Network Reactions to Banking Regulations

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Abstract

Banks' optimal liquidity requirements have to solve a delicate balance. Tighter requirements reduce the chances of liquidity shortages. Looser requirements foster the allocation of funds towards productive investments. With multiple banks characterizing optimal requirements is however more challenging. Banks form lending partnerships in the overnight interbank lending market to meet their liquidity needs. We show that the interbank network may suddenly collapse once reserve requirements are pushed above a critical level, with a discontinuous increase in systemic risk as cross-insurance possibilities for banks collapse.

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1 Introduction

Banks provide a key intermediation function in the economy. They extend loans to agents with productive investment opportunities using funds from depositors without access to those opportunities. As banks' investments may fail, depositors only provide funds if they are ex-ante compensated for those potential losses, forcing bankers to internalize failure and to invest efficiently, both in terms of scale and risk exposure. The scenario changes though, in the presence of governments with ex-post incentives to cover the losses of depositors with distortionary taxation proceedings. These bailouts provide an implicit insurance for depositors that allow banks to obtain funds at a subsidy, then investing excessively, either at a larger scale or taking more risks, relative to the efficient allocation. As highlighted by Nosal and Ordonez (2016), when the government lacks commitment not to bail out depositors in distress, banks tend to invest inefficiently.

To discourage inefficient investments that increase the likelihood and the size of failures and the need for distortionary bailouts, the government regulates banking activity. One of these regulations involve liquidity requirements that impose a direct upper bound on the amount of loans that can be extended by a bank per U.S. dollar kept physically in possession of the bank. Too tight liquidity requirements reduce excessive investments and risk-taking and then the needs for bailouts, but may end up choking the banks' role to channel funds to productive investment opportunities. When these two forces change smoothly in the level of liquidity requirements for an individual bank, it is possible to obtain the optimal regulation that delicately balances banks' incentives to take on excessive risk and banks' efficiency on extending loans.

This logic is sound when considering a single bank, but we argue fails when considering multiple banks that interact with each other. Banks do not operate in isolation and react to liquidity requirements by making loans to each other, in the overnight interbank lending market, at relatively low rates to cover liquidity shortages. After extending loans for a lucrative investment opportunity, a bank may fall short of the liquidity needed to fulfill potential refinancing needs to continue, and even expand, its investments to fruition and refer to other banks for short-term loans in order to satisfy these needs. This implies that the level of liquidity requirements may have a first order impact on the vibrancy and interconnectedness of the interbank market. How does the interbank network react to changes in liquidity requirements? How should the liquidity requirements be determined in a setting with interacting banks? How does the endogenous formation of interbank networks affect the welfare effects of liquidity requirements?

In this paper we explore these questions and show that the topology and the level of interconnectedness of the interbank network reacts discontinuously to liquidity requirements, adding an additional layer of complexity to the delicate balance that optimal banking regulations should strive to achieve. The reaction of an interbank network features a “phase transition": beyond a tipping point of liquidity requirements, the network becomes disproportionately less interconnected, with systemic risk increasing discontinuously in response to this abrupt change in the network architecture.

The benefit of having an interbank counterparty is to have additional "insurance" in case of a refinancing shock that requires extra funds to continue operating, and sometimes even expanding, some of bank's investments. Even though a fraction of the bank's counterparties is expected to become insolvent, a complementary fraction can provide extra liquidity to the bank if suffering a refinancing shock that requires extra liquidity. On the one hand, the marginal benefit of having an extra counterparty decreases as liquidity requirements tight, as the loan sizes decline and banks obtain less benefits from insuring continuation of those projects. On the other hand, the marginal cost of an extra counterparty is assumed independent on the level of liquidity requirement or the investment scale.

As liquidity requirements tighten, we do not only show that the desired level of counterparties decreases, but also that, after a critical regulation point, a bank would discontinuously prefer to reduce its counterparties, with interbank lending network jumping from very dense to very sparse, with the aggregate level of interbank activity collapsing. In other words, as liquidity requirements become tighter, not only the level of investment declines but also systemic risk, measured by the fraction of banks that choose to close operations, increases (discontinuously after a certain threshold), inducing a discontinuous increase in distortionary bailouts: a discontinuous reduction in both output and welfare.

This result suggests important policy implications. We show that the highest welfare that takes into account endogenous network formation, and then takes into account the network reaction to banking regulations, is achieved exactly at the tipping point after which networks discontinuously collapse. Just setting liquidity requirement at the tipping point that sustains networks and cross-insurance assumes the absence of exogenous shocks to fundamentals. When fundamentals are subject to shocks, setting liquidity requirements a the tipping point can induce crises after small changes in fundamentals. This suggests that the optimal distance between the regulatory liquidity requirements and the tipping point should balance the loss in welfare from being farther away from the tipping point and the gains from reducing the probability of the system moving beyond the tipping point, a crisis.

Based on these insights we compare the liquidity requirements that a *network-conscious* regulator, who understands the reaction of interbank insurance possibilities to changes in regulation, would impose with those of a *network-blind* regulator, who does not understand that networks change with regulation. We show that the potential welfare losses from ignoring network reactions can be sizable.

Literature Review: The recent financial crisis has been a catalyst for the interests both on liquidity regulations and on interbanking networks. Even though there has been a recent rich literature on these two topics, our paper is one of the few, if not the first, to combine these two policy-relevant topics and analyze the effects of liquidity requirements on interbank network formation.

On liquidity requirements, most of the literature has focused on the optimal level of a *single bank*'s liquidity requirement to prevent bank runs, such as Cooper and Ross (1998), Ennis and Keister (2006), Calomiris et al. (2014) and Santos and Suarez (2015). Diamond and Kashyap (2016) highlight a novel, informational, role of liquidity regulations. Liquidity requirements can induce banks to disclose their superior information about the likelihood of facing a run if they are set such that banks hold liquidity that goes unused, as this is the extra liquidity needed to deter runs. Interestingly, in their case the lender of last resort and liquidity requirements complement each other in eliciting information, while in our paper liquidity requirements are introduced because the lender of last resort is inevitable.

In spite of these efforts, Allen (2014) surveys the recent literature on liquidity regulation, concluding that "much more research is required in this area. With capital regulation there is a huge literature but little agreement on the optimal level of requirements. With liquidity regulation, we do not even know what to argue about." The confusion on the role of liquidity requirements to face liquidity shortages comes in large part from the lack of consensus of what a liquidity shortage is. While the referred previous literature focuses on using liquidity regulation to curb the effects of liquidity shocks on banks' short-term liabilities, as in Diamond and Dybvig (1983), our paper focuses on using of liquidity regulations to curb the effects of liquidity shocks on banks' assets refinancing needs, as in Holmström and Tirole (1998).

A more recent literature focus on the effects of liquidity regulations when banking interconnections are acknowledged. Farhi et al. (2009), for example, study the "externality" effects of regulation and the role of liquidity requirements to prevent free riding that occurs when the liquidity of one bank reduces the likelihood of a run in another bank. Aldasoro et al. (2015) construct a model of bank linkages and discuss the effects of banking regulations given the existence of banking interconnections. They show that liquidity requirements decreases systemic risks at the cost of lower efficiency. They study changes in regulation on risk and efficiency conditional on a network and not their effects on reshaping the network. This is an important difference, as we show that there exists a critical point beyond which liquidity requirements both increase systemic risks and reduce efficiency, with the single benefit of reducing distortionary bailouts. To the best of our knowledge ours is the first paper to tackle the question of optimal regulation not only when there are externalities and interbank lending, but also when banks strategically react to regulation by changing the way they connect and insure each other.

On liquidity regulation, there is also a recent debate, sparked mostly by new regulatory proposals by the Basel Committee on Banking Supervision and by the Dodd Frank Act, about liquidity requirements that are alternative to standard reserve requirements, such as the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR). The LCR extends the interpretation of “liquid assets" beyond reserves, including assets with higher interest rates. The costs, complexity and discretion vulnerability of LCR relative to the potentially small benefits relative to the use of reserve requirements have been discussed by Goodfriend (2016). The NSFR defines the portion of capital and reliable liabilities that financial intermediaries have to maintain over a specified time horizon relative to the amount of required stable funding in that time period. Even though our setting is constructed to capture reserve requirements, it could accommodate these two alternatives by including more assets classes (LCR) or a more involved timing of funding needs (NSFR).

There is also a recent literature on endogenous networks, such as Erol and Vohra (2016), but not a systematic study to understand the endogenous reaction of financial networks to changes in banking regulations, and their effects on systemic risk and welfare. Allen and Gale (2000) discuss bankruptcy contagion in an exogenous network (modeled as a ring) and, consistent to our results, show a negative relation between the degree of connectivity and systemic risk. Battiston et al. (2012) show that the relation between connectivity and systemic risk can be hump shaped as low connectivity implies less risk-sharing and high connectivity implies more exposure to default of partners. These are considerations our setting capture as well to identify the characteristics of endogenous networks. In a setting with heterogenous banks, Farboodi (2015) provides a model of banking networks that allow funds to flow from savers to investment opportunities, possibly over several links, and discusses the distribution of surplus depending on the network topology. None of these papers, however, have studied the change in networks induced by regulation and then optimal regulation taking networks into account.

Finally, while Ordonez (2016) study the unforeseen effects of banking regulations on financial innovations that allow banks to channel activities outside the scope of regulators (the so-called “shadow banking"), in this paper we focus on the unforeseen effects of banking regulations on networks, through the creation and destruction of banking interlinkages, affecting both systemic risk and welfare. Understanding the unforeseen effects of banking regulations are important considerations to factor in the discussion about optimal regulations.

We structure the paper as follows. In the next section we present a simple model with liquidity requirements and the possibility of creating relations with other banks to face possible refinancing needs. In Section 3 we study the effects of liquidity requirements on the anatomy of these banking linkages. In Section 4 we discuss the effect of liquidity requirements on systemic risk and we characterize optimal regulation of a network-conscious regulator that factors in network effects. In Section 5 we compare the welfare losses of network-blind regulators who does not consider the effects of regulations on networks. In the conclusions we highlight final remarks and considerations.

2 Model

2.1 Environment

We consider a single-period economy composed by k banks, k risk-neutral households and a government. We denote by $\mathcal{N} = \{n_1, ..., n_k\}$ the set of banks. Each bank n_i has access to a unique project (of maximum possible size, $\overline{L}_i$) to finance, and is associated with a unique household n_i^h. We assume that the household n_i^h deposits D_i in bank n_i at the beginning of the period and wishes to withdraw and consume at the end of the period.

The timing of actions and events within the period can be split in five stages. The first stage is the *regulation stage*. The government sets a liquidity requirement $\phi \in [0, 1]$, which imposes the fraction of the loan extended by each bank that the bank must keep as reserves in cash (or other highly liquid assets). This liquidity requirement is isomorphic to a leverage constraint.

The second stage is the *network formation stage*. Banks form links by mutual consent, that serve as credit lines to insure each other against future refinancing shocks that may put the maturity of the project at risk. A formed link between banks n_i and n_j is denoted $l_{ij} = l_{ji}$, and the resulting set of links is denoted $\mathcal{L} \subset [\mathcal{N}]^2$. $(\mathcal{N}, \mathcal{L})$ is the realized interbank network. Denote $\mathcal{N}_i = \{j : l_{ij} \in \mathcal{L}\}$ the set of counterparties of n_i and $d_i = |\mathcal{N}_i|$ its degree (the number of counterparties). Forming each link involves a utility cost κ_l to each counterparty.

The third stage is the *investment stage*. Each bank n_i extends in checks a loan of L_i to finance the project, keeping its deposits D_i as reserves. The liquidity requirement restricts the size of the loan (and then the size of the project) L_i, as $D_i \geq \phi L_i$, where $L_i \leq \bar{L}_i$. The returns from the projects materialize at the end of the period, and banks' checks (that amount to total bank's liabilities of L_i) are also due at the end of the period.

The fourth stage is the *continuation stage.* The projects managed by all banks are publicly revealed to be $\theta_i \in \{B, G\}$, where B represents a *bad project* that never matures and G a *good project* that can succeed if appropriate actions are taken by the bank in the next stage. The types of projects, θ_i are independent across projects, G with probability α_i and B with probability $1 - \alpha_i$.

After the projects' types are publicly observed, each bank n_i chooses an action a_i. Banks managing good projects can choose to continue (C) operating the project, or not (N), then $a_i \in \{C, N\}$. Continuing operation has a management cost (an effort cost) of κ_c per unit of investment and κ_l' to maintain the counterparty, making up a total of $\kappa_c L_i + \kappa_l' d_i$. Banks with bad projects always choose N; this is $a_i = N$, as the bank would only pay the previous continuation costs, without recovering any payoffs form the project. Let g_i and f_i respectively denote the number of counterparties of n_i that chose to continue and not continue: $g_i = |\{n_j \in \mathcal{N}_i : a_j = C\}|$ and $f_i = |\{n_j \in \mathcal{N}_i : a_j = N\}|$.

A bank n_i that chooses N liquidates its project early and recovers $R_1 L_i$ where $1 > R_1 \geq 0$. This implies that the total assets of a non-continuing bank at the end of the period are $D_i + R_1 L_i$ while total liabilities are $D_i + L_i$, as the bank owes D_i to the "original depositors" (households) and L_i to "business depositors" (check holders).[1] We assume that the government faces a large (maybe political) disutility from depositors not obtaining their funds back at the end of the period. This simple assumption guarantees that the government would always bail out the defaulted depositors, for a total of $(1 - R_1)L_i$. Banks n_i that choose not to continue then receive 0.

A bank n_i that chooses C, and continue business, move to the fifth and last, *refinancing stage.* At that stage at most one of the projects of the continuing banks receives a refinancing shock, which is characterized by new funds that are needed in cash for the project to mature (a liquidity shock). The probability that a bank n_i, and only n_i, needs extra funds is η_i. With probability $\eta_0 = 1 - \sum_i \eta_i$ no project receives a refinancing shock. Conditional on the continuing bank n_i receiving a refinancing shock, the amount of funds needed are ρ_i, drawn from a distribution with c.d.f. F_i. Investors can obtain these extra funds only from their associated counterparty banks in the network.

The banks not facing a refinancing shock and that have invested at scale L_i obtain a return $R_2 L_i$, where $R_2 > 1$. As for the bank facing the refinancing need, if the bank obtains enough funds to refinance the liquidity need ρ_i, then the project's return scales by $m \geq 1$ and pays

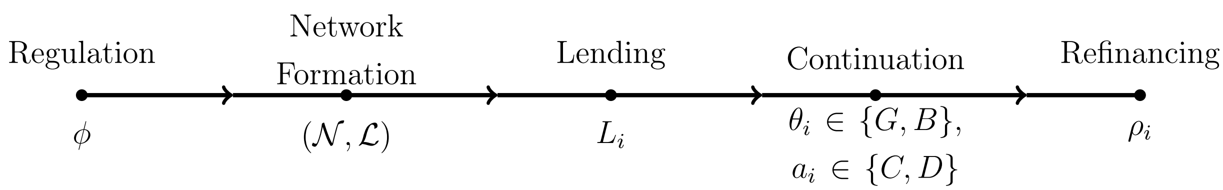
[1] The project uses n_i's checks in its investments, for example to buy raw materials or to pay workers. The recipients of these checks, for a total of L_i, eventually deposit them in the bank n_i, so we denote them as "business depositors."

an extra ρ_i, so that the return to n_i is $mR_2L_i + \rho_i$. If the bank does not obtain enough funds to refinance the liquidity need, the project fails and returns 0.

Finally, we assume that after the refinancing shock is observed, each household n_i^h receives extra funds (say wages) for W_i, which are deposited at bank n_i if n_i has continued and is still in business. Then, without interbank activity, a bank n_i facing a refinancing shock relies on its own available funds $W_i + D_i$ to ride the shock. This implies that in such case the bank would be able refinance the project to completion if and only if $\rho_i \leq W_i + D_i$.

Banks, however, may engage on interbank lending through credit lines (links) formed in advance at the network formation stage. We assume that funds do not travel further than one link as the main friction in the interbank market, that is a bank does not intermediate between two banks but rather intermediate between one household and one bank. Notice that due to the extra ρ_i return on top of R_2L_i, there is no risk in lending these funds to the bank facing a refinancing need, conditional on knowing that n_i will be able to borrow $\rho_i - D_i - W_i$ in total from its counterparties. A conterparty n_j, if it has decided to continue, has W_i excess liquidity to lend to n_i.[2] Therefore, n_i can provide the ρ_i if and only if $\rho_i \leq D_i + W_i + \sum_{j:n_j\in\mathcal{N}_i, a_j=C} W_j$.

The next timeline summarizes the sequence of events and actions and the main notation.



3 Network Formation and Contagion

Now we can solve the equilibrium, particularly focusing on the reaction of network formation to banking regulations. We work backwards and first solve the investment, continuation and refinancing stages. Then, based on these solutions, we characterize the equilibrium network (from the network formation stage) as a function of regulation ϕ.

[2] Motivated by the use of reserve requirements, banks are only allowed to lend above required reserves in the interbank market. Hence W_j can be used on the interbank market. Notice that if $D_j > \phi L_j$, n_j has excess liquidity out if its initial deposits D_j as well. In principal these could also be lended in the interbank market. We assume that n_j does not make loans in the interbank market out of its excess liquidity other than W_j. This can be thought of as the limit of the credit line. This assumption allows us to simplify the optimal choice of L_i by the bank n_i, which is otherwise intractable to pin down due to network externalities.

3.1 Investment, Continuation and Refinancing Stages

3.1.1 Payoffs

Payoffs at the end of the refinancing stage: A bank n_i that does not continue in the continuation stage had paid $\kappa_l d_i$ in terms of utility to form the network, has expost income $D_i + R_1 L_i$ and owes $D_I + L_i$. Then after paying all available assets to depositors, it obtains 0 return and depositors receive a bailout for $(1 - R_1)L_i$. Then the bank's expost payoffs are $-\kappa_l d_i$.

If the bank n_i continues and does not receive a refinancing shock, it pays back its depositors and has $(R_2 L_i + D_i + W_i) - (L_i + D_i + W_i) = (R_2 - 1)L_i$ return. The bank that continues had also incurred in utility costs to form the network, $\kappa_l d_i$, to maintain the network, $\kappa_l' d_i$ and to managing the project, $\kappa_c L_i$. Then the continuing bank's expost payoffs are $(R_2 - 1)L_i - (\kappa_l' d_i + \kappa_c L_i) - \kappa_l d_i$.

If the bank n_i continues and receives a refinancing shock, then payoffs depend on whether it can find funds to refinance or not. If the bank can refinance the shock ρ_i, it has $mR_2 L_i - L_i$ expost payoff since it receives ρ_i extra return on top of the $mR_2 L_i$. Notice that interbank lending does not enter a bank's payoffs since lending to a troubled bank only happens if certain of repayment. If the bank cannot refinance the shock ρ_i, it cannot fully repay its depositors and has expost payoffs $-(\kappa_l' d_i + \kappa_c L_i) - \kappa_l d_i$.

Expected payoffs at the end of the continuation stage: Let M_i denote expected the net return of a bank n_i on its unit loan aside of the management costs of having counterparties:

$$M_i = (1 - \eta_i)(R_2 - 1) + \eta_i F_i\left(D_i + W_i + \sum_{j:n_j \in \mathcal{N}_i, a_j = C} W_j\right)(mR_2 - 1) - \kappa_c \tag{1}$$

Accordingly, the bank n_i that continues has expected payoff (at the end of continuation stage)

$$M_i L_i - (\kappa_l' + \kappa_l)d_i. \tag{2}$$

If n_i does not continue, its payoff is $-\kappa_l d_i$.

Investment stage: At the continuation stage, bank n_i will best respond, so that payoffs are $-k_l d_i + max\{0, M_i L_i - \kappa_l' d_i\}$. M_i is independent of L_i since the continuation decision of n_i's counterparties depend only on the amount that n_i can lend to them in the interbank market, which is W_i, independent of L_i. Hence the expected payoff of n_i is weakly increasing in L_i. All banks prefer to extend as much loans as possible in the investment stage, that is $L_i = min\{\frac{D_i}{\phi}, \bar{L}_i\}$.

3.1.2 Assumptions

For simplicity in the exposition in what follows we focus on a symmetric scenario in which all banks are identical. Later we will discuss the implications of heterogeneity for network formation.

Assumption 1. $\bar{L}_i = \bar{L}$, $D_i = D$, $W_i = W$, $\alpha_i = \alpha$, $\eta_i = \eta$, $F_i \equiv F$ *for all* n_i.

Henceforth we denote $L(\phi) = min\{\frac{D}{\phi}, \bar{L}\}$. Recall that the total number of counterparties of n_i that continue is g_i, which are the banks that lend their excess liquidity to bank n_i when in need of refinancing. Morever define $T(d_i|\phi)$ as the "probability threshold" of n_i for continuing.

$$T(d_i|\phi) = T_0 + \frac{d_i \kappa_l'}{\eta(mR_2 - 1)L(\phi)}, \quad T_0 = \frac{\kappa_c - (1-\eta)(R_2 - 1)}{\eta(mR_2 - 1)} \tag{3}$$

Under Assumption 1 we can simplify the expression of net return per unit loan in (1) and rewrite the expected payoff in (2) as

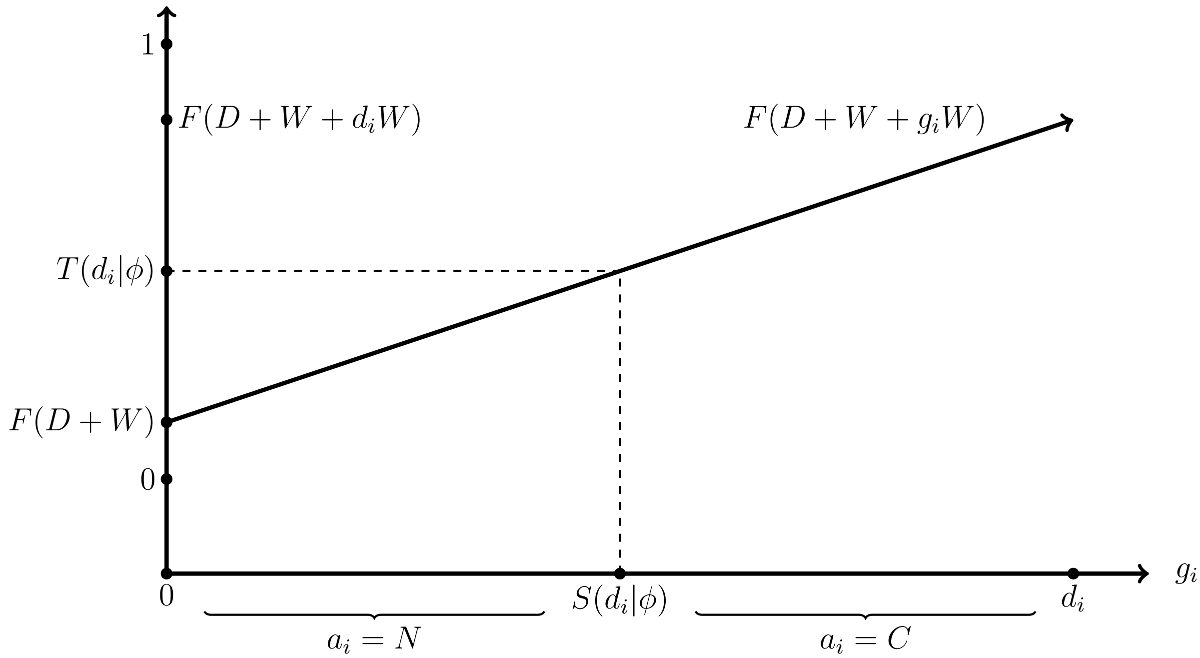
$$\Pi(C, g_i, d_i|\phi) = -\kappa_l d_i + (mR_2 - 1)\eta L(\phi)\big[F\big(D + W + g_i W\big) - T(d_i|\phi)\big], \tag{4}$$

$$\Pi(N, g_i, d_i|\phi) = -\kappa_l d_i \tag{5}$$

in case of continuation or not-continuation respectively.

If $T(d_i|\phi) > 1$, n_i plays N regardless, as $F(\cdot) \leq 1$. Similarly, if $T(d_i|\phi) < 0$ n_i plays C regardless, as $F(\cdot) \geq 0$. For $T(d_i|\phi) \in (0,1)$, if $g_i \geq (F^{-1}(T(d_i|\phi)) - D - W)/W$ the bank n_i's best response is to continue, as in such case $F(\cdot) > T(d_i|\phi)$.[3] The expression $S(d_i|\phi) \equiv \lceil (F^{-1}(T(d_i|\phi)) - D - W)/W \rceil - 1$ is a measure of the *fragility* of the bank n_i with degree d_i. n_i plays N if $g_i \leq S(d_i|\phi)$. The higher is S, the more successful counterparties a bank requires to refinance projects and continue then to maturity. Conditional on the the total number of counterparties and those successful counterparties that continue, a bank n_i is more likely to continue as $S(d_i|\phi)$ declines, which occurs when the bank has more funds on its own (higher D and W), or when the threshold for refinancing increases and the bank has less incentives to continue (higher $T(d_i|\phi)$). In this last case, the bank has less incentives to continue when the net present value of the project is relatively small (low R_2), when the cost of continuation is relatively large (high κ_c or κ_l') or when the probability that the project needs refinancing is relatively large (high η).

[3]Here we follow the convention that $F^{-1}(t) = \infty$ for $t > 1$ and $-\infty$ for $t < 0$.



For expositional simplicity we assume that F is uniform on $[0, P]$, which implies that $F(D + W + gW) = max\{0, min\{1, \frac{D+W+gW}{P}\}\}$. Moreover, we assume that the largest possible refinancing need P cannot be met at any level of connectivity. The latter makes sure that every additional counterparty that continues always strictly improves the expected payoff.

Assumption 2. *$F \sim U[0, P]$, $P > D + W + kW$.*

Before moving to solving for the equilibrium network, we specify the solution concepts.

3.2 Solution concepts

In the continuation stage, banks with bad shocks are forced into playing N. Banks with good shocks play a binary game among each other. This game is supermodular. The solution concept is the cooperating equilibrium: it is the Nash equilibrium in which the largest set of agents, with respect to set inclusion, play C among all Nash equilibria. Due to supermodularity, this equilibrium notion is well-defined. Supermodularity, by Tarski's Theorem, implies that the set of Nash equilibria is a complete lattice, and the cooperating equilibrium is the highest element of the lattice.

An alternative definition of the cooperating equilibrium can be given via a rationalizable contagion argument, which has a natural interpretation in financial contagion setup. Banks who receive bad shocks are forced into not continuing. After some banks become insolvent in this way, some solvent banks who are tightly connected to insolvent banks will become illiquid, and find N iteratively strictly dominant, and so on. The iterated dominance argument resembles the financial contagion black-boxed into a single period. Along the iteration, at the point that remaining banks can rationalize C, the contagion stops. The resulting profile is the rationalizable strategy profile in which anyone who can rationalize C do play C. Supermodularity ensures that this profile is also a Nash equilibrium.

In the network formation stage, banks evaluate each network with the expectation (over the shocks θ's) of their payoffs in the cooperating equilibrium in the subsequent periods. Agents form a strongly stable network which is defined as follows. Consider a candidate interbank network $(\mathcal{N}, \mathcal{L})$ and a subset $\mathcal{N}'$ of agents. A feasible deviation by $\mathcal{N}'$ from $\mathcal{L}$ is one in which i) $\mathcal{N}'$ can add any missing links or cut any existing links that stays within $\mathcal{N}'$, and ii) $\mathcal{N}'$ can cut any of the links between $\mathcal{N}'$ and $\mathcal{N}/\mathcal{N}'$. A profitable deviation by $\mathcal{N}'$ from $\mathcal{L}$ is a feasible deviation in which the resulting network yields strictly higher expected payoff to every member of $\mathcal{N}'$. An interbank network $(\mathcal{N}, \mathcal{L})$ is strongly stable if there are no subsets of $\mathcal{N}$ with a profitable deviation from $\mathcal{L}$.[4]

3.3 Network Formation Stage

Payoff functions $\Pi(C)$ in (4) and $\Pi(N)$ in (5) are special cases of Erol (2016) which proves the existence and almost uniqueness of strongly stable networks in a more general setup.

Define the functions V and d^* as follows.

$$V(d|\phi) := \mathbb{E}_{\tilde{g}}[(1-\alpha) \times \Pi(N, \tilde{g}, d|\phi) + \alpha \times max\{\Pi(C, \tilde{g}, d|\phi), \Pi(N, \tilde{g}, d|\phi)\}]$$

$$= -\kappa_l d + \beta L(\phi) \mathbb{E}_{\tilde{g}} \big[max\{0, F\big(D + W + \tilde{g}W\big) - T(d_i|\phi)\} \big]$$

where $\beta = \alpha(mR_2 - 1)\eta$ and $\tilde{g} \sim G[\cdot, d, \alpha]$ for G defined as the c.d.f. of binomial distribution with d trials and α success probability. V can be thought of as a hypothetical payoff function for a single bank with degree d supposing that its counterparties default if and only if they received bad shocks. In other words, V is the payoff function absent contagion.

$$d^*(\phi) := argmax_{d \in \mathbb{Z}^*} V(d|\phi).$$

[4]This network formation solution concept can be microfounded by a proposal game. Each bank pays $c > 0$ to make a proposal to another bank. Mutual proposals turn into links, and cost c is refunded to both. One-sided proposals do not turn into links, and c is lost. After being formed, links represent secondary market trades that serve to provide liquidity to banks. The strong Nash equilibria of this game corresponds to strongly stable networks. See Erol and Vohra (2016) for more details.

Since $F(\cdot) \leq 1$, d^* is generically well-defined. $d^*(\phi)$ is the hypothetical optimal degree for a single bank if there was no contagion. The following network formation result uses the main theorem of Erol (2016):

Proposition 1 (Network Formation)**.** *Let $k(\phi) \in [0, d^*(\phi)]$ be given by $k \equiv k(\phi)(mod\ d^*(\phi)+1)$. If $T(d^*(\phi)|\phi) > F(D+2W)$, there exists a unique strongly stable network which consists of $\big(k - k(\phi)\big)/(d^*(\phi)+1)$ disjoint cliques of order $d^*(\phi)+1$ and some more disjoint cliques of smaller order.*[5] *If $T(d^*(\phi)|\phi) < F(D+2W)$, the network described above is strongly stable. In any other strongly stable network, all but at most $d^*(\phi)$ banks have degree $d^*(\phi)$.*

In the cooperating equilibrium, if less than $S(d^*(\phi)|\phi)$ banks in a clique get good shocks, all $d^*(\phi)+1$ banks in the clique play N. If more than or equal to $S(d^*(\phi)|\phi)$ banks in a clique get good shocks, banks with bad shocks play N and banks with good shocks play C.

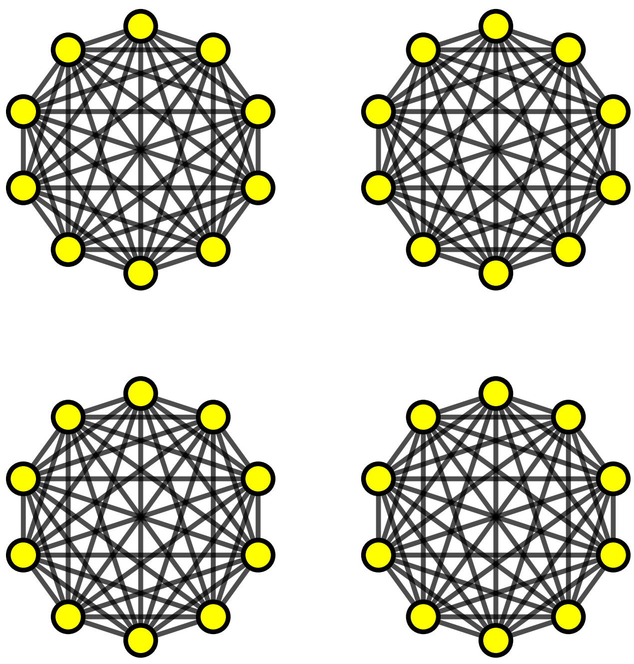


Figure 1: Cliques

The proof of Proposition 1 is in the Appendix. Figure 1 illustrates the structure of the network formed. Notice that this structure eliminates second order counterparty risk: the risk that a bank may incur losses due to some of its counterparties that choose to not continue because of their own counterparties that do not continue. By forming cliques of an optimal size, banks reach a satiation point in terms of desired number of counterparties while they

[5]See Erol (2016) for the exact order(s) of the remaining smaller clique(s). It needs heavy notation to describe the remainder of the network.

do not face any risk of contagion further and beyond what is inevitable, i.e. the risk that their immediate counterparties get bad shocks.

If $T(d^*(\phi)|\phi) < F(D+W)$ there is no contagion (meaning that there is naturally no second order counterparty risk) as the available funds from the bank facing the shock is enough to cover the refinancing need, then the bank continues even if none of its counterparties continue. If $F(D+W) < T(d^*(\phi)|\phi) < F(D+2W)$ there is minimal contagion, as it is enough that one counterparty continues for the bank in need of refinancing continues, then the bank continues unless none of its counterparties continue. The more interesting case is $T(d^*(\phi)|\phi) > F(D+2W)$, as the bank with refinancing needs continues if and only if more than a counterparty continues. Proposition 1 shows that in this last and more relevant case the strongly stable network is unique. In what follows, we ignore integer problems between $d^*(\phi)+1$ and k and focus on networks that consist of disjoint cliques of order $d^*(\phi)+1$.[6]

From Proposition 1 we can compute the distribution of the number of banks that play N.

Corollary 1 (Systemic Risk)**.** *For a given ϕ, the expected fraction of banks that play N is*

$$1-\alpha+\alpha G[S(d^*(\phi)|\phi), d^*(\phi)+1, \alpha],$$

and the probability that all banks play N is

$$G[S(d^*(\phi)|\phi), d^*(\phi)+1, \alpha]^{\frac{k}{d^*(\phi)+1}}.$$

This corollary shows two measures that capture systemic risk. As the fragility of a bank, $S(d^*(\phi)|\phi)$, increases weakly with ϕ, a tighter regulation provides more liquidity buffers to face refinancing needs, reducing in principle systemic risk. This is only the case, however, if we neglect the effect on the network structure via $d^*(\phi)$. Critically, if tighter regulations also decreases the equilibrium number of counterparties in the network, $d^*(\phi)$, then the level of systemic risk in the economy may increase.

In what follows we characterize how the equilibrium degree of the network $d^*(\phi)$ depends on regulation ϕ. The optimal number of counterparties for the bank will be given by the degree that maximizes its value function.

Proposition 2. *Define resillience $R(d|\phi) := d - S(d|\phi) - 1$. $V(d|\phi)$ is given by:*

[6] The results formally follows by assuming that ϕ is selected from a finite subset X of $[0,1]$, and k is divisible by $\prod_{\phi\in X}(d^*(\phi)+1)$. Intuitively, though, the remainder cliques represent a small fraction of the economy and they are isolated from the rest of the network, hence they do have negligible impact on the comparative statics.

$$V(d|\phi) = A\ L(\phi)\ G\big[R(d|\phi), d, 1-\alpha\big] +$$
$$d \times \Bigg[-\kappa_l - \alpha\kappa_l'\ G\big[R(d|\phi), d, 1-\alpha\big] + B\ L(\phi)\ G\big[R(d|\phi), d-1, 1-\alpha\big]\Bigg],$$

where

$$A = \alpha(mR_2 - 1)\big(F(D+W) - T_0\big) = \alpha\Bigg[(1-\eta)(R_2-1) - \kappa_c + \frac{\eta(mR_2-1)(D+W)}{P}\Bigg],$$

$$B = \frac{\alpha^2\eta(mR_2-1)W}{P}.$$

This characterization of the value function as a function of the network degree, which according to Proposition 1 is based on cliques, allows to obtain which is the individual optimal number of counterparties that a bank would choose to borrow in the interbank lending market in case it faces a refinancing need.

Notice that $F(D+W) - T_0$ governs the behavior of a bank with degree 0. The coefficient of d nicely decomposes the costs and benefits of having additional counterparties.

This expression is suggestive of discontinous dynamics already. For a second, ignore the dependence of the probabilities G on ϕ and d. As ϕ increases the coefficient of d in brackets decreases since $L(\phi)$ decreases. At the point that the coefficient becomes negative, the net value of a link changes from positive to negative, and the network suddenly turns from a dense network into a sparse network. This is first source of phase transition. Of course, the probabilities in G can also play a role. Recall that $T(d|\phi)$ increases in both ϕ and d due to the management costs of links. Accordingly, $S(d|\phi)$ also increases in d and ϕ. Depending on parameter values, this can lead to $R(d|\phi)$ being decreasing in d and ϕ. As ϕ goes up resillience R goes down. This incentivizes banks to reduce d to keep resillience R high. At one point, if ϕ becomes too large, it might become impossible to keep R sufficiently high and the bank can give up trying to maintain a sufficiently large resilience. This is the second source of phase transition. The effect of ϕ on V is portrayed in Figure 2.

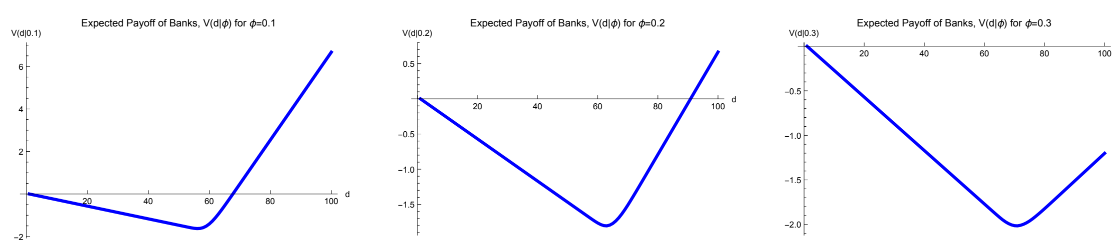


Figure 2: $V(d|\phi)$ as ϕ increases

As a consequence of the transition in qualitative features of V, the optimal degree d^* also features a transition as in Figure 3. For loose requirements, links are sufficiently beneficial and banks would like to form a dense network to achieve a sufficient chance of riding the refinancing shock. Beyond the transition region, links are too costly to achieve a sufficient likelihood of riding the shock. In other words, too tight requirements suffocate the banks by killing the interbank network.

Note that in this example, a bank with degree 0 does not continue meaning that banks own resources are not large enough to reduce the refinancing risk sufficiently. Banks with no counterparties never wait until maturity. Therefore, once the network is characterized by no links (a collapse in interbank markets), no bank continues. Banks with bad projects never continues anyways, while banks with good projects choose to liquidate their projects as the likelihood of a refinancing shock is large enough and the chances to refinance in such case are low enough that it does not compensate to pay continuations costs.

In Section ??, we discuss and illustrate the results for the case in which banks' own resources are sufficient for them to continue funding their projects in case of not having any counterparty.

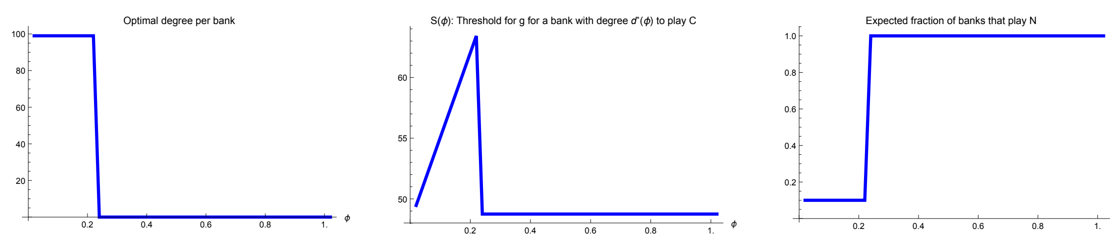


Figure 3: Optimal degree $d^*(\phi)$, induced fragility S, and resulting systemic risk

4 Optimality of a Network-Conscious Regulator

4.1 Notion of Welfare

The analysis so far has been positive and focused on understanding how the policy affects the network structure and the resulting systemic risk. Here we define the notion of welfare that allows us to obtain the optimal liquidity requirement in this environment, and to assess the losses from a naive regulation that disregard the effects of liquidity requirements on endogenous network formation and systemic risks.

We have assumed that banks have all the bargaining power and capture all the profits in case of continuation. We have also assumed that in case of not continuation, banks face limited liability and the government is forced to resort to distortionary bailouts to cover the banks' promises to depositors. Welfare is the sum of the bank's profits and social distortionary losses from bailouts.

Deposits are returned to households and consumed by them. The total amount $k(D+W)$ is constant so we don't count it for welfare. Then welfare has two components. One is the *banking component*: This is the part of welfare that is internalized by banks. It consists of banks' profits since all surplus of production is captured by banks. The other is the *bailout component*: This is the part of welfare that is not interbalized by banks. It consists of direct and indirect costs of providing the lost deposits back to the business depositors.

Banking component: The profits of all bad banks and for all good banks that do not continue are 0 and the expected profits for all good banks that continue are $(R_2-1)L$. Let g^* denote the total number of good banks that choose to continue: $g^* = |\{n_i : a_i = C, \theta_i = G\}|$. The expected total bank profits in the system is then $(g^* - \mathcal{I})(R_2-1)L$, where $\mathcal{I} = 1$ is an indicator function that takes the value 1 if a bank has a refinancing need that cannot be covered (and the project is then lost), and 0 otherwise. Including the total utility costs of forming the network $(k\kappa_l d^*)$, the utility costs of managing the network for all good banks continuing $(g^*\kappa_l' d^*)$ and the utility costs of managing the continuing projects $(g^*\kappa_c L)$. Hence the banking component is

$$(g^* - \mathcal{I})(R_2-1)L - k\kappa_l d^* - g^*(\kappa_c L + \kappa_l' d^*)$$

Bailout component: Let f^* denote the total number of good banks that choose not to continue: $f^* = |\{n_i : a_i = N, \theta_i = G\}|$. That means $k - f^* - g^*$ banks receive bad shocks, so that government covers $(k - f^* - g^*)L$ due to bad shocks. $f^*(1-R_1)L$ total amount is covered by government due to good banks that decide not to conitnue. Moreover, $\mathcal{I}L$ is covered due to the refinancing shock. The lost deposits are covered by the government which is the direct effect of bailouts which is not internalized by banks. Suppose that bailouts have a welfare cost on top of the direct resources provided, a total of $\kappa_d x$ for x total amount of resources used for bailouts $(\kappa_d \geq 1)$. The bailout component is then

$$-\kappa_d\big[(k - f^* - g^*)L + f^*(1-R_1)L + \mathcal{I}L\big].$$

Both components are influenced by policy ϕ via three main channels. First is the *loan channel*, as the loan depends on ϕ by $L(\phi) = min\{\frac{D}{\phi}, \bar{L}\}$. In words, the project scale per bank has direct effect on banks' profits and on the potential amount to be covered by bailouts. Second is the *contagion channel*, captured by the number of good banks who choose not to continue,

f^*, and for the good bank that continues and suffers a renegotiation need that cannot be covered in the network, $\mathcal{I}$. In words, the number of banks that liquidate good projects before maturity due to contagion of liquidation decisions affects the aggregate level of loans that end up maturing. That in turn affects the aggregate production, consumption, and the level of recovered deposits. Third channel consists of costs incurred by banks.

The number of banks with good shocks is $f^* + g^*$, which is αk in expectation at the regulation stage. Define $f^{**} = \mathbb{E}[f^*]$ and $\mathcal{I}^* = \mathbb{E}[\mathcal{I}]$ where expectations are taken at the regulation stage. Then expected welfare can be decomposed into components and channels as follows:

Welfare	Banking component	Bailout component
(Net) Loan channel	$\alpha k(R_2 - 1)L$	$-\kappa_d(1-\alpha)kL$
Contagion channel	$-f^{**}(R_2 - 1)L - \mathcal{I}^*(mR_2 - 1)L$	$-\kappa_d[f^{**}(1 - R_1)L + \mathcal{I}^*]L$
Management costs	$-k\kappa_l d^* - (\alpha k - f^{**})(\kappa_c L + \kappa_l' d^*)$	$-$

4.2 Optimal regulation and phase transition

Once we compute welfare for different levels of liquidity requirements ϕ, we can choose ϕ^* that maximizes welfare. Figure 4 shows two examples of total welfare. The first panel shows the case in which it is free to create a network $\kappa_l = 0$, but costly to manage the network upon continuation, $\kappa_l' > 0$. The second panel shows the case in which it is costly to create a network $\kappa_l > 0$, but free to manage the network upon continuation, $\kappa_l' = 0$. In both cases the optimal liquidity requirement ϕ^* falls into a region in which the network is transitioning from dense to sparse, as in Figure 3. In this case, optimal regulation involves systemic risk. The existence of a network allows more banks to continue, even when some banks will suffer refinancing shocks and may not continue. When liquidity requirements are large, they may suffocate banks enough such that they not only decline their loans but also choose not to continue.

This result suggests that, if the government has some uncertainty about fundamentals, it might be too risky to try to set the exact optimal policy at the kink since a slight overshooting can result in large unintended welfare cost and a collapse of interbank lending.

It is also important to understand the moving pieces for this result. In Figure 5, we dissect welfare into its components and channels. Banks' profits are increasing as ϕ increases because the loans are decreasing. Since loans decrease, the amount to covered by bailouts also decrease up until the transition point at which the number failing banks increase dramatically due to contagion.

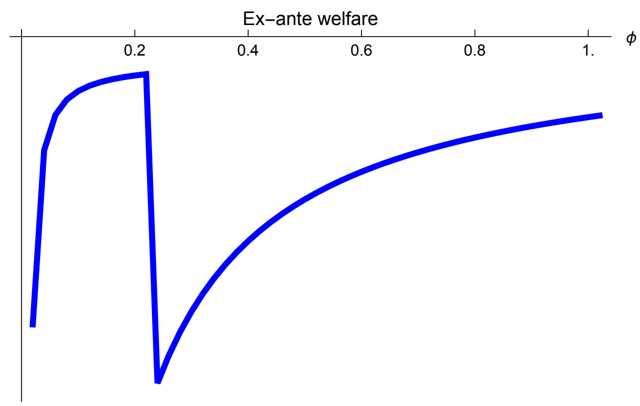


Figure 4: Welfare in ϕ

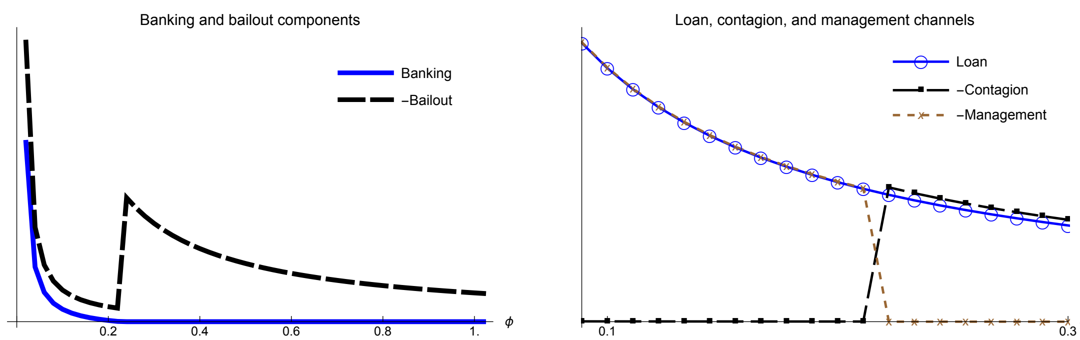


Figure 5: Components and channels of welfare in ϕ

Remark on what happens if isolated banks are liquid enough to continue: If $F(D+W) < PT_0$, isolated banks are destined to play N, which may seem to amplify the bailout costs. Here we illustrate in Figure 6 that even when $F(D+W) > PT_0$ it is possible that welfare features sharp falls around a transition point. However, the optimal is not at the transition point, rather at $\phi = 0$, which is still arguably close to the transition point.

Figure 6 shows banks payoffs sharply hit the lower bound which generates steep the fall and the recovery. Figure 7 shows that there is no spike in bailout costs.

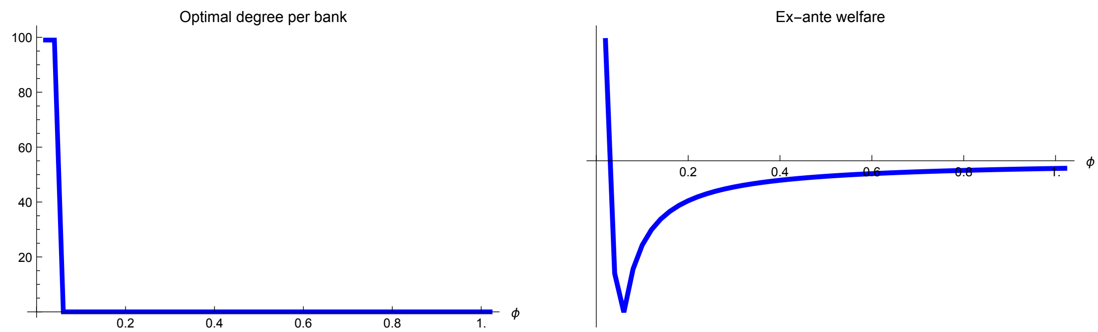


Figure 6: Optimal degree and induced welfare when isolated banks are sufficiently liquid

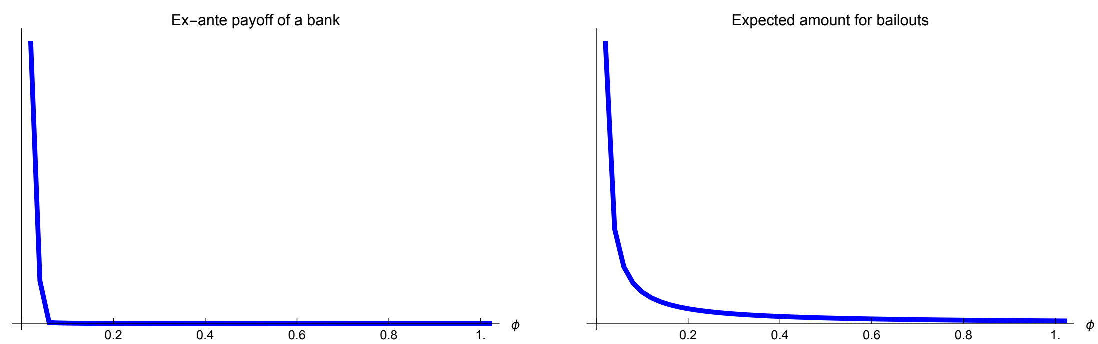


Figure 7: Optimal degree and induced welfare when isolated banks are sufficiently liquid

5 Mistakes of a Network-Blind Regulator

For any arbitrary network in cliques with degrees d and an arbitrary level of liquidity requirement ϕ, let $W(d,\phi)$ denote the resulting welfare. Suppose that the economy is at a status quo level of liquidity requirements ϕ_0 which is not necessarily optimal at this point. Banks in the market have also been structured into a strongly stable network in which all banks have degree $d_0 = d^*(\phi_0)$. The status quo welfare is $W(d^*(\phi_0),\phi_0)$. Now the government wishes to adjust ϕ to an optimal level. If network reactions are taken into account, government maximizes $W(d^*(\phi),\phi)$ over ϕ. The model predicts the changes as described in previous comparative statics: ϕ_0 is adjusted to $\phi^{**} = argmax_\phi\{W(d^*(\phi),\phi)\}$ and all banks have degree $d^{**} = d^*(\phi^*)$.

If, however, government does not acknowledge the reaction of the interbank network to the changes in policy and takes the network with degrees d_0 as given and fixed, what would be the unintended consequences of a so called optimal policy? In particlar, government takes the network in clique structure with degrees d_0 as given, and choose an optimal policy $\phi^*(d_0) = argmax_\phi\{W(d_0,\phi)\}$ which maximizes welfare for this fixed network with d_0. But then, the network reacts to this policy change and the degrees become $d^*(\phi^*(d_0))$. The realized welfare is $W(d^*(\phi^*(d_0)), \phi^*(d_0))$. The wedge

$$W(d_0, \phi_0) - W(d^*(\phi^*(d_0)), \phi^*(d_0))$$

is the immediate cost of the network-blind policy. On the other, the opportunity cost is

$$W(d^{**}, \phi^{**}) - W(d^*(\phi^*(d_0)), \phi^*(d_0)).$$

Figure 8 illustrates the main point. Following up on the previously given example and plots, a network-blind regulator who starts at a high status quo ϕ_0 takes the empty network $d^* = 0$ as given. Then he imagines that welfare in reaction to regulation follows the black solid line. A network-blind regulator who starts at a low status quo ϕ_0 on the other hand takes the complete network $d^* = k - 1$ as given. He imagines that welfare in reaction to regulation follows the brown dashed line. A network-conscious regulator follows the blue thick dotted line.

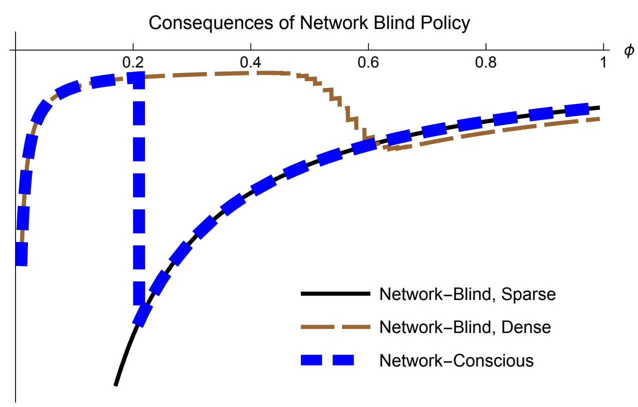


Figure 8: Network-conscious welfare, and network-blind welfare for given networks

Starting from a low ϕ_0, below the transition point 0.2 in the plot, the network is dense, and network-blind welfare would be maximized at around $\phi = 0.5$. But that falls beyond the transition region for ϕ, and the network reacts by becoming sparse. The realized welfare much lower than both the intended level and the starting status-quo level. Starting from a high ϕ_0, above 0.2, network is sparse and the network-blind welfare is maximized at $\phi = 1$. In this case the welfare is improved since ϕ starts and stays beyond the transition region so that the network does not react to a blind policy. The opportunity cost is still positive though since the optimal for a network-conscious policy is the transition point 0.2. These welfare consequences of network-blind policy for any status quo ϕ_0 are plotted in Figure 9.

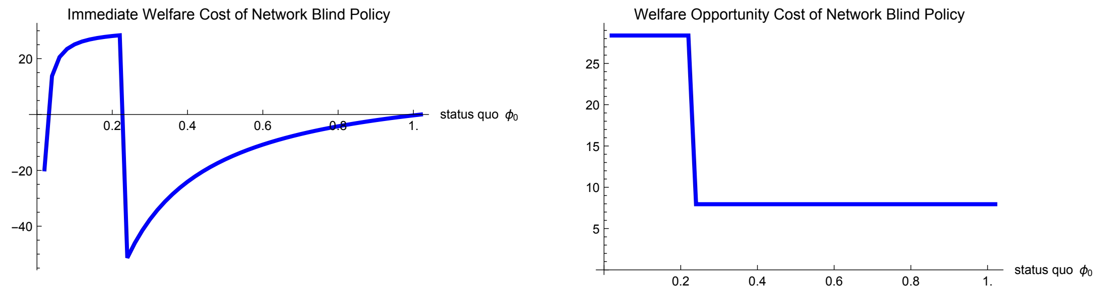


Figure 9: Welfare consequences of network-blind regulation for any status quo level ϕ_0

6 Conclusions

The discussion about the optimal level of liquidity requirements, and more generally about optimal banking regulations, has generated a fruitful recent debate among academics and policymakers alike, particularly in light of many creative proposals by Basel III and the Dodd-Frank Act. Given the prominent role that a dense interbank network and complex counterparty relationships have played during the recent crisis, it is surprising the scarce discussion about how the proposed regulations would affect the density and topology of interbank lending relations and the banking network more generally.

We show that tightening liquidity requirements above a critical threshold induces a sudden collapse in interbank relationships, discontinuously decreasing insurance across banks to face liquidity and refinancing shocks. This is an endogenous network reaction that should be taken into account by regulators when proposing further tightening of liquidity requirements. In other words, a network-blind tightening in regulation may induce a discontinuous increase in systemic risk as banks know they cannot ride refinancing shocks successfully, choosing to liquidity assets excessively.

The main goal of this paper is to highlight the potentially large unforeseen effects of liquidity requirements on network formation and systemic risks. Even though we have focused on liquidity requirements, as banks form networks to face liquidity needs to refinance and continue operations, similar results can be obtained when considering capital requirements and other forms of leverage constraints, as long as those restriction reduce investments to a level that discourages the formation of networks and gain from the cross-insurance against liquidity shocks that it provides.

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A Appendix

Proof of Proposition 1. Here we show how the payoff functions Π can be modified appropriately satisfy the assumptions in Erol (2016). We then sketch the main idea of the proof here.

We first need to establish the equivalence in contagion stage. In Erol (2016) bad banks are also allowed to contine. Define $\Pi^*(a_i, f_i, d_i, \theta_i)$ as

$$\Pi^*(a_i, f_i, d_i, \theta_i) = \begin{cases} \Pi(a_i, d_i - f_i, d_i) & \theta_i = G \\ \Pi(N, d_i - f_i, d_i) & \theta_i = B, a_i = N \\ \Pi(N, d_i - f_i, d_i) - 1 & \theta_i = B, a_i = C \end{cases}$$

For this altered function Π^*, even if bad banks were allowed to play C, they never would since it is strictly dominated to play C if $\theta_i = B$. Hence the cooperating equilibrium of the game with payoff function Π^* where bad banks are also allowed to play C correspond to the cooperating equilibrium of the game with payoff function Π where bad banks are forced to play N.

Now we check whether assumotions in Erol (2016) are satisfied by Π^*.

- Counterparty failures hurt: $\Pi^*(C, f_i, d_i, G)$ imust be strictfly decreasing in f_i. Notice that $P > D + W + kW$ and $D + W > 0$ makes sure that $F(D + W + g_i W)$ stays in the interior of the support of F. Hence $\Pi(C, g_i, d_i)$ is strictly increasing in g_i, making $\Pi^*(C, f_i, d_i, G)$ strictly decreasing in f_i.

- Playing N must give a payoff independent of f_i. This is clearly satisfied since $\Pi(N, g_i, d_i) = -\kappa_l d_i$ is independent of g_i.

- It must be a strictily dominant strategy to play N if $\theta_i = B$. This is guaranteed by the construction.

This means we can apply the theorem. The proof goes as follows in steps.

- A bank n_i with degree d_i cannot have strictly higher payoff than $V(d_i)$.

- A clique with d_i+1 banks give each of its member $V(d_i)$ payoff. The reason is as follows. If less than $S(d_i|\phi)$ other banks in the clique get bad shocks, a good bank n_i plays N and does not get hurt by the other good banks that play N due to the banks that get bad shocks. if more than $S(d_i|\phi)$ banks get good shocks, no other good banks in the clique are forced into playing N so that n_i incurs losses only due to bad counterparties, not more. Therefore, in a clique, a bank n_i incurs losses only due to bad shocks of its counterparties, which means it precisely achieves $V(d_i)$ by the definition of V.

- If banks form cliques at an optimal size maximizing the individual banks' payoff in the clique, that means they achieve the highest possible payoff they can achieve in any network. Hence such a network is strongly stable.

- In any other network structure, some deviations can always be constructured to strictly improve the deviating colation's payoff. The main idea is roughly as follows. If a bank has degree other than its degree at the strongly stable network described above, it cannot achieve this maximal payoff. If all banks have this "ideal degree" but are not formed in cliques, that means there are two banks each of which have counterparties that are not counterparties of the other bank. Then one of them can cut its link with its own secondary counterparty and connect to its counterparty's counterparty. This way they both reduce the risk incurring a loss due to second order counterparties.

□