A Model of Liquidity Hoarding and Term Premia in Inter-Bank Markets[1]

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Abstract

The recent financial crisis witnessed extreme levels of term inter-bank lending rates, such as one-month and three-month LIBOR, which have been proposed as the cause for large drops in lending to the real economy. We provide an explanation of such stress in term lending by modeling leveraged banks' precautionary demand for liquidity. When adverse asset shocks materialize, a bank's ability to roll over debt is impaired because of agency problems associated with high leverage. Hence, a bank's propensity to hoard liquidity, or conversely its willingness to provide term lending, is determined by its rollover risk over the term of the loan. In turn, each bank's ability to borrow in inter-bank markets is determined by leverage and risk of other banks. High levels of short-term leverage and risk of assets can lead to inefficiently low volumes and high rates for borrowing by banks with profitable lending opportunities. In extremis, there can be a complete freeze in inter-bank markets. Our model also provides novel testable implications for term inter-bank lending rates and volumes.

[1]The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of New York or the Federal Reserve System. We thank Sha Lu for excellent research assistance, Jamie McAndrews for valuable conversations, and participants at the Workshop on Money Markets and Payments organized by the Federal Reserve Bank of New York (October 2010).

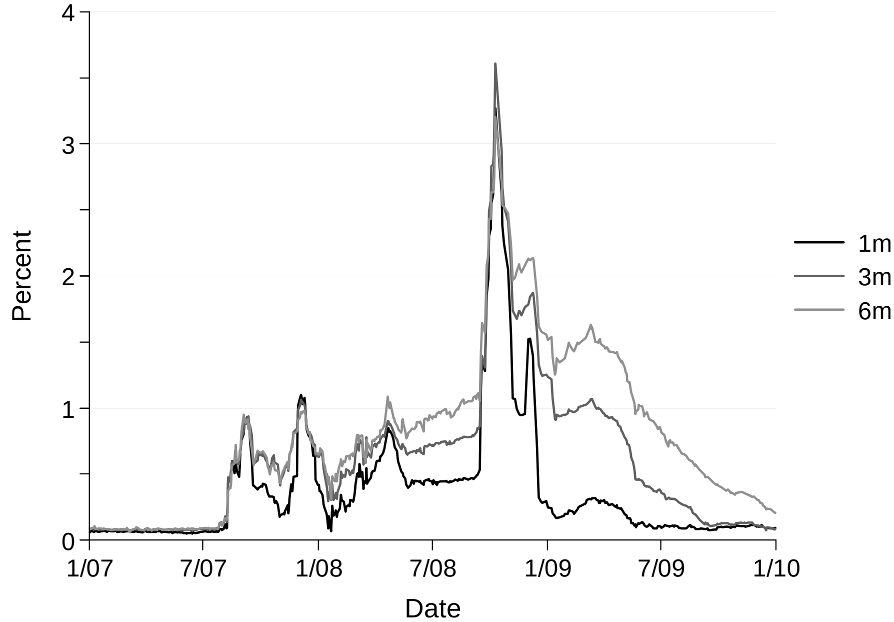
1 Introduction

Extreme levels of inter-bank lending rates, particularly at longer maturities, were seen as a principal problem of the recent financial crisis that caused intense financial distress among banks and resulted in large drops in lending to the real economy. Figure 1 shows that the spreads between London Interbank Offer Rate (LIBOR) and Overnight Indexed Swap (OIS) rate for 1-month, 3-month, and 6-month terms, seen as primary indicators of bank stress and the severity of the crisis, increased to over 300 bps at the peak of the crisis, in comparison to spreads of less than 10 bps before the crisis.[2] Theoretical and empirical studies have not been able to find agreement over to what extent these very large term LIBOR-OIS spreads were explained by increases in credit risk, liquidity risk, or risk premia. Banks with even the best credit quality borrowed at extremely high spreads to the risk-free rate, as shown by Kuo, Skeie and Vickery (2010). Further, Figure 2 shows the weighted-average maturity of inter-bank term lending estimated by Kuo, Skeie and Vickery (2010). Lending maturities fell from a peak average term of over 40 days before the start of the crisis in August 2007 to less than 20 days after the bankruptcy of Lehman Brothers in September 2008.

We provide an explanation of this stress in term inter-bank markets by building a model of banks' precautionary demand for liquidity. Our key insight is that each bank's willingness to provide term lending (for a given counterparty risk of its borrower) is determined by its own *rollover risk*, i.e., the risk that it will be unable to roll over its debt maturing before the term of the loan. If adverse asset shocks materialize in the interim, debt overhang can prevent highly leveraged banks from being able to raise financing required to pay off creditors. Thus, during times of heightened rollover risk ("crisis"), such banks anticipate a high cost of borrowing (or even credit rationing) to meet future liquidity shocks and "hoard" liquidity by lending less and more expensively at longer term maturities in the inter-bank markets. Elevated rates for term borrowing aggravate the debt overhang and rollover risk problems of other banks. Even strong banks are thus forced to cut back on borrowing term in inter-bank markets and bypass profitable investments such

[2] The LIBOR-OIS spread is a measure of the credit and liquidity term spread to the risk-free rate for inter-bank loans. LIBOR is a measure of banks' unsecured term wholesale borrowing rates. OIS is a measure of banks' expected unsecured overnight wholesale borrowing rates for the period of the fixed-for-floating interest rate swap settled at maturity, where the floating rate is the effective (average) fed funds rate for the term of the swap.

Figure 1: LIBOR-OIS Spread



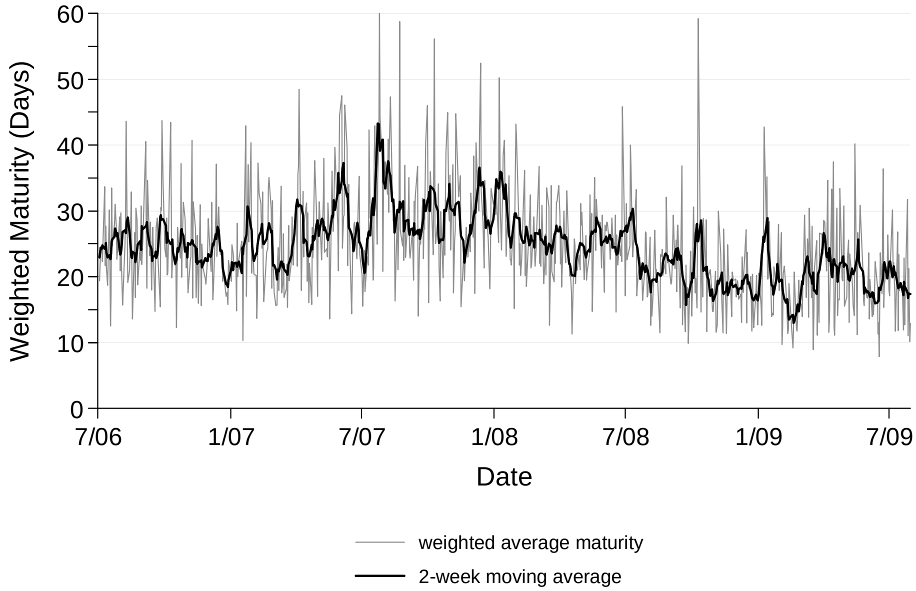
Source: British Banker's Association and Bloomberg

as real-sector lending for long-term and illiquid projects. Importantly, banks with greater short-term leverage and riskier assets are more reluctant to lend beyond the short term, resulting in a high term premium in inter-bank markets and reduced weighted-average maturity of lending terms. The distinction between term and overnight inter-bank rates was one of the crucial problems of the crisis but was largely ignored in the literature and in practice before the crisis.

Our benchmark model builds upon the asset-substitution or risk-shifting model of Stiglitz and Weiss (1981), Diamond (1989, 1991), and more recently, Acharya and Viswanathan (2008). In essence, these papers provide a micro-economic foundation for the funding constraints of a leveraged financial firm: the firm can switch to a riskier, negative net present value investment ("loan") after borrowing from financiers, in anticipation of which the financiers are willing to lend to the firm only up to a threshold level of funding so as to ensure there is enough equity to keep the firm's risk-shifting incentives in check.[3] If there is an adverse asset shock, the funding level can fall low enough that the firm is unable to

[3]The idea that equityholders may prefer negative NPV risky projects to transfer wealth away from creditors was first noted and modeled by Jensen and Meckling (1976).

Figure 2: Weighted Average Maturity of Term Inter-Bank Lending



Weighted average maturity is average maturity of estimated term interbank issuance, weighted by maturity for terms between one week and one year.

Source: Kuo, Skeie, and Vickery (2010)

roll over its existing debt. We use this building block of rollover risk to consider inter-bank transactions between two banks: a bank that has access to profitable investment but not enough arm's length financing to fund it (at least in the short run), and another bank that has surplus funds to potentially lend in the inter-bank market. Absent rollover risk and risk-shifting (or alternative debt overhang) problems, the inter-bank market achieves the efficient redistribution of liquidity that entails the surplus bank lending fully to the profitable one.

We show, however, that the risk-shifting problem and attendant funding constraints can produce a fundamental deviation in the equilibrium outcome from this efficient benchmark. In particular, we consider existing assets in place and leverage for both banks. This creates a debt overhang problem at the two banks in the following manner. For the bank with profitable investment, the willingness to borrow declines as the inter-bank rate rises in order to avoid triggering the risk-shifting problem. For the surplus bank, uncertainty about its asset quality and the ability to roll over debt in the future (when its payments come due or new investments materialize) induces a precautionary demand for retaining its liquidity; hence, it is willing to lend long-term against illiquid assets only if compensated by a suitably higher inter-bank rate. The equilibrium rate is determined by the clearing of this demand and supply of liquidity.

Our main result is that when the banking sector is healthy (more profitable investments, low uncertainty about asset quality and low short-term leverage), long-term inter-bank lending is at efficient levels and there is no term premium attributable to agency problems. In contrast, when the banking sector is weak, for example, in a crisis (fewer profitable investments, high uncertainty about asset quality and high short-term leverage), long-term inter-bank lending is at inefficiently low volumes and at inefficiently high rates. The deviation from the efficient benchmark arises both due to lower willingness of leveraged banks to lend given their precautionary demand for liquidity and of leveraged banks to borrow from such unwilling lenders at exorbitant rates. Indeed, ceteris paribus, the weaker is the health of the lending bank, the more stressed is the term inter-bank market: lower term lending and higher term premium in inter-bank markets, and in turn, lower investment in the economy.

We show that this market failure can lead in the extremis to a complete freeze in the inter-bank market, in which there is no interest rate at which inter-bank lending will occur. For large enough short-term leverage and moral hazard costs for banks, there is a range of interest rates for which banks with excess liquidity lend nothing and banks with excess capacity for investment will not borrow. Thus, we develop a new channel of financial contagion among banks in inter-bank markets with real consequences for credit to the economy. We show that even when the bank with profitable investment opportunities does not have solvency or liquidity risk, it may be unable to access liquidity on the inter-bank market when necessary because of the other banks' rollover risk and hoarding behavior.

The most important empirical implication of our model concerns the determinants of inter-bank lending rates and volumes. A bank's lending rate for a particular maturity in the inter-bank market and to the real sector increases with its own credit risk (e.g., balance-sheet leverage) and liquidity risk (e.g., nature of leverage – wholesale deposits relative to retail deposits), controlling for the credit risk of the counterparties that borrow. More uniquely to our model, a bank's borrowing rate for a particular maturity in the inter-bank market increases with the credit risk and liquidity risk of its lender, controlling for the borrower's own credit risk. In the same vein, bilateral inter-bank borrowing and lending is more likely to freeze when banks are more leveraged, especially short-term, and holding riskier and more complex or illiquid assets. These implications are worthy of detailed empirical investigation.

The remainder of the paper is organized as follows. Section 2 sets up our model and presents the analysis. Section 3 relates the results to existing empirical evidence and derives new implications. Section 4 discusses the related literature. Section 5 concludes with a brief description of future work concerning relative efficacy of different policy interventions in dealing with inter-bank market stress we modeled. Proofs are given in the appendix.

2 Model

There are three periods, dates $t = 0, 1, 2$, and two types of banks $i \in \{B, L\}$. At date 0, each bank has in place investment in one unit of a long-term illiquid asset. Bank $i = L$ also holds one unit of short-term "liquidity." Bank i has short-term debt due to depositors at date 1 with a face amount ρ^i, where $\rho^L \in [1, 2]$ and $\rho^B \in [0, 1]$. The amount ρ^i reflects the bank's effective short-term leverage in place. At the minimum value of ρ^i in its range, bank i has sufficient liquidity to repay all short-term debt at date 1. At the maximum value of ρ^i in its range, the bank i's one unit of the long-term asset is entirely financed by short-term debt. At date 1, bank i attempts to roll over its short-term debt by issuing new debt to depositors with a face amount f^i due at date 2.

At date 2, the long-term asset for each bank has a common payoff y with a random probability θ realized at date 1, where θ has a distribution $G(\theta)$ and density $g(\theta) > 0$ over $[\underline{\theta}, \bar{\theta}]$. The asset has a common payoff of zero with probability $1 - \theta$. At date 1, the bank is subject to moral hazard. The bank can increase risk, while decreasing expected return, by costlessly and unverifiably risk-shifting from the asset-in-place, which we call the "safer" asset, to a different long-term asset, which we call the "riskier" asset. Compared to any realization of θ for the safer asset, the riskier asset has a bank-specific, higher payoff $y^i_R > y$; higher risk $\theta^i_R < \underline{\theta}$, where θ^i_R is known at date 0 and is uncorrelated with θ; and a lower expected return $\theta^i_R y^i_R \leq \underline{\theta} y$. The common payoff of the safer asset reflects that the asset has only systematic risk. The bank-specific payoff of the riskier asset reflects idiosyncratic risk.

Bank $i = B$ is called the "borrowing bank" because it has an opportunity at date 0 for additional investment of one unit into the long-term asset but has no excess liquidity or available borrowing sources from outside depositors (at least not in the very short term). Bank $i = L$ is called the "lending bank" because at date 0 it has an excess unit of liquidity but has no opportunity for additional investment in the long-term asset, which provides a natural opportunity for lending to the borrowing bank in the inter-bank market. At date 0, the lending bank can store its liquidity, which gives a return of one the next period that can be used to pay off part of its short-term debt. Alternatively, the lending bank can make a two-period inter-bank loan to the borrowing bank, which invests any borrowed amount into the long-term asset. The lending bank has a supply for lending $l^*(r) \in [0, 1]$ and the borrowing bank has a demand for borrowing $b^*(r) \in [0, 1]$, where r is the interest rate on the inter-bank loan.

At date 1, bank i defaults if it cannot rollover its short-term debt with new debt that has an expected value of θf^i, and the proceeds the bank's asset-in-place have no utility to either the bank's depositors nor itself. For instance, these assets are rendered worthless by disintermediation of the bank or illiquid for a while due to its bankruptcy. Each bank maximizes its expected profit subject to the incentive constraint on the risk-shifting problem and the resulting rollover constraint, which we turn to next.

2.1 Lending bank

We model the incentive constraint for the banks as simply as possible by focusing the risk-shifting problem on the asset-in-place. For this asset, depositors can verify at date 2 whether the asset return is positive or zero. But depositors cannot verify or enforce whether the bank holds the safer asset rather than the riskier asset, and depositors cannot distinguish between whether a positive asset return is y or y_R^L. For the inter-bank loan, depositors cannot verify anything. In particular, depositors of the lending bank cannot verify whether the inter-bank loan pays off a positive amount at all.[4]

The lending bank chooses its lending supply curve $l^*(r)$ at date 0. The bank can repay part of its debt at date 1 with liquidity held $1 - l$. It needs to roll over the remaining debt

[4]See Acharya and Viswanathan (2009) for a related model showing that debt is the optimal contract with risk-shifting asset substition and coarseness of verifiable information on asset payoffs.

$\rho^L - (1-l)$ by issuing new debt to depositors with a face amount f^L due at date 2. Under no risk shifting by the bank, the bank's expected payoff conditional on rolling over debt is

$$\theta(y + lr - f^L). \tag{1}$$

Under risk shifting, the bank's expected payoff conditional on rolling over debt is

$$\theta_R^L(y_R^L - f^L) + \theta lr. \tag{2}$$

With probability $\theta_R^L(1-\theta)$, the lender receives $y_R^L - f^L$, and with probability $(1-\theta_R^L)(1-\theta)$, the lender receives zero. With probability $\theta_R^L\theta$, the lender receives $y_R^L - f^L + lr \geq 0$, and with probability $(1-\theta_R^L)\theta$, the lender receives lr. The payoff to the lender on the interbank loan lr that occurs with probability θ is unverifiable to depositors and is not used to repay depositors. The lender only repays f^L in the state of the lender's asset payoff y_R^L.

Given that the borrowing bank rolls over its liabilities at date 1 and it does not risk shift, the lending bank's incentive constraint not to risk shift is

$$\theta(y - f^L) + \theta lr \geq \theta_R^L(y_R^L - f^L) + \theta lr, \tag{3}$$

which can be simplified as

$$\theta(y - f^L) \geq \theta_R^L(y_R^L - f^L). \tag{4}$$

We interpret the incentive constraint as that a bank can "pledge" to pay depositors only up to y from the primary asset when it pays off a positive amount, which allows the bank to shift from holding the safer to the riskier asset. But a lending bank cannot pledge any returns from the inter-bank loan. An alternative interpretation of the risk-shifting problem is not that the bank substitutes which asset is held, but rather that the bank may decrease its risk management and monitoring of the asset, which leads to the decrease in probability and increase in return of payoff.

An increase in lending increases the severity of the moral hazard problem for the bank on its asset-in-place by decreasing the liquidity $1-l$ that the bank has available to reduce borrowing from depositors at date 1. The bank's incentive constraint (4) holds if and only if the realization of θ is large enough that

$$f^L \leq \frac{\theta y - \theta_R^L y_R^L}{\theta - \theta_R^L}. \tag{5}$$

For tractability, we assume the risk-shifting payout y_R^i to bank i increases as the probability of success θ_R^i decreases. In the limit, as $\theta_R^i \to 0$, we assume that $y_R^i \to \infty$ and $\theta_R^i y_R^i \to k^i$, where k^i equals the expected return of the risk-shifting assets and represents the severity and cost of the moral hazard problem. We consider the limiting case and write the incentive constraint (4) as

$$f^L \leq \frac{\theta y - k^L}{\theta}. \tag{6}$$

Subject to the incentive constraint holding, the depositors' individual rationality constraint for rolling over the short-term debt amount $\rho^L - (1-l)$ is

$$\rho^L - (1-l) \leq \theta f^L. \tag{7}$$

The rollover constraint depends on both the incentive constraint (4) and individual rationality constraint (7) holding:

$$\rho^L - (1-l) \leq \theta y - k^L. \tag{8}$$

We define $\hat{\theta}^L(l)$ as the bankruptcy cutoff value for the lending bank such that for a large enough realization $\theta \geq \hat{\theta}^L(\cdot)$, the rollover constraint (8) holds. The cutoff $\hat{\theta}^L(l)$ is implicitly defined by the rollover constraint (8) binding:

$$\rho^L - (1-l) = \hat{\theta}^L y - k^L. \tag{9}$$

Solving for $\hat{\theta}^L$,

$$\hat{\theta}^L(l) = \frac{\rho^L - (1-l) + k^L}{y}. \tag{10}$$

The rollover risk for the bank increases in leverage ρ^L and the severity of moral hazard k^L, which is equivalent to an amount of expected profits at date 2 that cannot be pledged at date 1.[5]

[5] We could instead assume that creditors of a bank that defaults at date 1 could collect the return on

Lemma 1. *The lending bank cannot rollover its debt at date 1 if* $\theta < \hat{\theta}^L(l) = \frac{\rho^L-(1-l)+k^L}{y}$. *The bankruptcy cutoff value for the lending bank* $\hat{\theta}^L$ *is increasing in leverage* ρ^L, *the severity of moral hazard* k^L, *and the inter-bank lending amount* l, *and is decreasing in the payoff of the asset* y *and liquidity held* $(1-l)$.

The bank chooses l for a given r to maximize utility u^L, which equals expected profits:[6]

$$u^L \equiv \int_{\hat{\theta}^L(l)}^{\bar{\theta}} [\theta(y+lr) - (\rho^L - (1-l))]g(\theta)d\theta. \tag{11}$$

The first order condition is

$$\frac{\partial u^L}{\partial l} = \int_{\hat{\theta}^L(l)}^{\bar{\theta}} (\theta r - 1)g(\theta)d\theta - (k^L + \hat{\theta}^L lr)g(\hat{\theta}^L)\frac{1}{y} \leq 0 \qquad (= \text{ if } l > 0). \tag{12}$$

The second term of the first order condition is always negative, reflecting the increase in the bank's probability of default at date 1 with an increase in lending. The first term of the first order condition is the expected rate of return to the bank on lending, which equals the expected rate of return on the loan conditional on the bank meeting its liquidity rollover needs at date 1. This term is positive iff $E[\theta r|\theta > \hat{\theta}^L] \geq 1$. If $E[\theta r|\theta > \hat{\theta}^L] < 1$, such that the expected rate of return paid to the bank under survival is greater from holding liquidity than from lending, then both terms are negative. The bank prefers to hold liquidity to lending it and $l^*(r) = 0$. If $E[\theta r|\theta > \hat{\theta}^L]$ is sufficiently greater than one such that $\frac{\partial u^L(r)}{\partial l} > 0$, then the expected rate of return to the bank on lending outweighs the increased probability of bankruptcy risk and the bank lends fully $l^*(r) = 1$.

An interior solution $l^*(r) \in (0,1)$ requires from the first order condition that

$$\int_{\hat{\theta}^L(l)}^{\bar{\theta}} (\theta r - 1)g(\theta)d\theta = (k^L + \hat{\theta}^L lr)g(\hat{\theta}^L)\frac{1}{y}. \tag{13}$$

For a marginal increase in l, the LHS of this condition gives the marginal increased lending benefit and the RHS gives the marginal increased lending cost. The LHS benefit is the

assets and the inter-bank loan at date 2. Under this assumption, the creditors might prefer $\theta < \hat{\theta}^L$ for θ not too low. Under bank default, the creditors' claims might be worth more than $\rho^L - (1-l)$ since the bank is not in control to risk-shift, but this has no qualitative consequence on the results.

[6] We can confine our analysis to considering $\hat{\theta}^i \leq \bar{\theta}$. The lending bank would not choose $l(r) > 0$ such that $\hat{\theta}^B(l > 0) > \bar{\theta}$, and in the section below, the borrowing bank would not choose $b(r) > 0$ such that $\hat{\theta}^B(b > 0) > \bar{\theta}$. For the case of $\hat{\theta}^i < \underline{\theta}$, we define $g(\theta < \underline{\theta}) \equiv 0$.

expected rate of return on the marginal increase in l. The RHS cost is the increase in bankruptcy risk, $g(\hat{\theta}^L)\frac{1}{y}$, applied to the borrowing bank's moral hazard cost on assets in place, k^L, and to the expected gross lending return, $\hat{\theta}^L lr$.

Assumption 1. *We assume that the second order condition holds for an interior lending solution. We show in the appendix that a uniform distribution $g(\cdot)$ and sizable enough frictions for leverage ρ^L and moral hazard k^L are sufficient.*

Lemma 2. *The lending bank's supply of lending $l^*(r) \in [0,1]$ is increasing in r and is decreasing in leverage ρ^L and the severity of moral hazard k^L.*

2.2 Borrowing bank

The borrowing bank chooses its demand curve $b^*(r)$ at date 0 to increase investment in the asset in place. At date 1, the bank needs to roll over short-term debt ρ^B by issuing new debt to depositors with a face amount f^B due at date 2. Depositors of the borrowing bank can verify whether the $1+b$ quantity invested in the asset pays off a positive amount but not whether the bank risk-shifts. The bank's incentive constraint not to risk shift is

$$\theta[(1+b)y - br - f^B] \geq \theta_R^B[(1+b)y_R^B - br - f^B]. \tag{14}$$

Greater amounts of inter-bank borrowing and additional investment into the asset increase the borrowing bank's moral hazard problem and tighten the incentive constraint.

Similar to the lending bank, the borrowing bank's incentive constraint (14) holds if and only if the realization of θ is large enough that

$$f^B \leq \frac{(1+b)(\theta y - \theta_R^B y_R^B)}{\theta - \theta_R^B} - br. \tag{15}$$

We consider the limit as $\theta_R^B \to 0$ and $\theta_R^B y_R^B \to k^B$, and write the incentive constraint (4) as

$$f^B \leq \frac{(1+b)(\theta y - k^B)}{\theta} - br. \tag{16}$$

Subject to the incentive constraint holding, the depositors' individual rationality constraint for rolling over the short-term debt amount ρ^B is

$$\rho^L \leq \theta f^B. \tag{17}$$

The rollover constraint depends on both the incentive constraint (4) and individual rationality constraint (7) holding:

$$\rho^B \leq \theta[(1+b)y - br] - (1+b)k^B. \tag{18}$$

We define $\hat{\theta}^B(b)$ as the bankruptcy cutoff value for the lending bank such that for a large enough realization $\theta \geq \hat{\theta}^B(\cdot)$, the rollover constraint (8) holds. The cutoff $\hat{\theta}^B(b)$ is implicitly defined by the rollover constraint (8) binding:

$$\rho^B = \hat{\theta}^B[(1+b)y - br] - (1+b)k^B. \tag{19}$$

Solving for $\hat{\theta}^B$,

$$\hat{\theta}^B(b) = \frac{\rho^B + k^B(1+b)}{(1+b)y - br}. \tag{20}$$

The rollover risk for the borrowing bank increases in leverage ρ^B and the severity of moral hazard k^B, which is equivalent to an amount of expected profits at date 2 that cannot be pledged at date 1.

Assumption 2. *We assume large enough moral hazard k^B and not too large leverage ρ^B such that the bankruptcy cutoff is increasing in borrowing, $\frac{\partial \hat{\theta}^B}{\partial b} \geq 0$, as shown in the appendix.*

Lemma 3. *The borrowing bank cannot rollover its debt at date 1 if $\theta < \hat{\theta}^B(b) = \frac{\rho^B + k^B(1+b)}{(1+b)y - br}$. The cutoff value $\hat{\theta}^B$ is increasing in the severity of moral hazard k^B, the inter-bank borrowing amount b, and the interest rate r, and is decreasing in leverage ρ^B and the payoff of the asset y.*

The borrowing bank chooses b for a given r to maximize utility u^B, which equals expected profits:

$$u^B \equiv \int_{\hat{\theta}^B(b)}^{\bar{\theta}} \{\theta[y + b(y-r)] - \rho^B\} g(\theta) d\theta. \tag{21}$$

The first order condition is

$$\frac{\partial u^B}{\partial b} = \int_{\hat{\theta}^B(b)}^{\bar{\theta}} \theta(y-r)g(\theta)d\theta - [(k^B+\rho^B)r-\rho^B y]\frac{k^B(1+b)g(\hat{\theta}^B)}{[(1+b)y-br]^2} \leq 0 \qquad (= \text{if } b>0). \quad (22)$$

Remark 1. *The borrowing bank does not borrow at an interest rate greater than the return on the asset:* $b^*(r > y) = 0$.

An interior solution $b^*(r) \in (0,1)$ requires from the first order condition that

$$\int_{\hat{\theta}^B(b)}^{\bar{\theta}} \theta(y-r)g(\theta)d\theta = [(k^B+\rho^B)r-\rho^B y]\frac{k^B(1+b)g(\hat{\theta}^B)}{[(1+b)y-br]^2}. \quad (23)$$

For a marginal increase in b, the LHS of this condition gives the marginal increased borrowing benefit and the RHS gives the marginal increased borrowing cost. The LHS benefit is the expected rate of return on the marginal increase in b, which equals the expected rate of return on investing the borrowing in the asset minus the rate of return on the borrowing, conditional on the borrowing bank meeting its liquidity rollover needs at date 1. This term is positive iff $y > r$. The RHS cost is the increase in bankruptcy risk, $g(\hat{\theta}^B)\frac{k^B}{y}$.

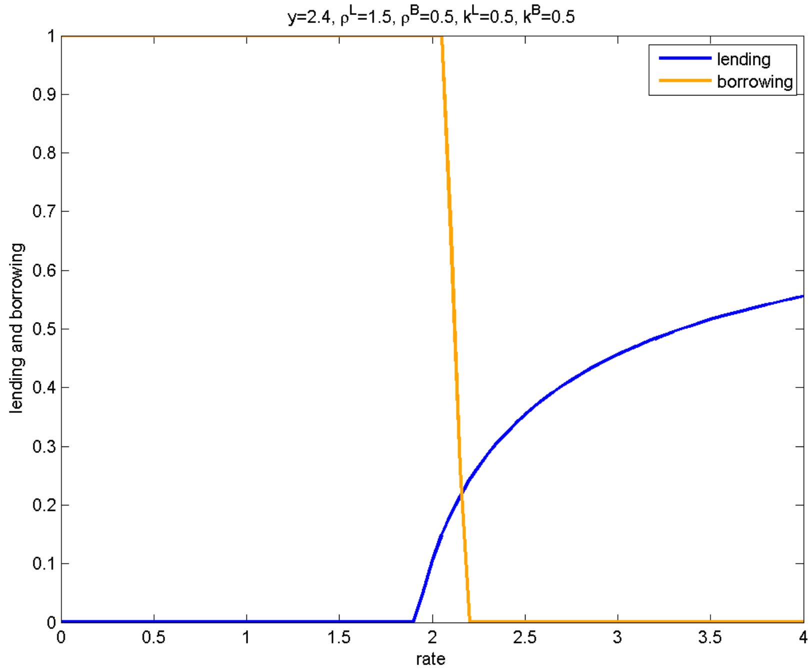
Lemma 4. *The borrowing bank's demand for borrowing* $b^*(r) \in (0,1)$ *is decreasing in the inter-bank rate* r*, leverage* ρ^B*, and the severity of moral hazard* k^B*.*

2.3 Equilibrium

An equilibrium is a lending quantity and interest rate pair (l^*, r^*) such that l^* satisfies the optimization problem for the lending bank given r^* and $b^* = l^*$ satisfies the optimization problem for the borrowing bank given r^*.

Example: Interior equilibrium. *Figure 3 illustrates a case of the lender supply curve and borrower demand curve that gives an interior equilibrium. The deviation from the full lending amount of one unit arises because the lending bank is unwilling to lend a full unit even at the rate of return on investment of* y*. The lending bank has a precautionary demand for liquidity to reduce its rollover risk tied to leverage and the attendant moral hazard. Additionally, the borrowing bank is unwilling to pay for any amount of borrowing the full rate of return* y *that it can receive by funding new investment with the inter-bank*

Figure 3: Inter-Bank Market Interior Equilibrium



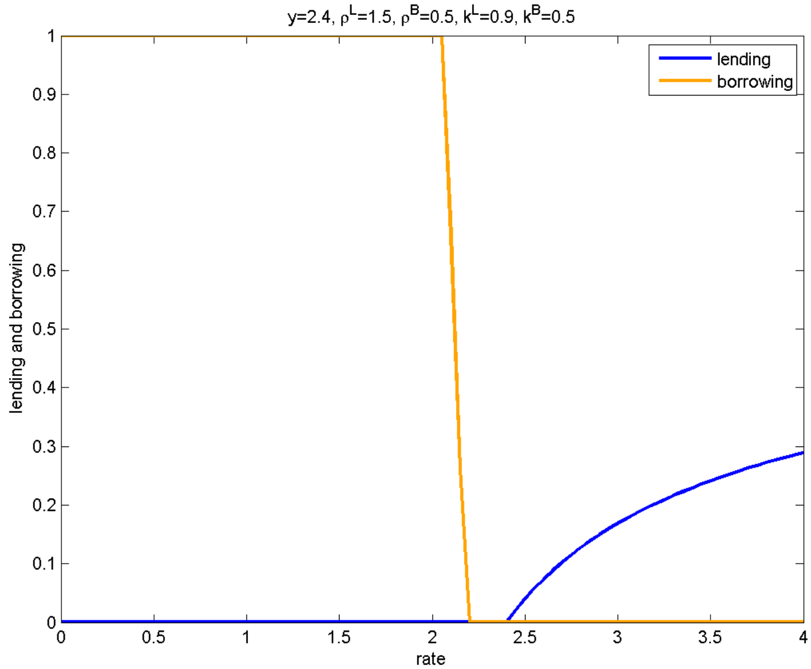
loan. This is because of the borrowing bank's own rollover risk induced by its leverage and the attendant moral hazard.

This decrease in inter-bank lending can lead ultimately to a complete freeze in the inter-bank market, in which there is no interest rate at which inter-bank lending will occur. For large enough leverage and moral hazard costs for banks, there is a range of interest rates at which banks with excess liquidity lend nothing and banks with excess capacity for investment will not borrow. For the condition

$$\min\, r(b=0) \leq \max r(l=0), \tag{24}$$

there exist equilibrium interest rates $r^* \in \{r : \min\, r(b=0) \leq \max r(l=0)\}$ for which the market freezes and equilibrium lending is zero ($l^* = 0$). This result highlights that when no lending occurs in the market, market clearing interest rates may not be determined. The next proposition states that if condition (24) holds with equality, a market freeze is the unique market equilibrium, for which the equilibrium rate is uniquely determined but equilibrium lending is zero.

Figure 4: Lending Freeze



Proposition 1. Freeze in the inter-bank market. *There is an inter-bank market freeze with no lending $l^* = 0$ for parameters such that condition (24) is satisfied.*

There is a particularly extreme version of an inter-bank market freeze that we call a lending freeze, in which the lender does supply any amount of the inter-bank loan at an interest below the return on the investment assets.

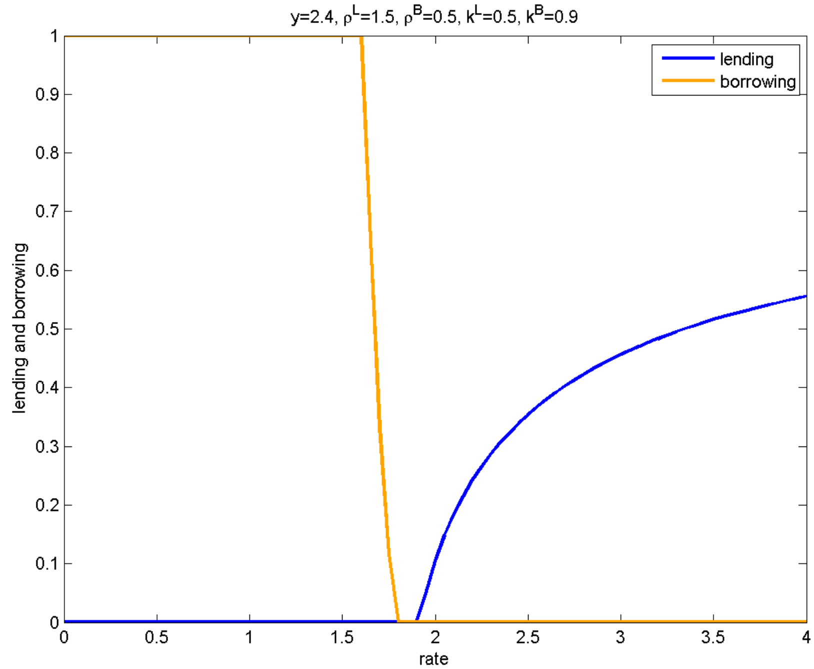
Example: Inter-bank lending freeze. *For illustration, Figure 4 illustrates an inter-bank lending freeze. The lending bank does not supply lending at any interest below the return on assets: $l^*(r \leq y) = 0$.*

In particular, consider the case of no leverage or moral hazard frictions for the borrowing bank, $\rho^B = k^B = 0$. The borrowing bank does not borrow at $r > y$. For a uniform distribution of $g(\theta)$ on the interval $[\underline{\theta}, \bar{\theta}]$, there is an explicit solution

$$r(l) = \frac{2[y(\bar{\theta} - \hat{\theta}^L) + k^L]}{y[\bar{\theta}^2 - (\hat{\theta}^L)^2] - 2\hat{\theta}^L l}. \tag{25}$$

For parameters such that $r(0) > y$, there is a lender freeze. Substituting into equation

Figure 5: Market Equilibrium Freeze



(25) and simplifying, $r(0) > y$ iff

$$(\bar{\theta}y - 1)^2 < (\rho^L + k^L - 2)^2 + k^L, \tag{26}$$

which is the condition for a lending freeze. But a freeze can arise for a greater parameter range as $b(r)$ can be zero for $r < y$ as well, as illustrated in Figure.*5.*

In this case, an inter-bank market freeze occurs because at rates that clear the market with zero lending, the lending bank prefers to hoard liquidity for precautionary reasons rather than lend. This is even though the rate is greater than its return on storage of one. The borrowing bank is unwilling to borrow at the rate, even though the rate is less than the rate that additional investment pays. This is because the borrowing bank is concerned about its own rollover risk at high enough interest rates.

Our primary result is that the inter-bank lending quantity is decreasing as the lending bank or the borrowing bank is more leveraged. As the moral hazard problem becomes more severe or as leverage increases for the lending bank, its *supply* of term lending decreases, driving the equilibrium inter-bank loan amount down. As the moral hazard problem becomes more severe or leverage increases for the borrowing bank, its *demand* for term borrowing decreases, driving the equilibrium inter-bank loan quantity down as well. As a result, the equilibrium inter-bank lending quantity falls (*ceteris paribus*) as the moral hazard becomes more severe and as leverage increases for either bank. Neither bank risk-shifts in equilibrium, but the possibility for risk-shifting moral hazard in the future leads the lending bank to hoard liquidity in advance.

Proposition 2. Stress in inter-bank lending. *The equilibrium inter-bank lending quantity* l^* *is decreasing in the lending and borrowing banks' leverage* ρ^i *and moral hazard* k^i: $\frac{dl^*}{d\rho^i} \leq 0$ *and* $\frac{dl^*}{dk^i} \leq 0$ *for* $i = B, L$.

It is also the case that as moral hazard or leverage increases for the lending bank, the equilibrium inter-bank rate increases. As the moral hazard problem becomes more severe or as leverage increases for the lending bank and its supply of term lending decreases, the inter-bank rate is driven up. As the moral hazard problem becomes more severe or leverage increases for the borrowing bank, its demand for term borrowing decreases, and drives the inter-bank rate down.

Proposition 3. Stress in inter-bank rates. *The equilibrium inter-bank rate* r^* *is increasing in the lending bank's leverage* ρ^L *and moral hazard* k^L, $\frac{dr^*}{d\rho^L} \geq 0$ *and* $\frac{dr^*}{dk^L} \geq 0$, *and decreasing in the the borrowing bank's leverage* ρ^B *and moral hazard* k^B, $\frac{dr^*}{d\rho^B} \leq 0$ *and* $\frac{dr^*}{dk^B} \leq 0$.

Example: Inter-bank market stress. *The decrease in equilibrium lending* l^* *with the increase in bank leverage* ρ^L, ρ^B *and in bank moral hazard* k^L, k^B *is illustrated in Figures 6 and 7, respectively. The increase in the equilibrium inter-bank rate* r^* *with the increase in lending bank leverage* ρ^L *is demonstrated in Figure 8.*

In Figure *6,* equilibrium lending l^*decreases gradually in the borrowing bank's leverage ρ^L, decreases sharply in the lending bank's leverage ρ^B, and entirely freezes for large enough lending bank leverage ρ^B. Figure *7* shows that equilibrium lending increases in both banks moral hazard k^B and k^L. Figure *8* shows that the equilibrium interbank rate increases sharply in lending bank leverage and increases only very gradually in the borrowing bank leverage.

Figure 6: Equilibrium Lending Decreasing in Bank Leverage

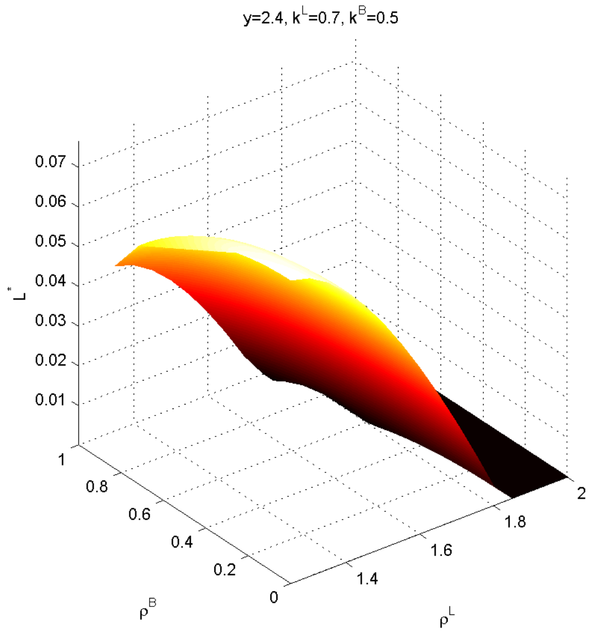


Figure 7: Equilibrium Lending Decreasing in Bank Moral Hazard

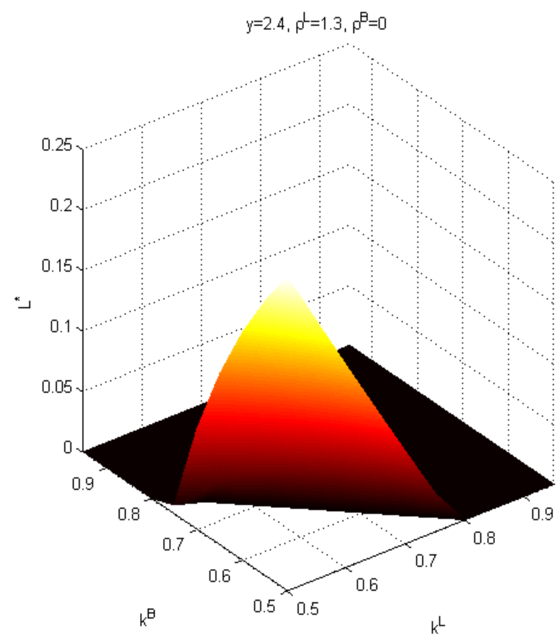
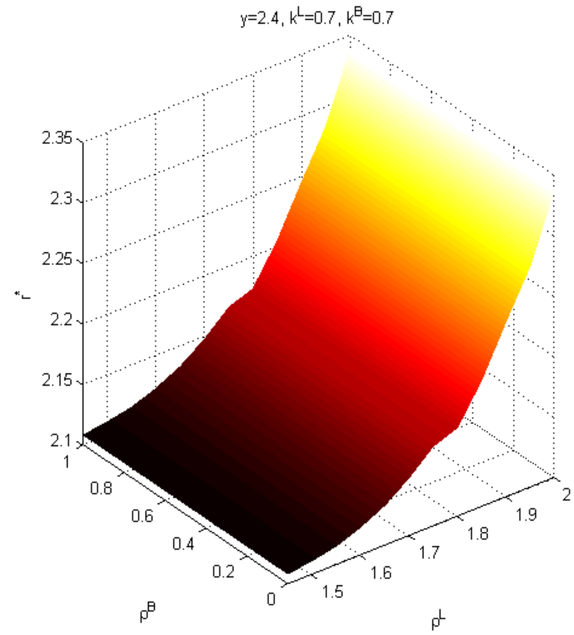


Figure 8: Equilibrium Rate Increasing in Lending Bank's Leverage



2.4 Discussion of results

We examine two benchmarks by considering a planner who can choose the lending quantity between banks but is otherwise subject to the rollover risk and risk-shifting moral hazard frictions in the economy. In particular, banks are subject to the same incentive constraints tied to the risk-shifting problem given by (4) and (14) and the resulting rollover constraints (8) and (18). First, we consider the lending chosen by a planner who maximizes total output. Second, we consider the lending chosen by a planner who maximizes the joint equity of the lending and borrowing banks.

To consider a planner's choice of total output, we write

$$u^L \equiv V^L - D^L + R^L, \tag{27}$$

where

$$V^L \equiv \int_{\hat{\theta}^L(l)}^{\bar{\theta}} \theta y g(\theta) d\theta, \tag{28}$$

$$D^L \equiv \int_{\hat{\theta}^L(l)}^{\bar{\theta}} [\rho^L - (1-l)] g(\theta) d\theta, \tag{29}$$

$$R^L \equiv \int_{\hat{\theta}^L(l)}^{\bar{\theta}} \theta l r g(\theta) d\theta, \tag{30}$$

and we write

$$u^B \equiv V^B - D^B - R^B, \tag{31}$$

where

$$V^B \equiv \int_{\hat{\theta}^B(b)}^{\bar{\theta}} \theta(1+b) y g(\theta) d\theta, \tag{32}$$

$$D^B \equiv \int_{\hat{\theta}^B(b)}^{\bar{\theta}} \rho^B g(\theta) d\theta, \tag{33}$$

$$R_2^B \equiv \int_{\hat{\theta}^B(b)}^{\bar{\theta}} \theta b r g(\theta) d\theta. \tag{34}$$

We will show that output, measured as $V^L + V^B$, is maximized for an interest rate of $\widehat{r} = 0$ and $\widehat{l}$ such that

$$\frac{dV^L}{dl} + \frac{dV^B}{db} = 0 \text{ for } l = b = \widehat{l}. \tag{35}$$

We can see that $\frac{dV^L}{dr} = 0$, since the lender's default cutoff $\hat{\theta}^L$ is independent of r, whereas $\frac{dV^B}{dr} < 0$ as the borrower's default cutoff $\hat{\theta}^B$ is increasing in r. Thus,

$$\frac{dV^L}{dr} + \frac{dV^B}{dr} < 0 \tag{36}$$

for all r, and the planner's choice is

$$\widehat{r} = 0. \tag{37}$$

The first order condition with respect to l is given by (35). To summarize, maximizing total output entails making liquidity transfers at the lowest possible rate of transfer (in other words, only making lump-sum transfers between borrower and lender, if necessary). The choice of lending quantity $\widehat{l}$ reflects the tradeoff that liquidity transferred by the lender increases its rollover risk $\hat{\theta}^L$, lowering its expected output, whereas that transferred to the borrower reduces its rollover risk $\hat{\theta}^B$ (given that rate of transfer is zero). In relation to this benchmark, a market equilibrium inter-bank rate of $r^* > 0$ does not maximize overall output of the banking sector. A lower rate $\widehat{r} = 0 < r^*$ reduces the probability of default of the borrowing bank. We also consider whether the market equilibrium level of inter-bank lending $l^*$is lower than the planner's choice of $\widehat{l}$. If $\widehat{l} > l^*$, then

$$\begin{aligned} \left.\frac{dV^L}{dl} + \frac{dV^B}{db}\right|_{l=b=\widehat{l}} &= 0 \qquad (38) \\ &< \left.\frac{dV^L}{dl} + \frac{dV^B}{db}\right|_{l=b=l^*}. \qquad (39) \end{aligned}$$

Output would be increased by increasing from the market equilibrium lending level l^* up to the planner's choice $\widehat{l}$. We consider this case. Note that at the private optimum (l^*, r^*),

$$\begin{aligned} \frac{dV^L}{dl} - \frac{dD^L}{dl} + \frac{dR^L}{dl} &= 0, \qquad (40) \\ \frac{dV^B}{db} - \frac{dD^B}{db} + \frac{dR^B}{db} &= 0. \qquad (41) \end{aligned}$$

Then, adding these first order conditions from the lender's and borrower's individual market optimization, we have

$$\frac{dV^L}{dl} + \frac{dV^B}{db} = \frac{dD^L}{dl} + \frac{dD^B}{db} + \frac{dR^B}{db} - \frac{dR^L}{dl}. \qquad (42)$$

Therefore the condition $\frac{dV^L}{dl} + \frac{dV^B}{db} > 0$ at $l = b = l^*$ implies that

$$\left(\frac{dD^L}{dl} - \frac{dR^L}{dl}\right) + \left(\frac{dD^B}{db} + \frac{dR^B}{db}\right) > 0 \text{ at } l = b = l^*. \qquad (43)$$

This condition states that the the net cost to the lending bank from an increase in lending exceeds the net cost to the borrowing bank. When this condition holds, there is under-investment in lending in the market equilibrium relative to the transfer amount that maximizes overall output of the economy.

To consider a planner's choice of total bank equity value, we consider a planner's choice of $(\widetilde{l}, \widetilde{r})$ for the problem of

$$\max_{l,r} u^L + u^B. \tag{44}$$

The first order conditions are

$$\frac{du^L}{dl} + \frac{du^B}{dl} = 0, \tag{45}$$
$$\frac{du^L}{dr} + \frac{du^B}{dr} = 0, \tag{46}$$

whereas in the market equilibrium, individual bank optimization gives (l^*, r^*) such that

$$\frac{du^L}{dl} = \frac{du^B}{dl} = 0. \tag{47}$$

For the planner's problem, we might obtain $\frac{du^L}{dl} < 0 < \frac{du^B}{dl}$ for certain parameter ranges when $\hat{\theta}^B < \hat{\theta}^L$.

3 Empirical predictions and relevance of results

Our results on hoarding of liquidity by banks and its effect on inter-bank rates are corroborated by empirical findings in the extant literature. Acharya and Merrouche (2009) show empirically that during the crisis, some settlement banks in the United Kingdom started precautionary hoarding of liquidity, measured as voluntary upward revisions of reserve balance targets with the Bank of England following critical dates of the crisis, such as the asset-backed commercial paper market freeze of August 8, 2007, the collapse of Northern Rock in mid-September 2007 and that of Bear Stearns in mid-March of 2008. In the cross-section of banks, hoarding was greater for banks that had suffered greater equity losses in the crisis, had lower reliance on retail deposits (relative to wholesale financing), in line with Lemma 2, and were most exposed to payment and settlement uncertainty. In time-series, weaker banks hoarded more reserves on days with greater payment and settlement shocks. Finally, they also document that this hoarding caused increases in inter-bank lending rates for all other banks (and in unabridged version of their paper, also led to increased rates and decreased volumes in lending to businesses and consumers), in line with Proposition 3. All of these effects are entirely consistent with predictions of our model.

Ashcraft, McAndrews and Skeie (2010) show similar evidence for the United States that during the crisis, larger precautionary reserve balances were held by weaker banks facing credit and liquidity frictions as a method to insure themselves against intraday liquidity shocks. They show evidence that banks sponsoring ABCP conduits had increased payments shocks, and that greater payments shocks led to an increase in bank's precautionary reserves. In addition, banks appear to have responded to higher uncertainty about payments during the crisis by becoming more reluctant to lend excess reserves when reserves were high. Both of these results are consistent with Lemma 2.

Ashcraft, McAndrews and Skeie (2010) and Afonso, Kovner and Schoar (2010) show evidence that overnight inter-bank lending in the fed funds and Eurodollar market increased through much of the early crisis and held up well after the Lehman bankruptcy. However, this has *not* been the case for maturities longer than overnight, especially one-month onwards, for which inter-bank lending volumes generally fell during the heart of the crisis while term spreads increased. Kuo, Skeie and Vickery (2010) show evidence of the decline in the maturity-structure (as illustrated in Figure 2), in line with Proposition 2. The decrease in term lending provides an explanation for the steady and even increasing levels of overnight inter-bank lending as banks with borrowing constraints and liquidity risk shift from term to overnight lending. Ashcraft, McAndrews and Skeie (2010) show empirical evidence supporting their theory that banks with borrowing constraints will prefer to lend increased amounts of overnight fed funds because it facilitates their precautionary hoarding of reserves intraday and acts as a source of liquid assets with overnight maturity that can be redeployed when liquidity is needed.

The level of term inter-bank lending spreads not only reflect bank funding stress but also were a significant cause of stress in the broader financial system and economy during the crisis. One, three and six month LIBOR rates play a central role in fixed income markets as they are used to index over $360 trillion of notional financial contracts as estimated by the British Bankers' Association (BBA), ranging from interest rate swaps and other derivatives to floating-rate residential and commercial mortgages. Large one-month and three-month LIBOR-OIS spreads during the crisis measure the cost of interbank borrowing for term maturities relative to the expected cost of rolling-over overnight borrowing and have been studied by several authors. McAndrews, Sarkar and Wang (2008), Michaud and Upper (2008) and Schwarz (2009) attribute most of the spread to liquidity risk, which supports Proposition 3. Taylor and Williams (2008a, 2008b) attribute the spread primarily to counterparty credit risk. Smith (2010) argues that time-varying risk premia explains half of the variation in spreads.[7] Our model clarifies that the term inter-bank spread consists of not just counterparty risk of borrowers but also rollover risk of lenders (which varied over the course of the crisis).

We also show that market failure can lead in the extremis to a complete freeze in the inter-bank market, in which there is no interest rate at which inter-bank lending will occur. For large enough short-term leverage and moral hazard costs for banks, there is a range of interest rates for which banks with excess liquidity lend nothing and banks with excess capacity for investment will not borrow, shown in Proposition 1. This result highlights that when no lending occurs in the market, market clearing interest rates may not be determined. High term premia do not indicate to what extent quantities of inter-bank borrowing have decreased. Put another way, lending volume is also an important determinant of market efficiency. Consistent with this observation, Kuo, Skeie and Vickery (2010) document the difficulty of using only spreads to measure the stress in the inter-bank market without also measuring volumes.

The inter-bank channel of financial contagion that we propose has real consequences for credit to the economy. In evidence that the financial crisis affected real-sector lending adversely, Ivashina and Scharfstein (2010) document that quarterly total (syndicated) loan issuance in the United States fell from 250 billion dollars in the two quarters prior to August 2007, steadily downward to just around 50 billion dollars in August to October 2008, for both investment grade and non-investment grade corporations. Initiations of revolving credit facilities followed an identical pattern. Acharya, Almeida and Campello (2009) show that on an average basis across firms in the United States (covered in the Compustat database), initiations of lines of credit fell from over 1 percent of corporate assets in 2007 to just about 0.43 percent in 2008, a drop of about one and a half standard

[7] Angelini, Nobili and Picillo (2009) show that bank characteristics began to affect rates in the Euro term inter-bank market enduring the crisis. Bartolini et. al (2010) and McAndrews (2009) also examine spreads between overnight rates on LIBOR, Eurodollar and fed funds. Kuo, Skeie and Vickery (2010) show that the dispersion of term inter-bank borrowing rates increased greatly during the crisis. Their work highlights the importance of studying the cross-section of liquidity risk and leverage, as well as credit risk, across both lending and borrowing banks in the inter-bank market.

deviations in initiations relative to corporate assets (measured over 1989 to 2008). They also document that average maturity of lines of credit fell from over 13 quarters in 2007 to just over 10 quarters in 2008, a reduction in maturity of about 9 months. While these effects on real-sector bank lending have not yet been explicitly linked to rollover risk and stress in inter-bank markets, our model results of Proposition 2 suggests a strong link that is important to investigate in future empirical work.

In addition, our model offers several new testable implications:

First, Lemma 2 shows that a bank's lending rate for a particular maturity in the inter-bank market and to the real sector increases with its own credit risk (e.g., balance-sheet leverage) and liquidity risk (e.g., nature of leverage – wholesale deposits relative to retail deposits), controlling for the credit risk of the counterparties that borrow. More uniquely to our model, Proposition 3 shows that a bank's borrowing rate for a particular maturity in the inter-bank market increases with the credit risk and liquidity risk of its lender, controlling for the borrower's own credit risk. In the same vein, bilateral inter-bank borrowing and lending is more likely to freeze when banks are more leveraged, especially short-term, and holding riskier and more complex or illiquid assets, following our Proposition 1.

Second, our model suggests that an increase in the risk of asset-level shocks increases term inter-bank rates, and reduces term inter-bank volumes, not just due to an increase in borrower's credit risk but also due to an increase in the rollover risk of the lender were adverse asset shocks to materialize, as in Propositions 2 and 3. That is, there should be an interaction effect between risk (i.e. equity volatility realized or implied, as reflected in the VIX) and both borrower and lender leverage and rollover risk in determining term interbank rates and volumes.[8]

Third, our model suggests that the introduction of the Federal Reserve's Term Auction Facility (TAF) for 28-day and later 84-day loans should have decreased rates and increased volumes of lending to the real sector, not only for banks that used the facilities but also by other banks. In essence, by acting as a relatively risk-free intermediary, the central bank can intermediate liquidity hoardings of riskier banks to safer banks with profitable opportunities.

[8] Furfine (2010) finds that the LIBOR-OIS spread is related to VIX over time, supporting our prediction, and results could be further tested by examing separate borrower and lender effects.

Fourth, our results indicate that surveys such as LIBOR of inter-bank market rates do not necessarily indicate the full breakdown that may occur in the inter-bank market as they do not focus on volumes, which is illustrated in Figure *4*. When there is a complete breakdown for some borrowers and lenders, the inter-bank rate is not even well-defined. Hence, the measurement and reporting of term inter-bank markets volumes are crucial for understanding the stress and collapse in these markets.

4 Related literature

While several papers in the literature develop theories for generating spreads in the inter-bank market, most do not distinguish between term and overnight inter-bank markets as our paper does. Goodfriend and King (1988) argue that interbank lending allows for efficient provision of lending among banks based on inter-bank monitoring such that central banks need only provide aggregate liquidity. Rochet and Tirole (1996) show that peer monitoring can be necessary for decentralized inter-bank lending, and moreover that term rather than overnight inter-bank lending is necessary for peer monitoring to take place. Calomiris and Kahn (1996) give empirical evidence that in the historical Suffolk system, banks' peer monitoring was motivated by holding each other banks' notes and led to peer discipline that took effect before legal discipline. Ashcraft, McAndrews and Skeie (2009) show how increases in interbank lending can be an efficient mechanism to distribute liquidity when some banks cannot be monitored by other banks and have limited participation in the interbank market. Keister and McAndrews (2009), Martin et al. (2008), and Bech and Klee (2009) examine how the interbank market is affected by the implementation of monetary policy and banks' management of reserves.

Interbank markets can also increase lending in response to liquidity shocks to provide the optimal distribution of liquidity under appropriate central bank interest rate policy, as shown by Freixas, Martin and Skeie (2010). Allen, Carletti and Gale (2009) show how a central bank can intervene in the interbank market and use its balance sheet to provide optimal liquidity when idiosyncratic bank liquidity shocks cannot be contracted upon. Bhattacharyya and Gale (1987) argue, however, that banks may free-ride on each other's liquidity in presence of inter-bank markets, and Repullo (2005) stresses free-riding on central bank liquidity.

Unlike these papers, we do not consider peer monitoring issues or efficiency (or inefficiency) consequences of central bank policy, but instead focus on positive implications on the terms (quantity and interest rates) of liquidity transfers in inter-bank markets when there are agency problems related to bank leverage.

Diamond and Rajan (2005) show how liquidity problems leading to insolvency at some banks can cause a decrease in the endogenous amount of aggregate liquid resources available to even fundamentally healthy banks. The contagion in their paper also operates through an increase in inter-bank market rates and results in a decrease lending to the real sector. This is, however, an *ex post* contagion rather than one in anticipation of insolvency or rollover risk (as in our model). Acharya, Shin and Yorulmazer (2008) derive a strategic motive for holding cash. When banks' ability to raise external financing is low, they anticipate fire sales of assets by troubled banks and as a result hoard liquidity and forego profitable but illiquid investments. Diamond and Rajan (2009) also study long-term credit contraction that operates through a channel of asset fire sales. During a crisis, banks delay asset sales as part of their efforts to stay alive (a version of the risk-shifting problem). In turn, high rates are required ex ante on term loans to the real sector.

While these papers focus on aggregate liquidity shortages and strategic demand for liquidity by banks, we derive instead a precautionary demand for liquidity by (weak) banks as contributing to heightened borrowing costs for (even safe) banks. In a contemporary paper, Gale and Yorulmazer (2010) model both the precautionary and the strategic motive for holding cash and show that banks may hoard liquidity and lend less than is constrained efficient, as in our model. In alternative channels for a decrease and possible freeze in inter-bank lending, Flannery (1996), Freixas and Jorge (2007), Freixas and Holthausen (2005) and Heider, Hoerova and Holthausen (2009) consider asymmetric information among banks whereas Donaldson (1992) and Acharya, Gromb and Yorulmazer (2007) consider imperfect competition in inter-bank markets and strategic behavior by relationship-specific lenders.

Finally, we take the presence of short-term debt maturing before bank's term lending as given. Several papers have justified this from first principles. Calomiris and Kahn (1991) and Diamond and Rajan (2001) present models in which short-term debt disciplines bankers and increases ex-ante bank liquidity. Acharya and Viswanathan (2010) explain why giving strong control rights to lenders is efficient in terms of generating ex-ante liquidity, provided that there are no ex-post liquidity costs. Brunnermeier and Oehmke (2009) provide a model in which short-term debt is sometimes inefficient but arises in equilibrium due to a "maturity rat-race" between creditors. Acharya, Schnabl and Suarez (2009) show empirically that the onset of the financial crisis of 2007-09 was due to bank exposures to off-balance sheet vehicles (conduits and SIVs) that were funded with extremely short-dated asset-backed commercial paper (ABCP). Acharya, Gale and Yorulmazer (2008) show how rollover risk can arise upon adverse news even in absence of agency problems. In their model, small liquidation costs can get amplified if debt has to be rolled over frequently relative to the likelihood of arrival of better news. Such rollover risk would also suffice to generate the effects on term-lending we derive in our model. Morris and Shin (2009) consider the credit spread of a borrowing firm and show how short-term debt can amplify credit risk component of the spread due to the likelihood of a "run." Finally, Caballero and Krishnamurthy (2008) derive a propensity of firms to hoard liquid assets and reduce risk-sharing when there is Knightian uncertainty about their risks.

5 Concluding remarks

In this paper, we provide an explanation for stress, and potentially freezes, in term inter-bank lending due to rollover risk of highly leveraged lenders. The term inter-bank lending rate and volume are jointly determined, reflecting the precautionary demand for liquidity of lenders and aversion of borrowers to trade at high rates of interest. The model is consistent with phenomena observed during financial crises and also provides implications for future empirical work, especially through its main results that the borrowing rate for a bank is also tied to the lender's rollover risk and leverage.

In future work, it seems interesting and important to analyze possible interventions that can address the excessive hoarding of liquidity by highly leveraged banks. Is an unconditional (traditional) lender of last resort (LOLR) in which a central bank provides liquidity to strong as well as weak banks desirable? Or would it be better to have a solvency-contingent LOLR in which the central bank provides liquidity only to sufficiently strong banks? And should there instead (or in addition) be a resolution authority that forces weak banks to reduce their rollover risk? We conjecture that a resolution authority to address weak banks' rollover risk and/or a solvency-contingent LOLR are more efficient interventions than the traditional, unconditional LOLR. Such analysis can also help comparisons with the type of interventions that were put in place during the crisis, including the TAF by the Federal Reserve and several policy interventions by the European Central Bank (ECB).

Appendix

Assumption 1. We make two assumptions that ensure that the second order condition is satisfied. First, we assume a uniform distribution for $g(\cdot)$, which is always sufficient to satisfy the condition needed for $g'(\hat{\theta}^L)$ to be not too small. This ensures that the lending bank has a minimal enough increase in its marginal bankruptcy risk for marginal increases in its bankruptcy cutoff value $\hat{\theta}^L$. Second, we assume large enough parameters for k^L and ρ^L relative to y such that

$$l > \frac{y}{2r} - \frac{1}{2}(\rho^L + k^L - 1). \tag{48}$$

■

Proof of Lemma 2. To study the second order condition,

$$\begin{aligned}
\frac{\partial^2 u^L}{\partial l^2} &= -\frac{1}{y}(2\hat{\theta}^L r + \frac{lr}{y} - 1)g(\hat{\theta}^L) - \frac{1}{y^2}(\hat{\theta}^L lr + k^L)g'(\hat{\theta}^L) &(49)\\
&= -\frac{g(\hat{\theta}^L)}{y}\left[2\hat{\theta}^L r + \frac{lr}{y} - 1 + \frac{1}{y}(\hat{\theta}^L lr + k^L)\frac{g'(\hat{\theta}^L)}{g(\hat{\theta}^L)}\right]. &(50)
\end{aligned}$$

For $g'(\hat{\theta}^L) \geq 0$, which is satisfied by a uniform distribution for $g(\cdot)$, condition (48) is sufficient for $\frac{\partial^2 u^L}{\partial l^2} < 0$. For $l \in [0, 1]$, we can see that lending is increasing in r, since

$$\begin{aligned}
\frac{\partial^2 u^L}{\partial l \partial r} &= \int_{\hat{\theta}^L(l)}^{\bar{\theta}} \theta g(\theta) d\theta - \hat{\theta}^L \frac{l}{y} g(\hat{\theta}^L) &(51)\\
&\geq \hat{\theta}^L g(\hat{\theta}^L)(1 - \frac{l}{y}) &(52)\\
&\geq 0, &(53)
\end{aligned}$$

where the last inequality holds since $l \leq 1 < y$. Lending is decreasing in ρ^L, since

$$\frac{\partial^2 u^L}{\partial l \partial \rho} = -(\hat{\theta}^L r - 1)g(\hat{\theta}^L)\frac{1}{y} - \frac{lr}{y}g(\hat{\theta}^L)\frac{1}{y} \leq 0, \tag{54}$$

which is satisfied by condition (48). Lending is also decreasing in k^L, since

$$\frac{\partial^2 u^L}{\partial l \partial k^L} = -(\hat{\theta}^L r - 1)g(\hat{\theta}^L)\frac{1}{y} - (1 + \frac{lr}{y})g(\hat{\theta}^L)\frac{1}{y} \leq 0, \tag{55}$$

which is always satisfied for $l \leq 1$ and $r \leq y$; Remark 1 shows that borrowing bank demand is never positive for $r > y$, which can be excluded. ■

Assumption 2. Writing

$$\frac{\partial \hat{\theta}^B}{\partial b} = \frac{(k^B + \rho^B)r - \rho^B y}{[(1+b)y - br]^2}, \tag{56}$$

$\frac{\partial \hat{\theta}^B}{\partial b} > 0$ iff k^B is large enough and ρ^B is not too large relative to y such that

$$r > \frac{\rho^B}{\rho^B + k^B} y. \tag{57}$$

■

Proof of Remark 1. We will show that $b^*(r > y) = 0$. To prove by contradiction, suppose instead that $b(r > y) > 0$. Positive borrowing $b^*(r) > 0$ requires that $\frac{\partial u^B}{\partial b} \geq 0$. However, with $r > y$, both terms in the RHS of equation (22) are negative, which implies $\frac{\partial u^B}{\partial b} < 0$, a contradiction. Thus, $b^*(r > y) = 0$. ■

Proof of Lemma 4. To study the second order condition, for $g'(\cdot) = 0$,

$$\frac{\partial^2 u^B}{\partial b^2} = \frac{-g(\hat{\theta}^B)[(k^B r)^2 - (\rho^B)^2 (y-r)^2}{[(1+b)y - br]^3}, \tag{58}$$

For $r \leq y$, $\frac{\partial^2 u^B}{\partial b^2} < 0$ iff $r > \frac{\rho^B}{\rho^B + k^B} y$, which holds by Assumption 2. Continuing assuming $g'(\cdot) = 0$ and $r > \frac{\rho^B}{\rho^B + k^B} y$, we can see that borrowing is decreasing in r, since

$$\frac{\partial^2 u^B}{\partial b \partial r} = - \int_{\hat{\theta}^B(b)}^{\bar{\theta}} \theta g(\theta) d\theta - \hat{\theta}^B g(\hat{\theta}^B)(y - r) \frac{\partial \hat{\theta}^B}{\partial r} - \frac{k^B(1+b) g(\hat{\theta}^B)(k^B + \rho^B)}{[(1+b)y - br]^2} \tag{59}$$

$$- \frac{2k^B(1+b) g(\hat{\theta}^B)(k^B + \rho^B)r - \rho^B y)b}{[(1+b)y - br]^3} \leq 0. \tag{60}$$

Borrowing is decreasing in ρ^B since

$$\frac{\partial^2 u^B}{\partial b \partial \rho^B} = -\frac{\rho^B (y - r) g(\hat{\theta}^B)}{[(1+b)y - br]^2} \leq 0. \tag{61}$$

Borrowing is decreasing in k^B since

$$\frac{\partial^2 u^B}{\partial b \partial k^B} = -\frac{(1+b)[k^B(1+b)y + k^B(1-b)r](\hat{\theta}^B)}{[(1+b)y - br]^2} \leq 0. \tag{62}$$

Also, note that when Assumption 2 does not hold, $r < \frac{\rho^B}{\rho^B+k^B}y$, then $\frac{\partial u^B}{\partial db} > 0$ and $b^*(r) = 1$. ■

Proof of Proposition 2 and 3. In equilibrium, $l^*(r^*, x) = b^*(r^*, x)$ and hence $\frac{dl^*}{dx} = \frac{db^*}{dx}$ for $x \in \{k^B, k^L, \rho^B, \rho^L\}$. Thus,

$$\frac{\partial l^*}{\partial x} + \frac{\partial l^*}{\partial r^*}\frac{dr^*}{dx} = \frac{\partial b^*}{\partial x} + \frac{\partial b^*}{\partial r^*}\frac{dr^*}{dx}, \tag{63}$$

so we have

$$\frac{dr^*}{dx} = -\frac{[\frac{\partial b^*}{\partial x} - \frac{\partial l^*}{\partial x}]}{[\frac{\partial b^*}{\partial r^*} - \frac{\partial l^*}{\partial r^*}]}. \tag{64}$$

Now, $\frac{\partial b^*}{\partial r^*} \leq 0$ and $\frac{\partial l^*}{\partial r^*} \geq 0$, therefore $\text{sign}(\frac{dr^*}{dx}) = \text{sign}(\frac{\partial b^*}{\partial x} - \frac{\partial l^*}{\partial x})$. For $x = k^B$, $\frac{\partial l^*}{\partial x} = 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{dk^B} \leq 0$. For $x = k^L$, $\frac{\partial l^*}{\partial x} \leq 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{dk^L} \geq 0$. For $x = \rho^B$, $\frac{\partial l^*}{\partial x} = 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{d\rho^B} \leq 0$. For $x = \rho^L$, $\frac{\partial l^*}{\partial x} \leq 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{d\rho^L} \geq 0$.

Consider

$$\frac{dl^*}{dx} = \frac{\partial l^*}{\partial x} + \frac{\partial l^*}{\partial r^*}\frac{dr^*}{dx}. \tag{65}$$

For $x = k^L$, as shown above,

$$\frac{dr^*}{dk^L} = \frac{\frac{\partial l^*}{\partial k^L}}{\frac{\partial b^*}{\partial r^*} - \frac{\partial l^*}{\partial r^*}}, \tag{66}$$

therefore

$$\frac{dl^*}{dk^L} = \frac{\partial l^*}{\partial k^L}\left[1 - \frac{\frac{\partial l^*}{\partial r^*}}{\frac{\partial l^*}{\partial r^*} - \frac{\partial b^*}{\partial r^*}}\right]. \tag{67}$$

Now

$$\frac{\frac{\partial l^*}{\partial r^*}}{\frac{\partial l^*}{\partial r^*} - \frac{\partial b^*}{\partial r^*}} \leq 1 \tag{68}$$

as $\frac{\partial b^*}{\partial r^*} \leq 0$; hence, $\frac{dl^*}{dk^L} \leq 0$. Similarly, as $\frac{\partial l^*}{\partial \rho^L} \leq 0$, $\frac{\partial l^*}{\partial r^*} \geq 0$, and $\frac{dr^*}{d\rho^L} \geq 0$, we have $\frac{dl^*}{d\rho^L} \leq 0$. As $\frac{\partial l^*}{\partial k^B} = 0$, $\frac{\partial l^*}{\partial r^*} \geq 0$, and $\frac{dr^*}{dk^B} \leq 0$, we have $\frac{dl^*}{dk^B} \leq 0$. Finally, as $\frac{\partial l^*}{\partial \rho^B} \leq 0$, $\frac{\partial l^*}{\partial r^*} \geq 0$, and $\frac{dr^*}{d\rho^B} \leq 0$, we have $\frac{dl^*}{d\rho^B} \leq 0$. ■

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