The Causal Structure of the Vector Autoregression in Economics:

A Case Study of U.S. M2

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Workshop: Case Studies of Causal Discovery with Model Search

Source

"Empirical Identification of the Vector Autoregression: The Causes and Effects of U.S. M2," in Jennifer L. Castle and Neil N. Shephard, editors, *The Methodology and Practice of Econometrics: A Festschrift in Honour of David F. Hendry*. Oxford: Oxford University Press, 2009, pp. 37-58.

Coauthors:

- Selva Demiralp, *Koç University*, Istanbul
- Stephen J. Perez, *California State University*, *Sacramento*

The Real World Problem

M2 =

liquid deposits		65%
= demand deposits	5%	
+ other checkable deposits	5%	
+ savings deposits	55%	
+ small time deposits		16%
+ retail money funds		11%
+ currency in circulation		11%

M2 and the Fed: Monetary and Reserve Analysis Section of the Division of Monetary Affairs of the Board of Governors of the Federal Reserve System – a minimally theoretical and informal analysis of M2 growth with a money-demand orientation.

Minimal Theory

The Quantity Theory of Money: the growth rate of money should equal the growth rate of nominal income, adjusting for the trend in velocity.

Special Factors:

i) Interest rate effects

ii) Equity market effects:

iii) Other special factors including:
activity in mortgage-backed securities;
tax effects; and
currency shipments abroad.

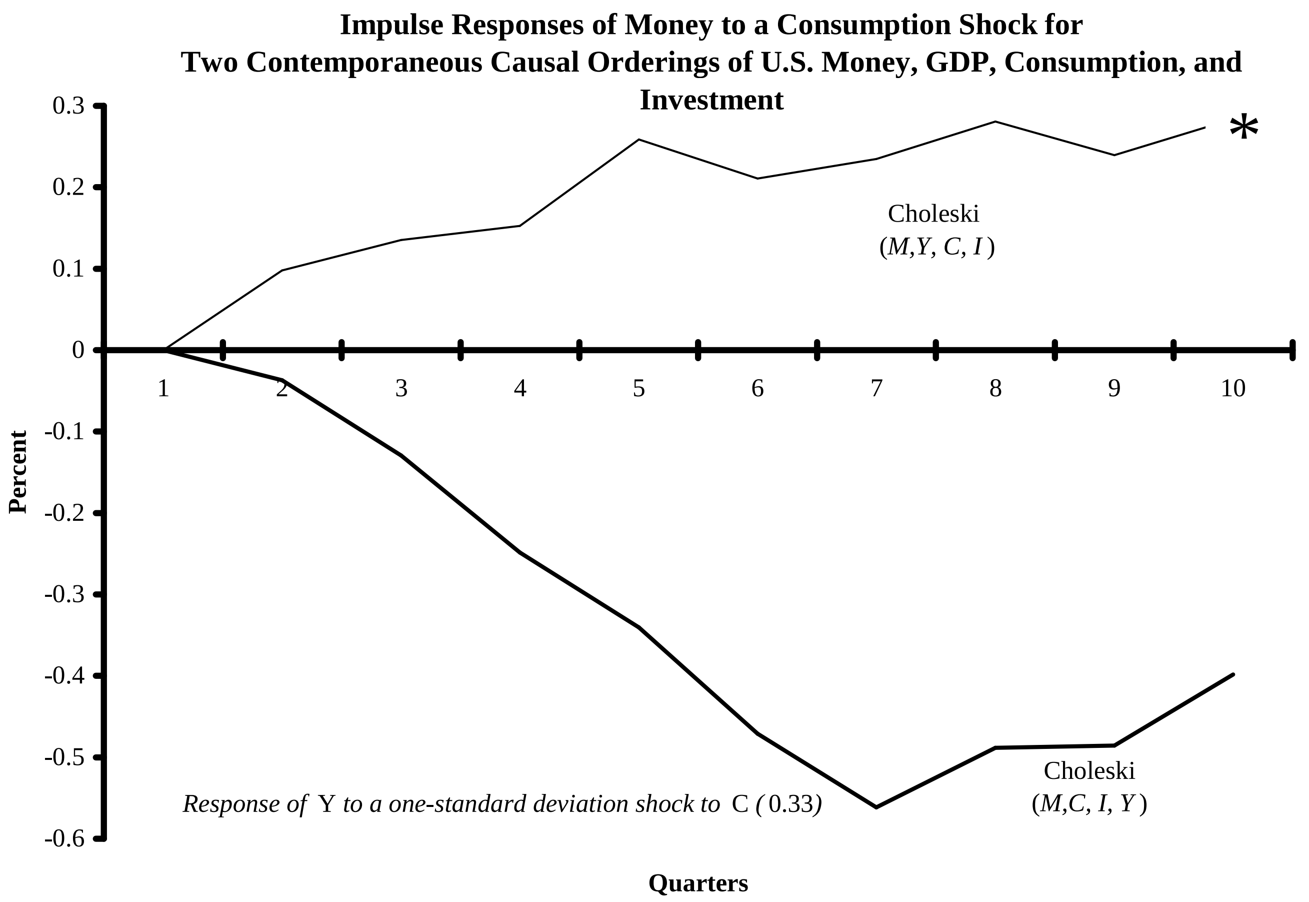
How far can we get with only such minimal prior theory?

An Illustration – 1

Object of Study: Structural Vector-Autogression (SVAR)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{YM} & 1 & 0 & 0 \\ \alpha_{CM} & \alpha_{CY} & 1 & 0 \\ \alpha_{IM} & \alpha_{IY} & \alpha_{IC} & 1 \end{bmatrix} \begin{bmatrix} M_t \\ Y_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} \beta_{MM} & \beta_{MY} & \beta_{MC} & \beta_{MI} \\ \beta_{YM} & \beta_{YY} & \beta_{YC} & \beta_{YI} \\ \beta_{CM} & \beta_{CY} & \beta_{CC} & \beta_{CI} \\ \beta_{IM} & \beta_{IY} & \beta_{IC} & \beta_{II} \end{bmatrix} \begin{bmatrix} M_{t-1} \\ Y_{t-1} \\ C_{t-1} \\ I_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^Y \\ \varepsilon_t^C \\ \varepsilon_t^I \end{bmatrix}$$

- Data are time series
- Contemporaneous relations are meant to be structural; lagged relationships are meant to be reduced-form (i.e., summaries of autocorrelations)
- Contemporaneous matrix can be read as a directed graph
- Errors are mutually independent:
  - would not generally obtain if contemporaneous coefficient matrix = the identity matrix
  - implicitly requires *causal sufficiency* – though not well understood by current economists (better understood in the 1940s/50s by econometricians in the Cowles Commission tradition.
- Typical use: impulse-response functions and counterfactual analysis



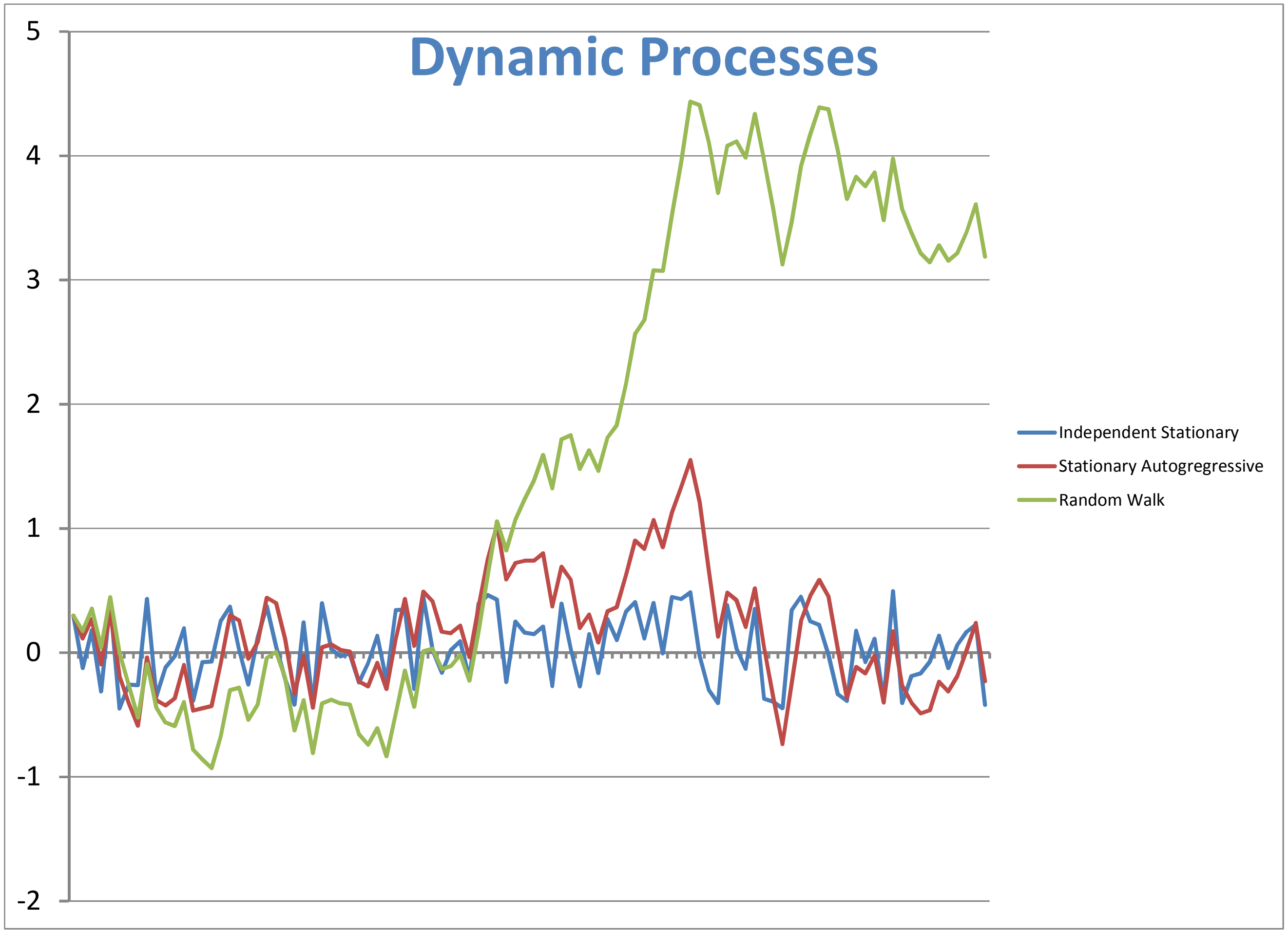
Impulse Responses of Money to a Consumption Shock for
Two Contemporaneous Causal Orderings of U.S. Money, GDP, Consumption, and Investment
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
Percent
1
2
3
4
5
6
7
8
9
10
*
Choleski
(M,Y, C, I)
Choleski
(M,C, I, Y)
Response of Y to a one-standard deviation shock to C (0.33)
Quarters

An Illustration – 2

Problem of Causal Inference in the SVAR:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{YM} & 1 & 0 & 0 \\ \alpha_{CM} & \alpha_{CY} & 1 & 0 \\ \alpha_{IM} & \alpha_{IY} & \alpha_{IC} & 1 \end{bmatrix} \begin{bmatrix} M_t \\ Y_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} \beta_{MM} & \beta_{MY} & \beta_{MC} & \beta_{MI} \\ \beta_{YM} & \beta_{YY} & \beta_{YC} & \beta_{YI} \\ \beta_{CM} & \beta_{CY} & \beta_{CC} & \beta_{CI} \\ \beta_{IM} & \beta_{IY} & \beta_{IC} & \beta_{II} \end{bmatrix} \begin{bmatrix} M_{t-1} \\ Y_{t-1} \\ C_{t-1} \\ I_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^Y \\ \varepsilon_t^C \\ \varepsilon_t^I \end{bmatrix}$$

- Algorithms normally work directly on variables without time series structure:
  - must account for it in all cases
  - particularly difficult in nonstationary cases: admissible probability distributions are nonstandard (e.g., distributions appropriate to Brownian motion) in which ordinary statistics do not apply


Dynamic Processes
5
4
3
2
1
0
-1
-2
Independent Stationary
Stationary Autogregressive
Random Walk

An Illustration – 3

Solution to time-series structure:

- Condition variables on past history:
  - regression problem involving only observables

$$\begin{bmatrix} M_t \\ Y_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} \gamma_{MM} & \gamma_{MY} & \gamma_{MC} & \gamma_{MI} \\ \gamma_{YM} & \gamma_{YY} & \gamma_{YC} & \gamma_{YI} \\ \gamma_{CM} & \gamma_{CY} & \gamma_{CC} & \gamma_{CI} \\ \gamma_{IM} & \gamma_{IY} & \gamma_{IC} & \gamma_{II} \end{bmatrix} \begin{bmatrix} M_{t-1} \\ Y_{t-1} \\ C_{t-1} \\ I_{t-1} \end{bmatrix} + \begin{bmatrix} \omega_t^M \\ \omega_t^Y \\ \omega_t^C \\ \omega_t^I \end{bmatrix}$$

  - estimated error terms ($\tilde{\omega}_t^j$) = original variables purged of time-series structure:
    - stationary
    - preserves contemporaneous causal order (i.e., unlike the ε_t^j, the $\tilde{\omega}_t^j$ not independent, but preserve the causally induced covariances that serve as inputs to search algorithms.

An Illustration – 4

The new object of analysis:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{YM} & 1 & 0 & 0 \\ \alpha_{CM} & \alpha_{CY} & 1 & 0 \\ \alpha_{IM} & \alpha_{IY} & \alpha_{IC} & 1 \end{bmatrix} \begin{bmatrix} \tilde{M}_t \\ \tilde{Y}_t \\ \tilde{C}_t \\ \tilde{I}_t \end{bmatrix} = \begin{bmatrix} \tilde{\varepsilon}_t^M \\ \tilde{\varepsilon}_t^Y \\ \tilde{\varepsilon}_t^C \\ \tilde{\varepsilon}_t^I \end{bmatrix}$$

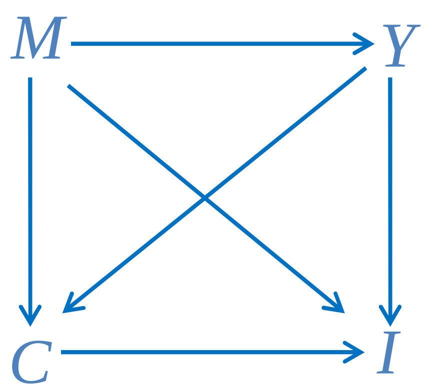
Further Problems of Causal Inference in the SVAR:

- How to specify contemporaneous causal structure?
  - usually (not necessarily) *lower triangular*
    - identification restriction: matrix must be full rank – i.e., $\geq n(n-1)/2$ restrictions
    - typically *just identified* (no *overidentifying* restrictions) – i.e., $= n(n-1)/2$ restrictions
    - typical appeal to "economic theory" (= just so stories) to determine order of variables

An Illustration – 5

Further Problems of Causal Inference in the SVAR:

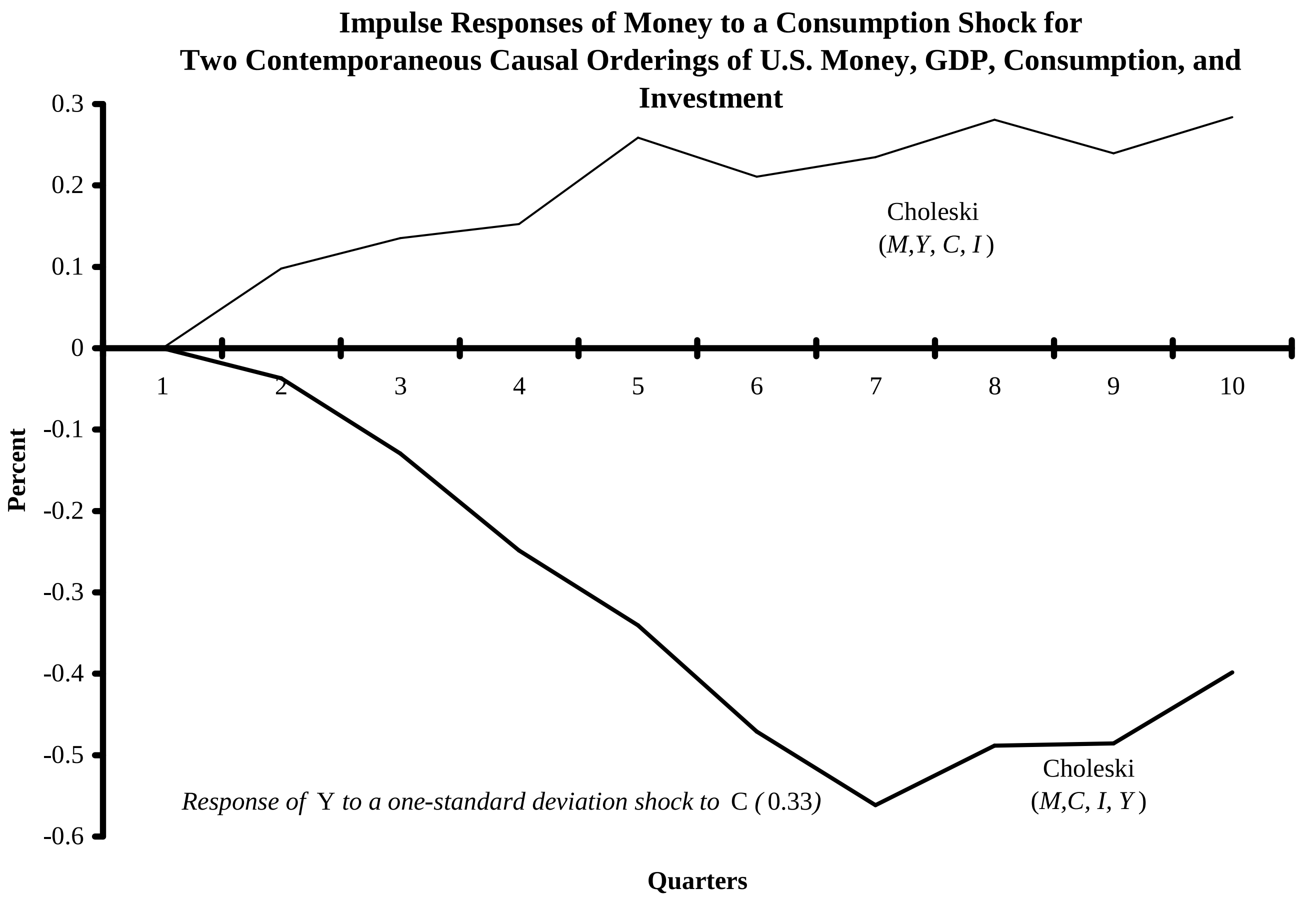
- Observational equivalence
  - all just-identified models have the same likelihood: no statistical basis for model choice
  - same point in causal graphs:
    - no unshielded colliers
    - any reordering of arrowheads (= reordering vector of variables) produces same skeleton
    - Pearl’s observational equivalence theorem



An Illustration – 6

Further Problems of Causal Inference in the SVAR:

- Nonequivalent impulse-response functions:



An Illustration – 7

- If data are actually overidentified, apply causal search algorithm to filtered data:

$$\begin{bmatrix} 1 & \alpha_{MY} & \alpha_{MC} & \alpha_{MI} \\ \alpha_{YM} & 1 & \alpha_{YC} & \alpha_{YI} \\ \alpha_{CM} & \alpha_{CY} & 1 & \alpha_{CI} \\ \alpha_{IM} & \alpha_{IY} & \alpha_{IC} & 1 \end{bmatrix} \begin{bmatrix} \tilde{M}_t \\ \tilde{Y}_t \\ \tilde{C}_t \\ \tilde{I}_t \end{bmatrix} = \begin{bmatrix} \tilde{\varepsilon}_t^M \\ \tilde{\varepsilon}_t^Y \\ \tilde{\varepsilon}_t^C \\ \tilde{\varepsilon}_t^I \end{bmatrix}$$

- Impose structure of contemporaneous matrix on original SVAR and estimate:

$$\begin{bmatrix} 1 & \alpha_{MY} & \alpha_{MC} & \alpha_{MI} \\ \alpha_{YM} & 1 & \alpha_{YC} & \alpha_{YI} \\ \alpha_{CM} & \alpha_{CY} & 1 & \alpha_{CI} \\ \alpha_{IM} & \alpha_{IY} & \alpha_{IC} & 1 \end{bmatrix} \begin{bmatrix} M_t \\ Y_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} \beta_{MM} & \beta_{MY} & \beta_{MC} & \beta_{MI} \\ \beta_{YM} & \beta_{YY} & \beta_{YC} & \beta_{YI} \\ \beta_{CM} & \beta_{CY} & \beta_{CC} & \beta_{CI} \\ \beta_{IM} & \beta_{IY} & \beta_{IC} & \beta_{II} \end{bmatrix} \begin{bmatrix} M_{t-1} \\ Y_{t-1} \\ C_{t-1} \\ I_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^Y \\ \varepsilon_t^C \\ \varepsilon_t^I \end{bmatrix}$$

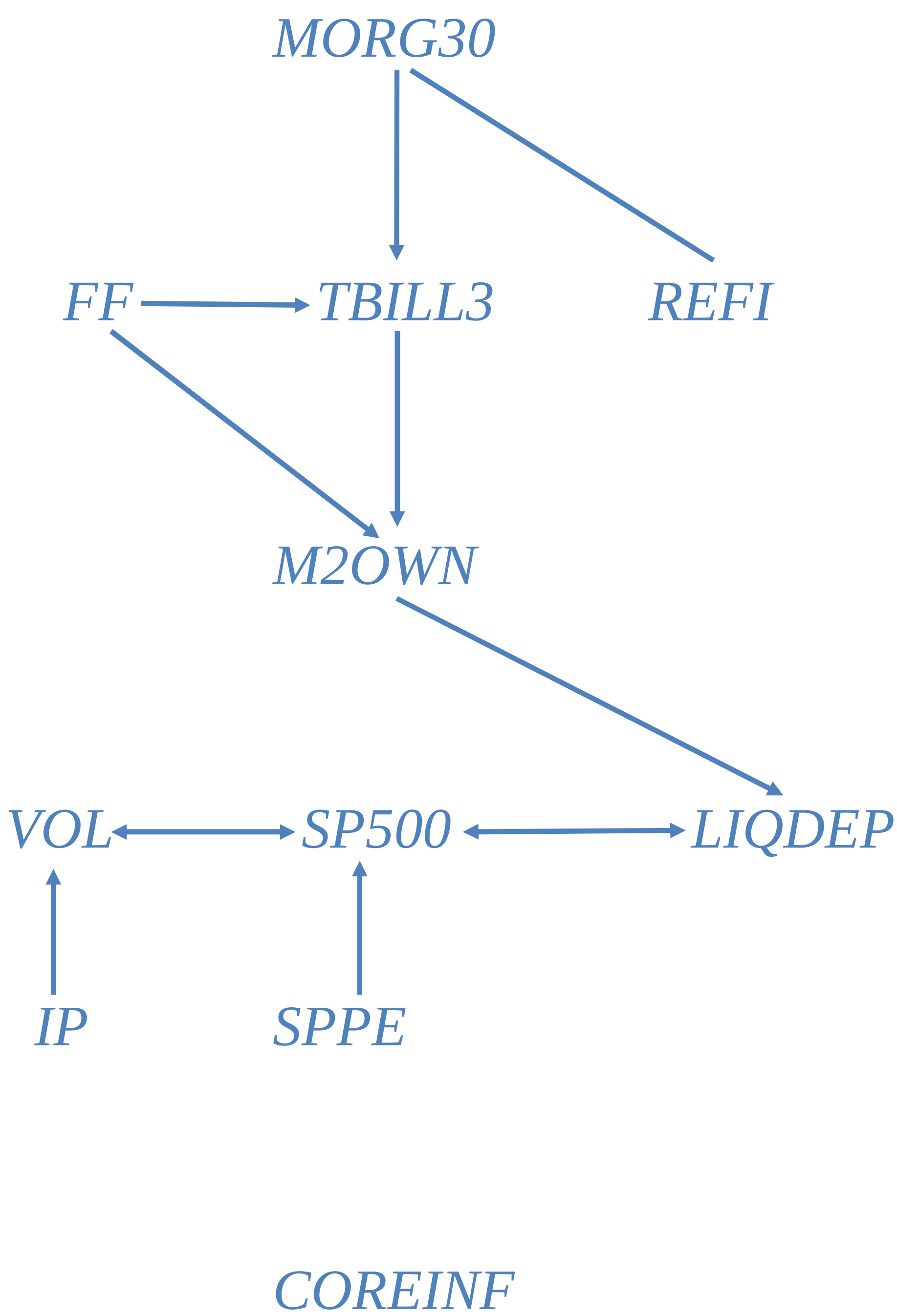
Back to the Actual Case

The Data

11 monthly series that run from 1990:02 to 2005:03;

- **liquid deposits (*LIQDEP*);**
- core CPI inflation (*COREINF*);
- industrial production (*IP*);
- S&P 500 stock market index (*SP500*)
- price-earnings ratio (*SPPE*);
- stock market volatility (*VOL*);
- index of mortage refinancing (*REFI*);
- interest rate on 30-year fixed-rate mortgages (*MORG30*);
- the Federal funds rate (*FF*);
- the opportunity cost of M2 (*M2OC*) [= *M2OWN* – *TBILL3*];
- the own rate of interest on M2 (*M2OWN*);
- the 3-month T-bill rate (*TBILL3*).

Graph from the PC Algorithm



How reliable is this graph?

Bootstrap Procedure: Demiralp, Hoover, and Perez *Oxford Bulletin of Economics and Statistics*, 2008

$$\begin{bmatrix} M_t \\ Y_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} \gamma_{MM} & \gamma_{MY} & \gamma_{MC} & \gamma_{MI} \\ \gamma_{YM} & \gamma_{YY} & \gamma_{YC} & \gamma_{YI} \\ \gamma_{CM} & \gamma_{CY} & \gamma_{CC} & \gamma_{CI} \\ \gamma_{IM} & \gamma_{IY} & \gamma_{IC} & \gamma_{II} \end{bmatrix} \begin{bmatrix} M_{t-1} \\ Y_{t-1} \\ C_{t-1} \\ I_{t-1} \end{bmatrix} + \begin{bmatrix} \omega_t^M \\ \omega_t^Y \\ \omega_t^C \\ \omega_t^I \end{bmatrix}$$

- Draw columns with replacement from estimated error matrix
- Use equation system to generate simulated variables
- Apply PC algorithm to simulated data and record graph
- Repeat 10,000 times

16

Table 3
Bootstrap Evaluation of the Causal Graph

Causal Order Selected by the PC Algorithm[1]			Edge Identification (percent of bootstrap realizations)[2]					Summary Statistics for Bootstrap Distribution[3]		
			—	←	no edge	→	↔	exists	directed	net direction
REFI	—	*MORG30*	65	22	0	11	3	100	35	-11
SP500	←	*SPPE*	17	52	0	11	20	100	83	-41
FF	→	*M2OWN*	47	14	0	38	1	100	53	25
FF	→	*TBILL3*	7	5	1	79	8	99	93	73
VOL	↔	*SP500*	8	20	5	28	39	95	92	8
MORG30	→	*TBILL3*	7	3	5	75	10	95	93	72
LIQDEP	↔	*SP500*	0	0	45	30	24	55	100	30
IP	→	*VOL*	2	1	47	37	14	53	97	37
M2OWN	←	*TBILL3*	0	10	64	16	10	36	99	6
LIQDEP	←	*M2OWN*	2	8	72	7	12	28	94	-1
REFI	no edge	*TBILL3*	0	0	82	15	3	18	98	14
REFI	no edge	*M2OWN*	3	6	83	3	4	17	80	-3
IP	no edge	*SPPE*	1	0	85	7	7	15	94	7
SP500	no edge	*M2OWN*	1	4	86	1	9	14	95	-3
COREINF	no edge	*SP500*	1	0	87	10	2	13	95	10
IP	no edge	*M2OWN*	0	0	88	5	7	12	99	5

[1] 16 of 55 candidate edges; only edges that are identified as existing in 12 percent or more of the bootstrap replications are shown.

[2] Values indicate percentage of 10,000 bootstrap replications in which each type of edge is found. Based on the procedure in Demiralp, Hoover, and Perez (2008) with critical value of 2.5 percent for tests of conditional correlation (corresponding to the 10 percent critical value used in the PC algorithm).

[3] *exists* = the percentage of bootstrap replications in which an edge is selected (= 100 – no edge); *directed* = edges directed as a percentage of edges selected; *net direction* = difference between edges directed right (→) and left (←).

Bootstrap Evaluation of the Causal Graph

Causal Order Selected by the PC Algorithm			**Summary Statistics for Bootstrap Distribution**		
			exists	**directed**	**net direction**
REFI	—	*MORG30*	100	35	-11
SP500	←	*SPPE*	100	83	-41
FF	→	*M2OWN*	100	53	25
FF	→	*TBILL3*	99	93	73
VOL	↔	*SP500*	95	92	8
MORG30	→	*TBILL3*	95	93	72
LIQDEP	↔	*SP500*	55	100	30
IP	→	*VOL*	53	97	37
M2OWN	←	*TBILL3*	36	99	6
LIQDEP	←	*M2OWN*	28	94	-1
REFI	no edge	*TBILL3*	18	98	14
REFI	no edge	*M2OWN*	17	80	-3
IP	no edge	*SPPE*	15	94	7
SP500	no edge	*M2OWN*	14	95	-3
COREINF	no edge	*SP500*	13	95	10
IP	no edge	*M2OWN*	12	99	5

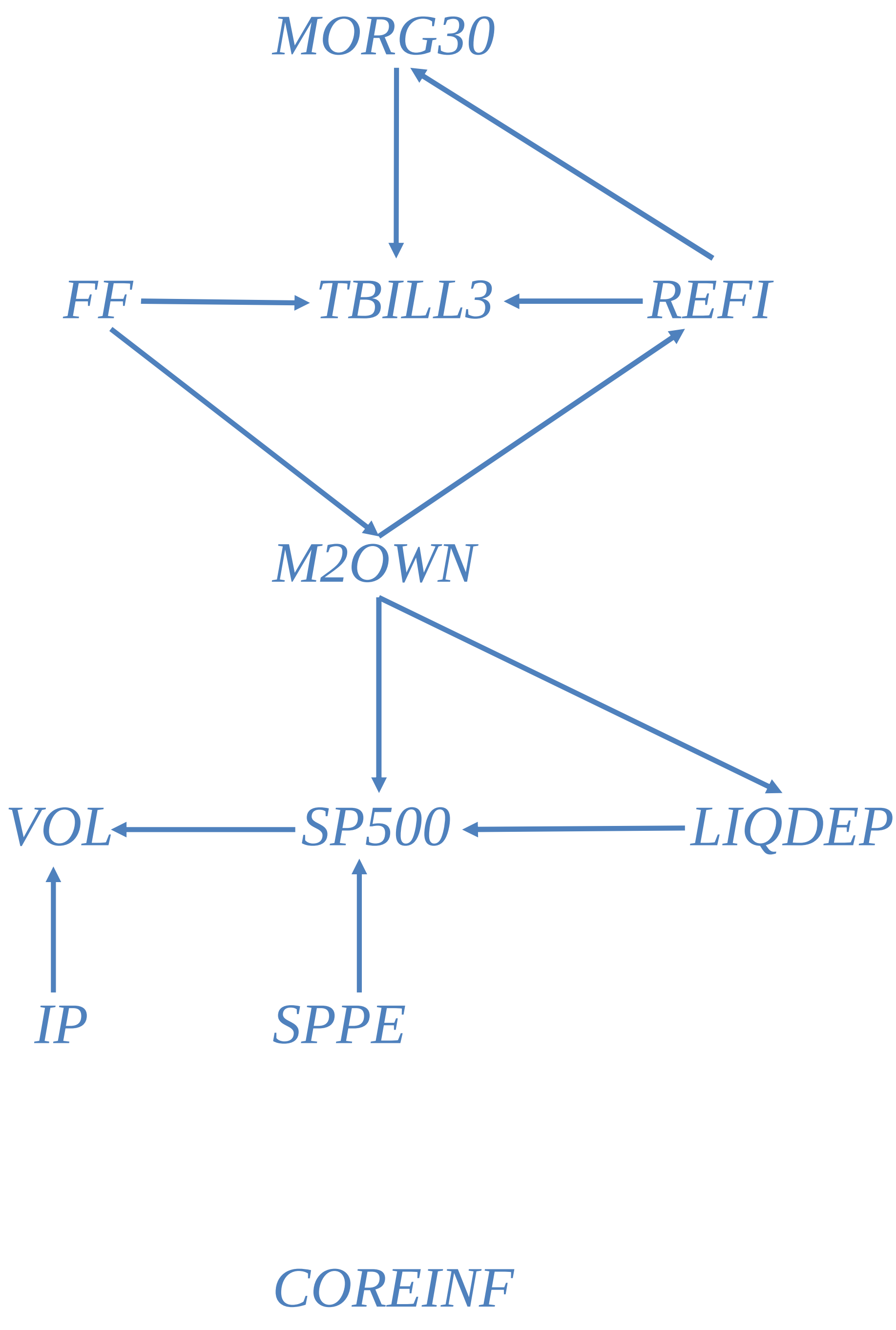
Notes on Initial Causal Order

- Unoriented edge from *REFI* $\rightarrow$ *MORG30* based on dominant order in Table 3.
- Bidirectional edges: *VOL* $\leftrightarrow$ *SP500* and *SP500* $\leftrightarrow$ *LIQDEP* despite acylicality assumption:.
    - possible reasons
        - i) small sample problem
        - ii) omitted latent variable
- Initial graph strongly rejected against a just-identified model: $p = 0.002$ with *VOL* $\leftrightarrow$ *SP500* and *SP500* $\leftrightarrow$ *LIQDEP* left bidirectional and either *REFI* $\rightarrow$ *MORG30* or *REFI* $\leftarrow$ *MORG30*
- Fear of omission greater than fear of commission.
    - Supplement the graph with all the *borderline* edges that appear in
        - > 10%
        - > 2.5% of bootstrap replications for *REFI* $\leftarrow$ *MORG30*

Table 4
Contemporaneous Causal Structure: Specification Search

	Specification	Likelihood Ratio Test against the Just-Identified Model (*p-value*)
Search I		
Initial Model	Graph in Figure 1 modified with *REFI* → *MORG30*	0.002
General Model I	As above, plus: *REFI* → *TBILL3*; *REFI* ← *M2OWN*; *IP* → *SPPE*; *M2OWN* → *SP500*; *IP* → *M2OWN*; *COREINF* → *REFI*	0.140
Tests of Restrictions		
1	omit *LIQDEP* ← *SP500*	no convergence
2	restore *LIQDEP* ← *SP500*; omit *VOL* → *SP500*	0.163
3	omit *LIQDEP* ← *SP500*	0.185
4	omit *IP* → *SPPE*	0.166
5	omit *M2OWN* ← *TBILL3*	0.146
6	omit *COREINF* → *SP500*	0.123
7	omit *IP* → *M2OWN*	**0.102**
8	omit *REFI* → *TBILL3*	0.068
Search II		
Initial Model	Graph in Figure 1 modified with *REFI* ← *MORG30*	0.002
General Model II	As above, plus: all edges that appeared in more than 2.5 percent of bootstrap replications	0.069

Figure 2
Final Contemporaneous Causal Graph



The $\mathbf{A}_0$ Matrix

Causes

Effects	*COREINF*	*FF*	*IP*	*SPPE*	*M2OWN*	*LIQDEP*	*SP500*	*REFI*	*MORG30*	*VOL*	*TBILL3*
COREINF	1	0	0	0	0	0	0	0	0	0	0
FF	0	1	0	0	0	0	0	0	0	0	0
IP	0	0	1	0	0	0	0	0	0	0	0
SPPE	0	0	0	1	0	0	0	0	0	0	0
M2OWN	0	a_{52}	0	0	1	0	0	0	0	0	0
LIQDEP	0	0	0	0	a_{65}	1	0	0	0	0	0
SP500	0	0	0	a_{74}	a_{75}	a_{76}	1	0	0	0	0
REFI	0	0	0	0	a_{85}	0	0	1	0	0	0
MORG30	0	0	0	0	0	0	0	a_{98}	1	0	0
VOL	0	0	a_{103}	0	0	0	a_{107}	0	0	1	0
TBILL3	0	a_{112}	0	0	0	0	*0*	a_{118}	a_{119}	0	1

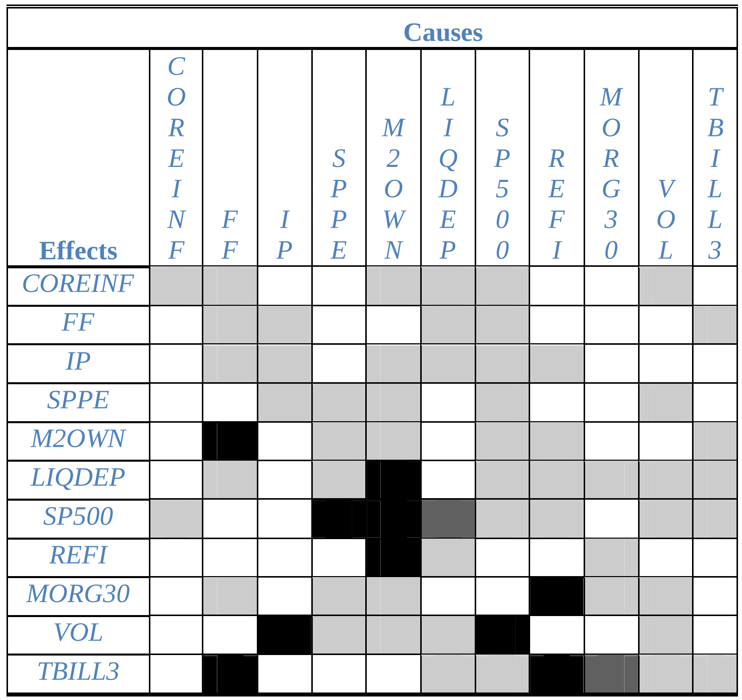
Specification of Lagged Dynamics

- Apply LSE (David Hendry)-style general-to-specific specification search techniques.
  - Automated search algorithms first developed and demonstrated to be effective in:
    - Hoover and Stephen Perez, "Data Mining Reconsidered: Encompassing and the General-to-Specific Approach to Specification Search, *Econometrics Journal*, vol. 2, no. 2, 1999, pp. 1-25
    - Hoover and Perez, "Truth and Robustness in Cross-country Growth Regressions," *Oxford Bulletin of Economics and Statistics* 66(5), 2004, pp. 765-798.
    - Hoover, "The Role of Hypothesis Testing in the Molding of Econometric Models," *Erasmus Journal for the Philosophy of Economics*, forthcoming.

  - Developed into commercially available software:
    - Hendry and Krolzig, *PcGets*
    - Doornik and Hendry, *Autometrics* package in *PcGive* version12 and above.

Summary of Results

The Causal Structure of the SVAR



Key:

= lagged causes only

= contemporaneous causes only

= lagged and contemporaneous causes

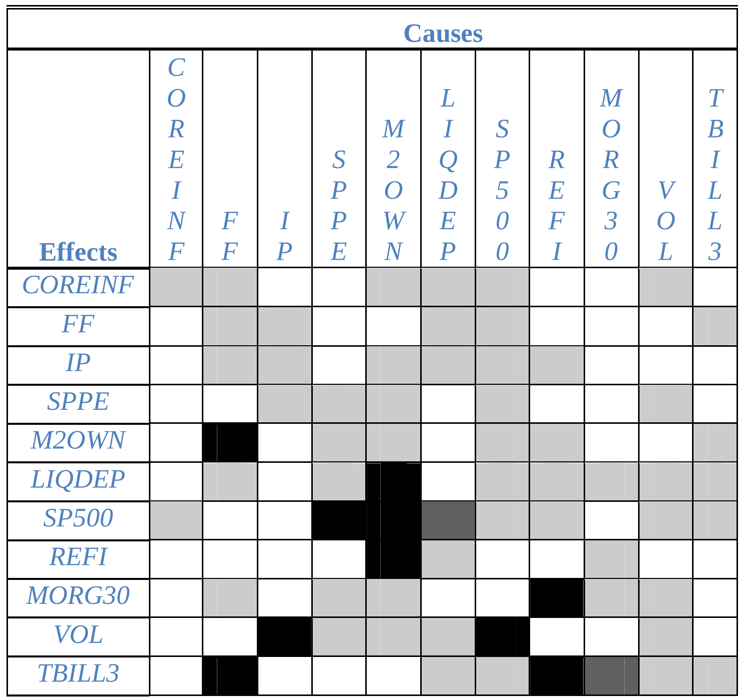
Notes: based on the detailed SVAR specification in Appendix B.

Compare to a Choleski (Lower-Triangular) Ordering:

The Causal Structure of the SVAR

	Causes										
Effects	*COREINF*	*FF*	*IP*	*SPPE*	*M2OWN*	*LIQDEP*	*SP500*	*REFI*	*MORG30*	*VOL*	*TBILL3*
COREINF											
FF											
IP											
SPPE											
M2OWN											
LIQDEP											
SP500											
REFI											
MORG30											
VOL											
TBILL3											

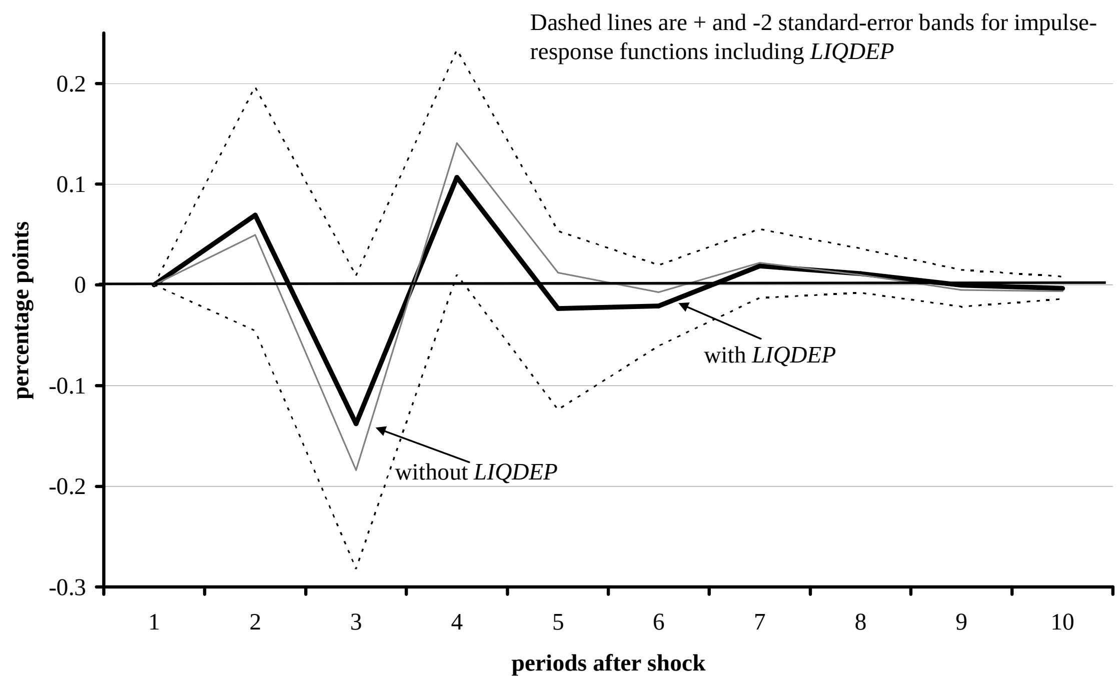
The Causal Structure of the SVAR



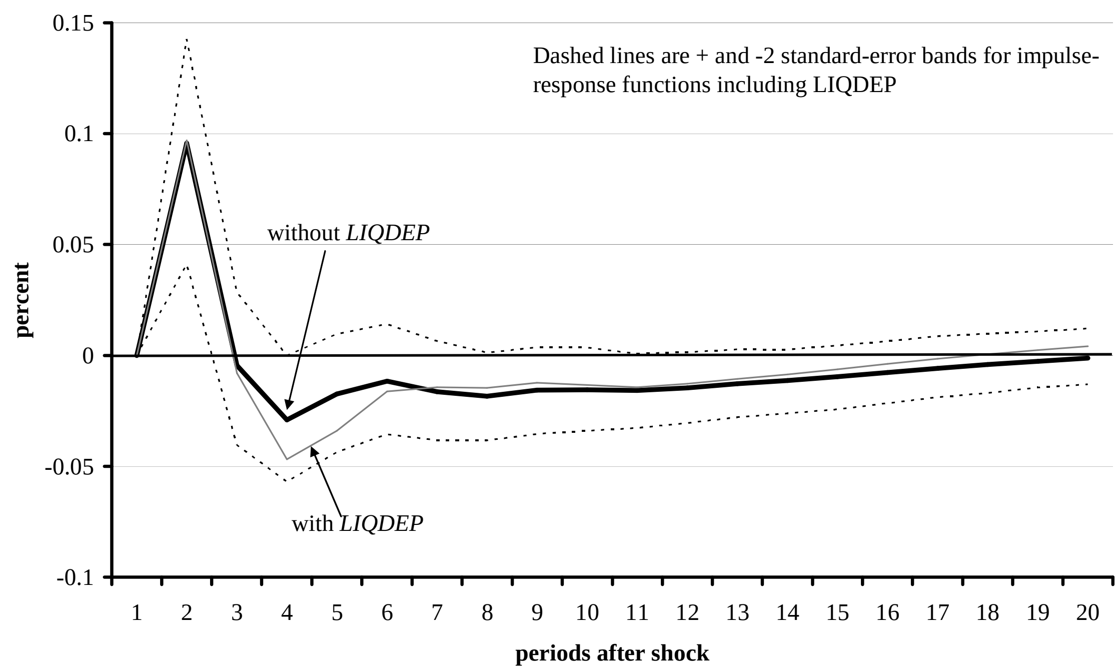
- *LIQDEP* depends on almost everything but what the quantity theory suggests: on *M2OWN*, but not *TBILL3*, *COREINF*, or *IP*;
- Nevertheless, *LIQDEP* causes everything but *SPPE*, *M2OWN*, and *MORG30*.

Counterfactual Experiment

Impulse Response of *COREINF* to permanent 25-basis-point shock to *FF*



Impulse Response of *IP* to permanent 25-basis-point shock to *FF*



Open Issues

- Causal search with simultaneous and cyclical cause
    - Piyachart Phiromswad and Kevin D. Hoover, "Selecting Instrumental Variables: A Graph-Theoretic Approach"
- Nonstationary variables frequently cointegrated:
    - implies latent common stochastic trends: number of trends + number of cointegrating relations = number of trending varaibles
    - opens possibility of causal relationships among the latent stochastic trends
    - work in progress

Thanks

The End