t+1}$. The first order conditions for problem (9) with respect to b_{t+1} and l_{t+1} are

$$\frac{1-\varphi_t}{\overline{R}_t^b}\mathbb{E}_t\frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} - \frac{\partial \varphi_t}{\partial b_{t+1}}\frac{\mathbb{E}_t\tilde{b}_{t+1}}{\overline{R}_t^b} - \beta\mathbb{E}_t\frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} - \beta(1-\theta_{t+1})\gamma_t = 0, \tag{27}$$

$$-\frac{1}{R_t^l} + \frac{1-\varphi_t}{\overline{R}_t^b}\mathbb{E}_t\frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}} - \frac{\partial \varphi_t}{\partial l_{t+1}}\frac{\mathbb{E}_t\tilde{b}_{t+1}}{\overline{R}_t^b} + \beta\mathbb{E}_t\left(1-\frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}}\right) + \beta(1-\theta_{t+1})\gamma_t = 0. \tag{28}$$

I now use the definition $\tilde{b}_{t+1}$ provided in (6) to derive the following terms

$$\begin{aligned}
\frac{\partial \varphi_t}{\partial b_{t+1}} &= \varphi'_{t+1}\frac{1}{l_{t+1}}, \\
\frac{\partial \varphi_t}{\partial l_{t+1}} &= -\varphi'_{t+1}\omega_{t+1}\frac{1}{l_{t+1}}, \\
\mathbb{E}_t\frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} &= 1-\theta_{t+1}, \\
\mathbb{E}_t\frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}} &= \theta_{t+1}\underline{\xi}, \\
\mathbb{E}_t\tilde{b}_{t+1} &= (1-\theta_{t+1})b_{t+1} + \chi_{t+1}\underline{\xi}l_{t+1}.
\end{aligned}$$

Substituting in (27) and (28) and re-arranging we obtain

$$\frac{1}{\overline{R}_t^b} = \beta\left[1+\frac{\varphi_{t+1}+\varphi'_{t+1}\hat{\omega}_{t+1}+\gamma_t}{1-\varphi_{t+1}-\varphi'_{t+1}\hat{\omega}_{t+1}}\right], \tag{29}$$

$$\frac{1}{R_t^l} = \beta\left[1+\frac{\varphi'_{t+1}\hat{\omega}_{t+1}^2(1-\theta_{t+1})(1+\gamma_t)}{1-\varphi_{t+1}-\varphi'_{t+1}\hat{\omega}_{t+1}} + \Big(1-\theta_{t+1}+\theta_{t+1}\underline{\xi}\Big)\gamma_t\right], \tag{30}$$

where $\hat{\omega}_{t+1} = \omega_{t+1} + \frac{\theta_{t+1}\underline{\xi}}{1-\theta_{t+1}}$.

The multiplier γ_t is zero if $\omega_{t+1} < 1$ and positive if $\omega_{t+1} = 1$. Therefore, the first order conditions can be written as

$$\begin{aligned}
\frac{1}{\overline{R}_t^b} &= \beta\left[1+\frac{\varphi_{t+1}+\varphi'_{t+1}\hat{\omega}_{t+1}}{1-\varphi_{t+1}-\varphi'_{t+1}\hat{\omega}_{t+1}}\right], \\
\frac{1}{R_t^l} &= \beta\left[1+\frac{\varphi'_{t+1}\hat{\omega}_{t+1}^2(1-\theta_{t+1})}{1-\varphi_{t+1}-\varphi'_{t+1}\hat{\omega}_{t+1}}\right],
\end{aligned}$$

which are satisfied with the inequality sign if $\gamma_t > 0$. Since they are all functions of ω_{t+1}, the first order conditions can be written as in (10) and (11).

E Proof of Lemma 2.2

Let's consider the first order conditions (29) and (30) when $\omega_{t+1} < 1$. In this case the lagrange multiplier γ_t is zero. Since $A_{t+1} > 0$ and φ_{t+1} and φ'_{t+1} are both positive for $\omega_{t+1} > \underline{\xi}$, conditions (29) and (30) imply that $\overline{R}^b_t$ and R^l_t are smaller than $1/\beta$.

The next step is to derive the return spread from (29) and (30) to obtain

$$\frac{R^l_t}{\overline{R}^b_t} = \frac{1}{1 - \varphi_{t+1} - \varphi'_{t+1} A_{t+1}[1 - (1 - \theta_{t+1})A_{t+1}]}. \tag{31}$$

Given the properties of the cost function (Assumption 1), to show that the spread is bigger than 1 I only need to show that $(1 - \theta_{t+1})A_{t+1} < 1$. Using $A_{t+1} = \omega_{t+1} + \frac{\theta_{t+1}\underline{\xi}}{1-\theta_{t+1}}$ and taking into account that $\omega_{t+1} < 1$ and $\theta_{t+1} < 1$, we can verify that $(1 - \theta_{t+1})A_{t+1} < 1$. Therefore, the spread is bigger than 1.

To show that the spread is increasing in the leverage, I differentiate (31) with respect to ω_{t+1} to obtain

$$\frac{R^l_t}{\overline{R}^b_t} = \frac{(\varphi''_{t+1} A_{t+1} + 2\varphi'_{t+1})[1 - (1 - \theta_{t+1})A_{t+1}]}{\left[1 - \varphi_{t+1} - \varphi'_{t+1} A_{t+1}(1 - (1 - \theta_{t+1})A_{t+1}\right]^2}$$

Given the properties of the cost function (Assumption 1), the derivative is zero for $\omega_{t+1} \leq \underline{\xi}$. To prove that the derivative is positive for $\omega_{t+1} > \underline{\xi}$, I only need to show that $(1 - \theta_{t+1})A_{t+1} < 1$, which has already been shown above. Therefore, the return spread is strictly increasing for $\omega_{t+1} > \underline{\xi}$.

F Proof of Proposition 2.2

Banks make decisions at two different stages. At the beginning of the period they choose whether to renegotiate the debt and at the end of the period they choose the funding and lending policies. Given the initial states, b_t and l_t, the renegotiation decision boils down to a take-it or leave-it offer made by each bank to its creditors for the repayment of the debt. Denote by $\tilde{b}_t = f(b_t, l_t, \xi^e_t)$ the offered repayment. This depends on the individual liabilities b_t, individual assets l_t, and the expected liquidation price of assets ξ^e_t. The superscript e is to make clear that the bank decision depends on the expected price in the eventuality of liquidation. Obviously, the best repayment offer made by the bank is

$$f(b_t, l_t, \xi^e_t) = \begin{cases} b_t, & \text{if} \quad b_t \leq \xi^e_t l_t \\ \\ \xi^e_t l_t, & \text{if} \quad b_t > \xi^e_t l_t \end{cases}, \tag{32}$$

which is accepted by creditors whenever the actual liquidation price is bigger than the expected price ξ^e_t.

After the renegotiation stage, banks choose the funding and lending policies, b_{t+1} and l_{t+1}. These policies depend on the two interest rates, $\overline{R}^b_t$ and R_l, and on the probability distribution of the next period liquidation price ξ_{t+1}. Since we could have multiple equilibria, the next period price could be stochastic. Suppose that the price could take two values, $\underline{\xi}$ and 1, with the probability of the low value defined as

$$\theta(\omega_{t+1}) = \begin{cases} 0, & \text{if} \quad \omega_{t+1} < \underline{\xi} \\ \lambda, & \text{if} \quad \underline{\xi} \leq \omega_{t+1} \leq 1 \\ 1, & \text{if} \quad \omega_{t+1} > 1. \end{cases}$$

The variable $\omega_{t+1} = b_{t+1}/l_{t+1}$ represents the leverage of all banks in a symmetric equilibrium, that is, they all choose the same leverage. For the moment the symmetry of the equilibrium is an assumption. I will then show below that in fact banks do not have incentives to deviate from the leverage chosen by other banks.

Given the above assumption about the probability distribution of the liquidation price, the funding and lending policies of the bank are characterized in Lemma 2.2 and depend on $\overline{R}^b_t$ and R^l_t. In short, if $\overline{R}^b_t/(1-\tau) = R^l_t$, then the optimal policy of the bank is to choose a leverage $\omega_{t+1} \leq \underline{\xi}$. If $\overline{R}^b_t/(1-\tau) < R^l_t$, the optimal leverage is $\omega_{t+1} > \underline{\xi}$.

Given the assumption that the equilibrium is symmetric (all banks choose the same leverage ω_{t+1}), multiple equilibria arise if the chosen leverage is $\omega_{t+1} \in \{\underline{\xi}, 1\}$. In fact, once we move to the next period, if the market expects $\xi^e_{t+1} = \underline{\xi}$, all banks are illiquid and they choose to renege on their liabilities (given the renegotiation policy (32)). As a result, there will not be any bank that can buy the liquidated assets of other banks. Then the only possible price that is consistent with the expected price is $\xi_{t+1} = \underline{\xi}$. On the other hand, if the market expects $\xi^e_{t+1} = 1$, banks are liquid and, if one bank reneges, creditors can sell the liquidated assets to other banks at the price $\xi_{t+1} = 1$. Therefore, it is optimal for banks not to renegotiate consistently with the renegotiation policy (32).

The above proof, however, assumes that the equilibrium is symmetric, that is, all banks choose the same leverage. To complete the proof, we have to show that there is no incentive for an individual bank to deviate from the leverage chosen by other banks. In particular, I need to show that, in the anticipation that the next period liquidation price could be $\xi_{t+1} = \underline{\xi}$, a bank do not find convenient to chose a lower leverage so that, in the eventuality that the next period price is $\xi_{t+1} = \underline{\xi}$, the bank could purchase the liquidated asset at a price lower than 1 and make a profit (since the unit value for the bank of the liquidated assets is 1.

If the price at $t+1$ is $\xi_{t+1} = \underline{\xi}$, a liquid bank could offer a price $\underline{\xi} + \epsilon$, where ϵ is a small but positive number. Since the repayment offered by a defaulting bank is $\underline{\xi} l_{t+1}$, creditors prefer to sell the assets rather than accepting the repayment offered by the defaulting bank. However, if this happens, the expectation of the liquidation price $\xi^e = \underline{\xi}$ turns out to be incorrect ex-post. Therefore, the presence of a single bank with liquidity will raise the expected liquidation price to $\underline{\xi} + \epsilon$. But even with this new expectation, a bank with liquidity can make a profit by offering $\underline{\xi} + 2\epsilon$. Again, this implies that the expectation turns out to be incorrect ex-post. This mechanism will continue to raise the expected price to $\xi^e_{t+1} = 1$. At this point the liquid bank will not offer a price bigger than 1 and the ex-post liquidation price is correctly predicted to be 1. Therefore, as long as there is a single bank with liquidity, the expected liquidation price must be 1. But then a bank cannot make a profit in period $t+1$ by choosing a lower leverage in period t with the goal of remaining liquid in the next period. This proves that there is no incentive to deviate from the policy chosen by other banks.

Finally, the fact that multiple equilibria cannot arise when $\omega_t < \underline{\xi}$ is obvious. Even if the price is $\underline{\xi}$, banks remain liquid.

G Proof of Proposition 2.3

Given a fixed interest rate R^b, the aggregate demand for bank liabilities, equation (13), has a converging fix point $B^*(R^b)$. The fixed point is increasing in R^b and converges to infinity as R^b converges to $1/\beta$. This implies that, if $\tau = 0$, then the leverage of banks is always bigger than $\underline{\xi}$. To show this, suppose that banks choose a leverage of $\omega < \underline{\xi}$. According to conditions (10) and (11), we have that $R^b = R^l = 1/\beta$. But when $R^b = 1/\beta$ the demand of bank liabilities is unbounded in the limit. This implies that to reach a stable equilibrium without renegotiation (that is, $\omega < \underline{\xi}$), R^b must be smaller than $1/\beta$. This requires τ to be sufficiently big. In fact, when $\tau > 0$ and $\omega < \underline{\xi}$, we have $R^b/(1-\tau) = R^l = 1/\beta$. Since the demand for bank liabilities is increasing in R^b, there must be some $\hat{\tau} > 0$ such that, for $\tau > \hat{\tau}$, the equilibrium is characterized by $\omega < \underline{\xi}$. This implies that the economy is not subject to crises and converges to a steady state. For $\tau < \hat{\tau}$, instead, the equilibrium is characterized by $\omega > \underline{\xi}$. In this case the economy is subject to self-fulfilling crises and, therefore, it does not converge to a steady state.

H Numerical solution

I describe first the numerical procedure without the structural break. I will then describe the numerical procedure when τ changes inducing a transition dynamics.

H.1 Stationary equilibrium without structural break

The states of the economy are given by the bank liabilities B_t, the bank loans L_t and the realization of the sunspot shock ε_t. These three variables are important in determining the renegotiation liabilities $\tilde{B}_t$. However, once we know the renegotiated liabilities $\tilde{B}_t$, this becomes the sufficient state for solving the model. Therefore, in the computation I will solve for the recursive equilibrium using $\tilde{B}_t$ as a state variable.

I will use the following equilibrium conditions:

$$H_t = \phi(w_t)\tilde{B}_t, \tag{33}$$

$$\frac{B_{t+1}}{R^b_t} = \beta A_t, \tag{34}$$

$$A_t = \tilde{B}_t + (1 - w_t)H_t \tag{35}$$

$$\alpha H_t^{\frac{1}{\nu}} = w_t, \tag{36}$$

$$1 = \beta R^l_t(1 + \mu_t), \tag{37}$$

$$p_t = \beta \mathbb{E}_t \Big[\chi + (1 + \eta\mu_t)p_{t+1}\Big], \tag{38}$$

$$L_{t+1} = \eta \mathbb{E}_t p_{t+1}, \tag{39}$$

$$\frac{1}{\overline{R}^b_t} = \beta \left[1 + \frac{\varphi_{t+1} + \varphi'_{t+1}\hat{\omega}_{t+1}}{1 - \varphi_{t+1} - \varphi'_{t+1}\hat{\omega}_{t+1}}\right], \tag{40}$$

$$\frac{1}{R^l_t} = \beta \left[1 + \frac{\varphi'_{t+1}\hat{\omega}^2_{t+1}(1 - \theta_{t+1})}{1 - \varphi_{t+1} - \varphi'_{t+1}\hat{\omega}_{t+1}}\right], \tag{41}$$

$$\overline{R}^b_t = \left[1 - \theta(\omega_{t+1}) + \theta(\omega_{t+1})\left(\frac{\xi}{\omega_{t+1}}\right)\right] R^b_t, \tag{42}$$

$$\omega_{t+1} = \frac{B_{t+1}}{L_{t+1}} \tag{43}$$

Equations (33)-(35) come from the aggregation of the optimal policies of entrepreneurs (labor demand, savings and end of periods wealth). Equations (36)-(39) come from the optimization problem of households (labor supply, optimal borrowing, optimal holding of the fixed asset, and borrowing constraint). Notice that the borrowing constraint of households (equation (39) is not always binding. However, when it is not binding and the multiplier is $\mu_t = 0$, households' borrowing is not determined. Therefore, without loss of generality I assume that in this case households borrow up to the limit. This explains why the borrowing constraint is always satisfied with equality. Equations (40)-(41) are the first order conditions of banks. These conditions are satisfied with equality if $\omega_{t+1} < 1$ and with inequality if $\omega_{t+1} = 1$. Equation (42) defines the expected return on bank liabilities given the price of these liabilities, that is, the inverse of R^b_t. The final equation (43) simply defines leverage.

One complication in solving this system of equations is that the expectation of the next period price of the fixed asset, $\mathbb{E}_t p_{t+1}$, is unknown. All we know is that the next period price is a function of $\tilde{B}_{t+1}$, that is, $p_{t+1} = P(\tilde{B}_{t+1})$. If I knew the function $P(\tilde{B}_{t+1})$, for any given state $\tilde{B}_t$, the above conditions would be a system of 11 equations in 11 variables: H_t, A_t, μ_t, w_t, p_t, R^b_t, R^l_t, $\overline{R}^b_t$, B_{t+1}, L_{t+1}, ω_{t+1}. Notice that $\tilde{B}_{t+1}$ is a known function of B_{t+1}, L_{t+1} and the realization of the sunspot shock ε. Therefore, I can compute the expectation of the next period price p_{t+1} if I know the function $P(\tilde{B}_{t+1})$. We can then solve the 11 equations for the 11 variables and this would provide a solution for any given state $\tilde{B}_t$.

The problem is that I do not know the function $P(\tilde{B}_{t+1})$. Therefore, the procedure will be based on a parametrization of an approximation of this function. In particular, I approximate $P(\tilde{B}_{t+1})$ with a piece-wise linear function over a grid for the state variable $\tilde{B}_t$. I then solve the above system of equations at each grid point for $\tilde{B}_t$. As part of the solution I obtain the current price p_t. I then use the solution for the current price to update the approximated function $P(\tilde{B}_{t+1})$ at the grid point. I repeat the iteration until convergence, that is, the values guessed for $P(\tilde{B}_{t+1})$ at each grid point must be equal (up to a small rounding number) to the values of p_t obtained by solving the model (given the guess for $P(\tilde{B}_{t+1})$).

H.2 Equilibrium with structural break

When the intermediation cost τ changes, the economy transits from a stochastic equilibrium to a new stochastic equilibrium. This requires to solve for the transition and the solution method is based on the following steps.

1. I first compute the stochastic equilibrium under the regime that proceeds the structural break (the operation cost of banks is constant at the initial level).

2. I then compute the stochastic equilibrium under the terminal regime (the operation cost of banks remains constant at the new level).

3. At this point I solve the model backward at any time t starting at the terminal period when the operation cost remain constant at the new level. At each t I solve the system (33)-(43) using the approximated function $P_{t+1}(\tilde{B}_{t+1})$ found at time $t+1$. In the first backward step (last period of the transition), $P_{t+1}(\tilde{B}_{t+1})$ is the approximated price function found in the stochastic stationary equilibrium after the break (see previous computational step).

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