Topology and Measure in Logics for Point-Free Space

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Abstract

Space, as we typically represent it in mathematics and physics, is composed of dimensionless, indivisible points. But philosophers have for a long time doubted whether there are any point-sized regions of space. On an alternative, region-based approach to space, extended regions together with the relations of parthood and contact are taken as primitive; points are represented as mathematical abstractions from regions.

Region-based theories of space have been traditionally modeled in regular closed (or regular open) algebras, in work that goes back to Whitehead. More recently, formal logics for region-based accounts of space were developed in, *e.g.*, [3] and [2], and it was shown that these logics have both a nice topological and relational semantics.

The present paper explores the question of completeness of these logics for individual topological spaces of interest: the real line, the rationals, Cantor space. A secondary aim is to study a different model of logics for region-based theories of space, based on the Lebesgue measure algebra (or algebra of Borel subsets of the real line modulo sets of Lebesgue measure zero). As a model for point-free space, the algebra was first discussed in [1]. The main results of the paper are that the minimal logic for contact algebras, $\mathbb{L}^{\text{cont}}_{\text{min}}$, is complete for the rationals and Cantor space; the extension $\mathbb{L}^{\text{cont}}_{\text{min}} + (Con)$ is complete for the real line and the Lebesgue measure algebra.

References

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