WORKSHOP ON LOGIC, INFORMATION, AND TOPOLOGY

Saturday, October 20, 2019
136A Adamson Wing, Carnegie Mellon University

Dynamic epistemic logic concerns the information conveyed by the beliefs of other agents. Belief revision theory studies rational belief change in light of new information. Formal learning theory concerns systems that learn the truth on increasing information. Topology is emerging as a particularly apt formal perspective on the underlying concept of propositional information. The talks in this workshop address the preceding themes from a range of overlapping perspectives.

8:15-9:00 Light breakfast

9:00-9:10 OPENING REMARKS

Kevin T. Kelly
Director, Center for Formal Epistemology, Carnegie Mellon University

9:10-10:25 THE TOPOLOGY OF STATISTICAL INQUIRY

Konstantin Genin, University of Toronto

Taking inspiration from Kelly's *The Logic of Reliable Inquiry* (1996), Baltag et. al. (2015) and Genin and Kelly (2015) provide a general topological framework for the study of empirical learning problems. Baltag et. al. (2015) prove a key result showing that it is almost always possible to learn by *stable* and *progressive* methods, in which the truth, once in the grasp of a learning method, is never relinquished. That work is grounded in a non-statistical account of information, on which information states decisively refute incompatible possibilities. However, most scientific data is statistical, and in these settings, logical refutation rarely occurs. Critics, including Sober (2015), doubt that the gap between propositional and statistical information can be bridged. In Genin (2018), I answer the skeptics by identifying the unique topology on probability measures whose closed sets are exactly the statistically refutable propositions. I also show that a statistical analogue of progressive learning can also be achieved in the more general setting. That result erects a topological bridge on which insights from learning theory can be ported directly into machine learning, statistics and the data-driven sciences.

10:25-10:45 Coffee break

10:45-12:00 KNOWABLE DEPENDENCY: A TOPOLOGICAL ACCOUNT

Alexandru Baltag, ILLC, University of Amsterdam

If *to be is to be the value of a variable,* then to know is *to know* a *functional dependence between variables*. (Moreover, the conclusion may still be arguably be true even if Quine's premise is wrong...) This points towards a fundamental connection between Hintikka's Epistemic Logic and Vaananen 's so-called Dependence Logic (itself anticipated by the Independence-Friendly Logic of Hintikka and Sandu). The connection was made precise in the Epistemic Dependence Logic introduced in my 2016 AiML paper. Its dynamics captures the widespread view of knowledge acquisition as a process of *learning correlations* (with the goal of eventually tracking causal relationships in the actual world). However, when talking about *empirical* variables in natural sciences, the exact value might not be knowable, and instead only inexact approximations can be known. This leads to a topological conception of empirical variables, *as maps from the state space into a topological space*. Here, the exact value of the variable is represented by the output of the map, while the open neighborhoods of this value represent the knowable approximations of the exact answer. I argue that knowability of a dependency amounts in such an empirical context to the *continuity* of the given functional correlation. To know (in natural science) is *to know a continuous dependence between empirical variables.*

2:00-3:15 BELIEFS, PROPOSITIONS and DEFINITE DESCRIPTIONS

Eric Pacuit, University of Maryland

In this paper, we introduce a doxastic logic with expressions that are intended to represent definite descriptions for propositions. Using these definite descriptions, we can formalize sentences such as:

- Ann believes that the strangest proposition that Bob believes is that neutrinos travel at twice the speed of light.
- Ann believes that the strangest proposition that Bob believes is false.

The second sentence has both de re and de dicto readings, which are distinguished in our logic. We motivate our logical system with a novel analysis of the Brandenburger-Keisler paradox. Our analysis of this paradox uncovers an interesting connection between it and the Kaplan-Montague Knower paradox. (This is joint work with Wes Holliday)

3:15-4:30 THE EPISTEMOLOGY OF NONDETERMINISM

Adam Bjorndahl, Carnegie Mellon University

Propositional dynamic logic (PDL) is a framework for reasoning about nondeterministic program executions (or, more generally, nondeterministic actions). In this setting, nondeterminism is taken as a primitive: a program is nondeterministic iff it has multiple possible outcomes. But what is the sense of "possibility" at play here? This talk explores an epistemic interpretation: working in an enriched logical setting, we represent nondeterminism as a relationship between a program and an agent deriving from the agent's (in)ability to adequately measure the dynamics of the program execution. More precisely, using topology to capture the observational powers of an agent, we define the nondeterministic outcomes of a given program execution to be those outcomes that the agent is unable to rule out in advance. In this framework, continuity turns out to coincide exactly with determinism: that is, determinism is continuity in the observation topology. This allows us to embed PDL into (dynamic) topological (subset space) logic, laying the groundwork for a deeper investigation into the epistemology (and topology) of nondeterminism.

4:30-5:00 Coffee break

5:00-6:15 SPATIAL MODELS OF HIGHER-ORDER S4

Colin Zwanziger, Carnegie Mellon University

Topological spaces provide a model for propositional S4 modal logic (McKinsey and Tarski 1944) in which the modal operators can be thought of as expressing verifiability and refutability (c.f. Schulte and Juhl 1996, Kelly 1996,...). It is natural to ask: is there a "spatial" notion of model which stands in the same relation to (modal S4) *predicate* logic as topology does to propositional logic?

Garner (2010) introduced ionads to provide a notion of "higher topological space". The sheaf semantics of Awodey and Kishida (2008) yields a special example of an ionad. A generalization of Garner's ionads is suggested here as a response to our question, in which the "points" will themselves often be mathematical structures (e.g. groups, rings,...), considered together with their isomorphisms. Any such generalized ionad is a model of (classical) higher-order S4 (by application of Zwanziger 2017). Furthermore, to any generalized ionad, we can associate a Grothendieck topos (analogous to the poset of opens of a topological space) that is generated canonically from a theory in the verifiable (geometric) fragment of first-order logic. Thus, generalized ionads may be of interest for applications to verifiability and refutability.