Building Latent Variable Models

by

Richard Scheines
Peter Spirtes
Clark Glymour

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Richard Scheines
Peter Spirtes
Clark Glymour

Dept. of Philosophy
Carnegie Mellon University

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1. Introduction

Researchers routinely face the problem of inferring causal relationships from large amounts of data, sometimes involving hundreds of variables. Often, it is the causal relationships between "latent" (unmeasured) variables that are of primary interest. The problem is how causal relationships between unmeasured variables can be inferred from measured data. For example, naval manpower researchers have been asked to infer the causal relations among psychological traits such as job satisfaction and job challenge from a data base in which neither trait is measured directly, but in which answers to interview questions are plausibly associated with each trait. By combining background knowledge with an algorithm that searches for causal structure among the unobserved variables, we have created a tool that can reliably extract useful causal information about latent variables from large data bases. In what follows we describe the class of causal models to which our techniques apply, the property that connects the causal structure of such models to measured data, the algorithm that searches for causal structures, its reliability and complexity, and simulation studies that attest to the algorithm's reliability on samples of realistic size.

2. Causal Models

2.1 Causal Graphs and Causal Models. Economists, psychologists, sociologists, and political scientists routinely employ "structural equation models," or "causal models," to represent the causal structure among a set of random variables. These include regression models, factor analytic models, and path models.[2] These models typically are expressed as systems of linear equations among a set of random variables **V** along with an intended causal interpretation and distributional assumptions about **V**. The causal structure can be represented by a directed graph G, called a **causal graph**, such that there is an edge from variable V1 to V2 in G just in case V1 is a direct cause of V2. For example, in the causal

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[2]See (Bollen 89) for a good introduction to "structural equation models."

graph in figure 1, both SAT and IQ scores are directly caused by intelligence and test-taking ability.

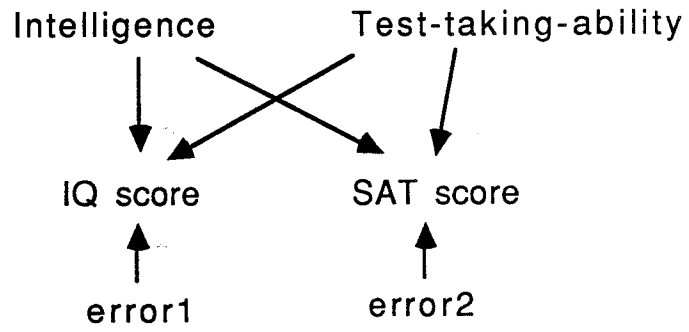

Figure 1

In structural equation models every effect is a linear combination of all of its direct causes. This assumption allows us to represent the equations in such models by attaching labels to the edges in the causal graph, where these labels represent the linear coefficients in the associated equations.

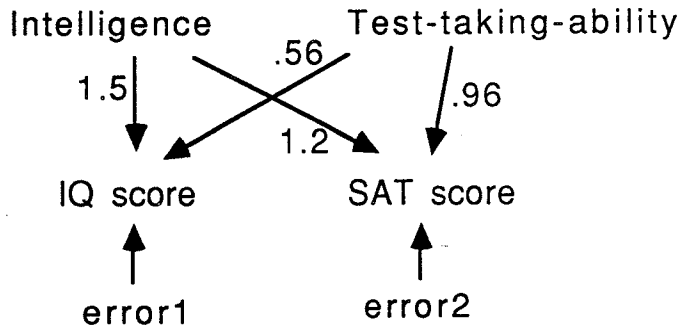

Figure 2

In figure 2, for example, IQ score = 1.5*Intelligence + .56*Test-taking-ability + error1. Let a **linear causal model** associated with a causal graph G be given by <D,θ>, where D is a distribution over the exogenous variables in G (those that are only causes and not effects) and θ is a vector of linear coefficients that correspond to the appropriate edge

labels in G.[3] Given a causal graph G, the variance/covariance matrix Σ among the measured variables in G is completely determined by $<D,\theta>$.

2.2 Connecting the Evidence to the Causal Structure: Vanishing Tetrad Constraints. There are, however, constraints on Σ that are determined *just by the causal graph G associated with a causal model.* That is, there are constraints on Σ that are satisfied regardless of the value of $<D,\theta>$. These constraints include **vanishing partial correlations** and **vanishing tetrad differences**, and they provide a connection between causal structure and measured data. For variables X, Y, W, and Z, there are three possible vanishing tetrad differences, any two of which are independent of each other:

$$\rho XY * \rho WZ - \rho XW * \rho YZ = 0$$
$$\rho XW * \rho YZ - \rho XZ * \rho YW = 0$$
$$\rho XZ * \rho YW - \rho XY * \rho WZ = 0$$

If the exogenous variables in D have non-zero variance, then we have a purely graphical characterization of the conditions under which a causal graph implies for all values of θ a vanishing tetrad difference among measurable correlations.[4] The characterization allows us to calculate the set of tetrad equations implied by a causal graph extremely quickly. We search for the causal graph that implies the set of vanishing tetrad differences that most closely matches the set of vanishing tetrad differences judged to vanish inthe population.[5]

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[3]We also assume that each variable has a unique associated "error" variable of non-zero variance, that is all variables of non-zero indegree are the effect of a variable of indegree 0 and outdegree 1.

[4]See (Spirtes 89).

[5]The statistical tests we use to judge whether a tetrad difference vanishes in the population, and the metric that we use to measure how closely a set of tetrad differences implied to vanish match the set of tetrad differences judged to vanish in the population are described in detail in (Spirtes 91), and (Scheines 91). It is possible that a tetrad difference vanishes in the population for a given graph for some but not all values of θ; however if a graph does not entail that a given tetrad

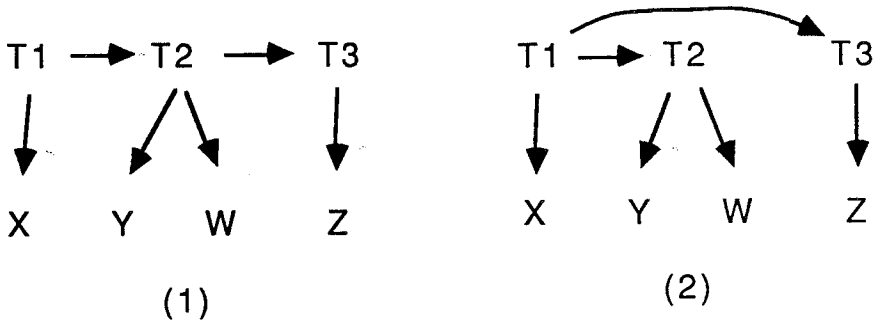

Figure 3

Tetrad equations allow us to discriminate among different latent variable models where partial correlations do not.[6] For example, suppose that in graphs (1) and (2) in figure 3 only X, Y , Z, and Z are measured. Neither graph implies any vanishing partial correlations, but

graph 1 implies:

$\rho X1,X3 * \rho X2,X4 = \rho X1,X4 * \rho X2,X3$

$\rho X1,X4 * \rho X2,X3 = \rho X1,X2 * \rho X3,X4$

$\rho X1,X3 * \rho X2,X4 = \rho X1,X2 * \rho X3,X4$

graph 2 implies:

$\rho X1,X2 * \rho X3,X4 = \rho X1,X3 * \rho X2,X4$

If the results of statistical tests indicate that only $\rho XW * \rho YZ = \rho XY * \rho WZ$ in the population, then graph 2 is a better model of the data, because it entails the only tetrad difference judged to vanish in the population, and it does not entail any vanishing tetrad differences that are judged not to hold in the population.

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difference vanishes for all values of θ, the Lebesgue measure of the set of θ's for which the tetrad does vanish is zero.

[6]For example, vanishing partial correlations. Although vanising partial correlations of any order give us as good a connection to causal structure as can be had when all the common causes of measured variables are themselves measured (Spirtes 91), they cannot distinguish among the two graphs in Figure 3, unlike tetrad constraints.

3. The Strategy

Structural equation models that involve latent variables are sometimes presented as containing two parts: the "measurement model", and the "structural model." The structural model involves only the causal connections among the latent variables, and the measurement model is the rest of the causal connections. Roughly, our strategy is to use certain tetrad equations to find a *pure measurement model,* i.e. one in which each measured variable is directly associated with only one latent. Having such a measurement model, we can then use different tetrad equations to constrain the structural model.

3.1 Pure Latent Variable Models.

A model is a **pure latent variable model** if and only if

> i) each latent variable has at least two measured variables as its direct effects,
> ii) each measured variable is the cause of no variable, and
> iii) each measured variable is the direct effect of exactly one latent variable and a unique error term.[7]

For example:

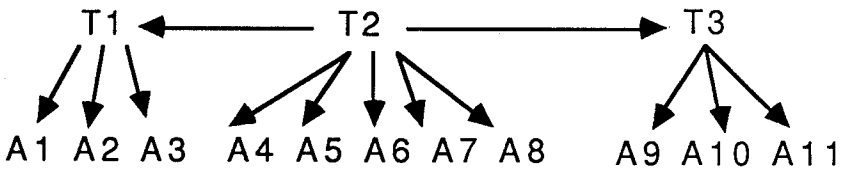

Figure 4

where the As represent measured variables and the Ts represent unmeasured variables, and for convenience the error terms have been omitted. The important fact is that *in a pure latent variable model the*

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[7]Models like these are also called "multiple indicator models."

correlations and vanishing tetrad constraints partially determine the causal structure among the unmeasured variables.

What if we are given a set of variables that do not form a pure latent variable model? It is possible that while a given set of variables do not form a pure latent variable model, some subset of the variables do. *It is possible to use vanishing tetrad differences to reliably select a subset of the variables that form a pure latent variable model (if one exists), even if the causal structure among the latent variables is unknown.* We then use vanishing tetrad constraints to partially determine the causal structure among the latent variables in the pure latent variable model. We will explain each of these procedures in more detail.

3.3 Finding Pure Latent Variable Models.

A measurement model is pure just in case it is the measurement model of a pure latent variable model, i.e., each measured variable is not a cause and is the direct effect of exactly one latent variable and its own independent error term. In actual research the set **V** of measured variables in a data base is often chosen to measure a particular set of latent variables **T**. That is, for each latent variable T_i, a subset of **V** is intended to measure T_i. We suppose the investigator can, by whatever means, form mutually exclusive subsets of **V** such that each subset measures a latent variable T_i, i.e. the researcher can partition **V** into $\mathbf{V_{Ti}}$, such that for each i the variables in $\mathbf{V_{Ti}}$ are direct effects of T_i.[8] The resulting model may not be a pure latent variable model, however, because some of the measured variables may be causes of other variables, or because some measured variables may be caused directly by more than one latent variable in **T**. To summarize, we assume that a researcher attempting to discover the causal relations among a set of latent variables **T** from measured variables **X**, can

1) Identify **T**, and

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[8] That is, there is an edge from T_i to each V in $\mathbf{V_{Ti}}$.

2) successfully partition the variables in **X** into groups that at least measure each T_i.

True Causal Structure

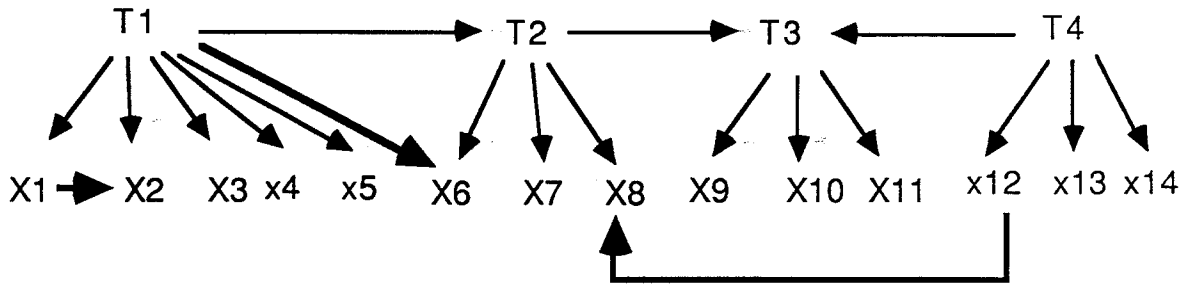

Figure 5

For example, suppose that the true causal graph G is the one depicted in figure 5. The we assume a researcher can identify **T** = {T1,T2,T3,T4}, and can successfully partition **X** = {X1-X14} into clusters as we show in figure 6.[9]

Clusters From Background Knowledge

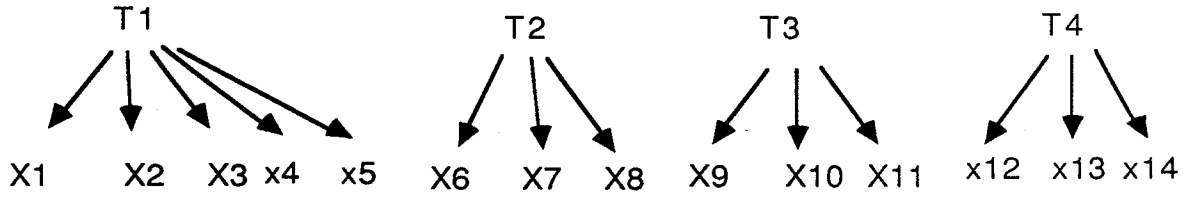

Figure 6

G (figure 5) is not a pure latent variable model because X1 causes X2, X6 is caused by both T1 and T2, and X12 causes x8. Consequently, the set of measured variables X1 through X14 do not constitute a pure measurement model of **T**. There are subsets of {X1-X14} that can constitute a pure measurement model of **T**, however, and figure 7 shows one.

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[9]The clusters in figure 6 are not uniquely correct. Another correct initial clustering might group X6 with the T1 cluster.

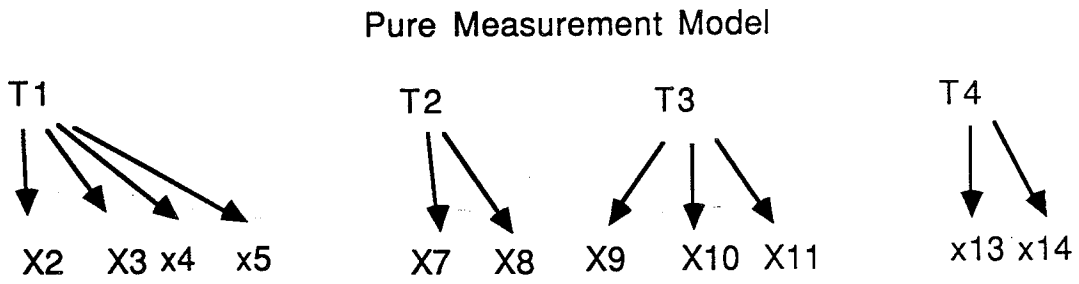

Figure 7

We have designed an algorithm that can reliably find a subset of variables that form a pure latent variable model, *regardless of what the (up to this point unknown) causal connections among the latent variables are.* We call this part of the procedure and a later part the SCALES procedure. SCALES eliminates those members of $\mathbf{V_{Ti}}$ that are impure measures of T_i, either because they are also the effects of some other unmeasured variable T_j or because they are also causes or effects of some other measured variable.

Impure Indicators

A measured variable can be an impure measure for three reasons, which are exhaustive but not exclusive.

1) If V_i measures T_i, but is also causally connected to a latent variable T_j in some way not mediated by T_i, then we say that V_i is **latent-measured impure.**

2) If a pair of measured variables V1, V2 from the same cluster $\mathbf{V_{Ti}}$ are causally connected in some way not mediated by T_i then we say V1 and V2 are **intra-construct impure.**

3) If a pair of measured variables V1, V2 from distinct clusters $\mathbf{V_{Ti}}$ and $\mathbf{V_{Tj}}$ are causally connected in some way not mediated by either T_i or T_j then we say V1 and V2 are **cross-construct impure.**

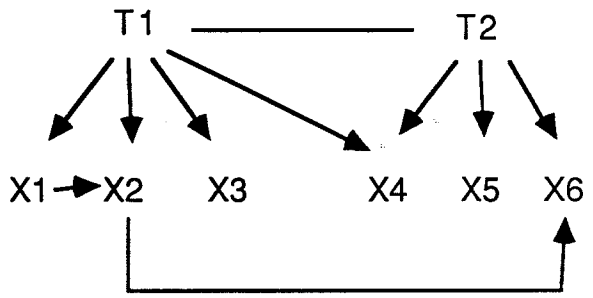

Figure 8

For example, in figure 8, X1, X2, and X6 are impure. The breakdown is as follows.

Intra-construct impure: { X1, X2}
Cross-construct impure: { X2, X6}
Latent-measured impure: { X4 }

The strategy for finding subsets of **V** that form a pure measurement model is to use tetrad equations of one type to eliminate those measured variables that are intra-construct impure, to use tetrad equations of a different type to eliminate those that are cross-construct impure, and finally those that are latent-measured impure. Provided the initial clusterings are correct, the strategy will correctly identify a subgraph that is a pure latent variable model (if one exists).

Foursomes

Tetrad equations involve foursomes of measured variables. Based on the clustering from background knowledge: call a foursome of measured variables an **intra-construct foursome** if all of the measured variables are effects of the same latent variable (see Figure 9). Call a foursome in which at least two measured variables are measures of two different latent variables, and in which there are exactly two latent variables, a **cross-construct foursome**. Call a foursome in which one indicator is from one latent variable, two indicators are from a second latent variable, and another indicator is from a third latent variable a **1-2-1 foursome**.

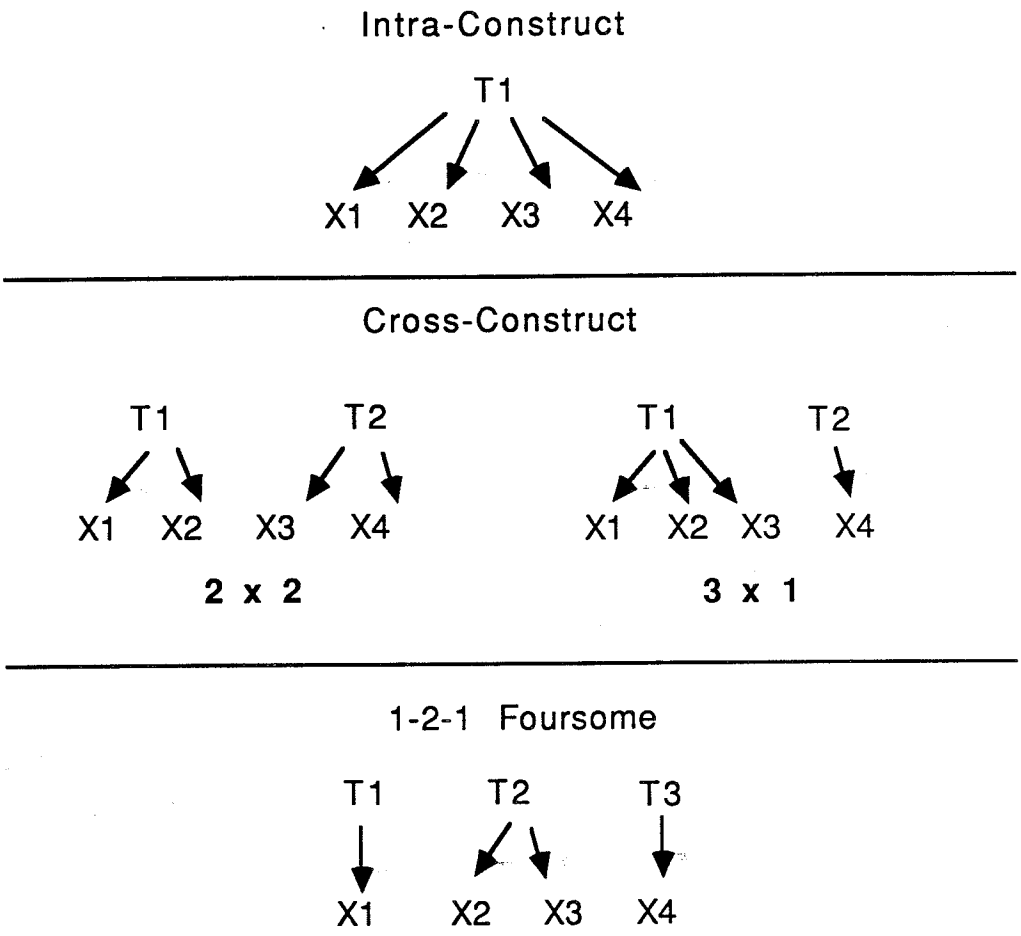

Figure 9

Tetrad equations among intra-construct foursomes identify intra-construct impurities. Those among 2x2 cross-construct foursomes identify cross-construct impurities, and those among 3x1 cross-construct foursomes identify measured variables that are latent-measured impure.

Tetrad Equations and Statistics

Let T_p be a population tetrad difference, say:

$$\rho X1,X2 * \rho X3,X4 - \rho X1,X3 * \rho X2,X4.$$

Let T_S be the corresponding tetrad difference in the sample. Let p(T) = the probability that a sample tetrad difference > T_S, given that $T_p = 0$. We calculate such a probability by using the Wishart statistic[10] for the variance of a tetrad difference under the assumption of multinormally distributed variables. Since the sample distribution of the tetrad difference is asymptotically normal, we can calculate p(T) straightforwardly. In the TETRAD II program, the decision to reject or accept the hypothesis $T_p = 0$ is based on a user set significance level. In what follows, we heuristically assume that the the higher p(T), the more likely that $T_p = 0$. To capture this intuition, we assign a score[11] to each tetrad equation:

> If p(T) > significance level, then Score = Score + p(T),
> else Score = Score - (1 - p(T))

Intra-Construct Foursomes

An intra-construct foursome X1-X4 implies all three tetrad equations:

$$\rho_{X1,X2} * \rho_{X3,X4} = \rho_{X1,X3} * \rho_{X2,X4} = \rho_{X1,X4} * \rho_{X2,X3}$$

if at least three of X1-X4 are intra-construct impure. If a *pair* among X1-X4, say <X1,X2> as in the top right of figure 10, is intra-construct impure, then the tetrad equations involving $\rho_{X1,X2}$ are not implied by the model. Unfortunately, both equations that involve $\rho_{X1,X2}$ also involve $\rho_{X3,X4}$, so we cannot, just from a single intra-construct foursome, distinguish which among two pairs is impure. Thus, in figure 10, we show in the top right the model that generates the data, the tetrad equation that implied by that model, and the two pairs <X1,X2> and <X3,X4> that we can postulate as candidates for intra-construct impure measures.

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[10]See (Glymour 87).

[11]For more detail on the scoring function, see (Spirtes 90a)

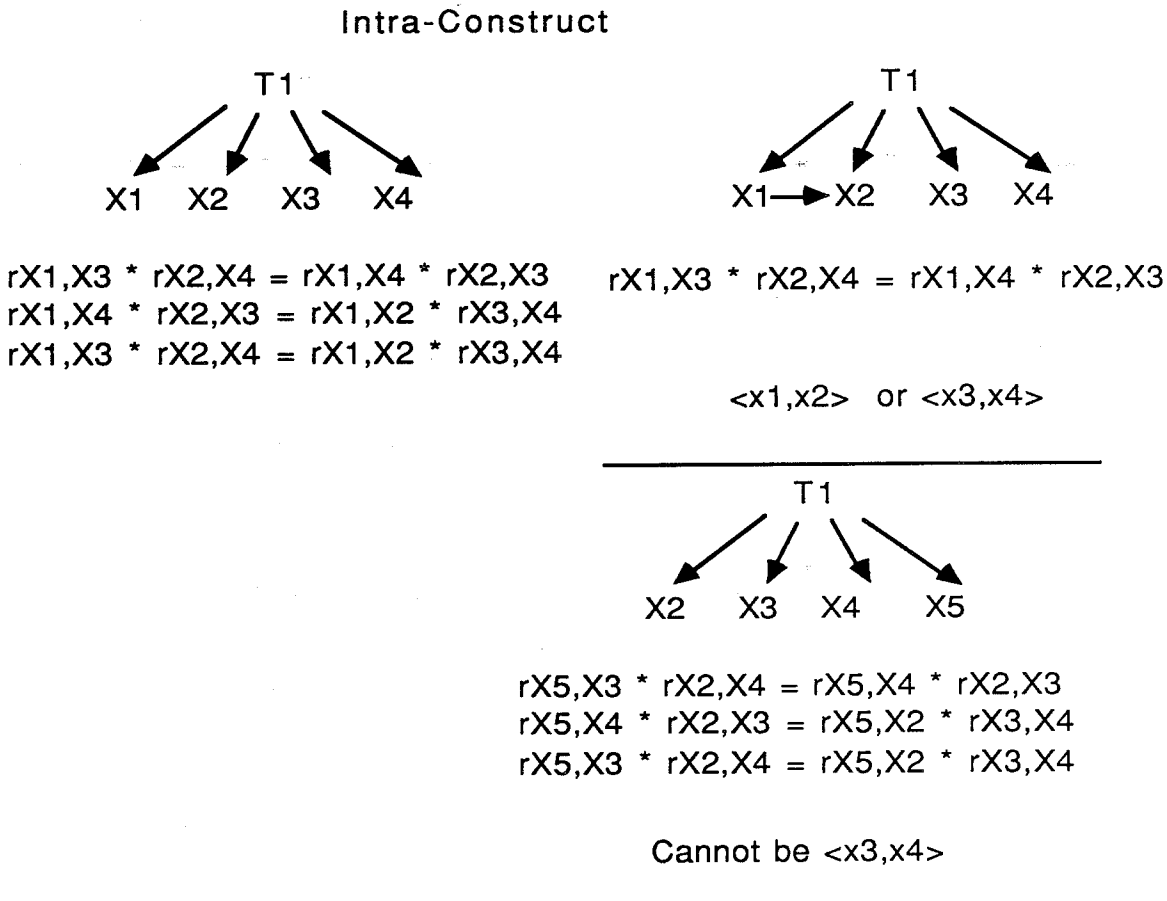


Figure 10

We can distinguish which pair is impure by checking other intra-construct tetrad equations that involve one pair and not the other. For example, if we check the foursome {X2,X3,X4,X5}, then only the pair <X3,X4> is involved. If the two tetrad equations in this foursome involving $\rho_{X3,X4}$ hold, then we cannot eliminate <X3,X4> from consideration. In essence, we use redundancy to overcome the underdetermination unavoidable from single foursomes. The algorithm for eliminating intra-construct impurities is as follows.

Repeat
  For each indicator Vi in $\mathbf{V_{Ti}}$,
    For each intra-construct foursome F involving Vi,
      For each tetrad equation T in F,
        If p(T) > significance level, then Score = Score + p(T),

else $Score_{Vi} = Score_{Vi} - (1 - p(T))$.

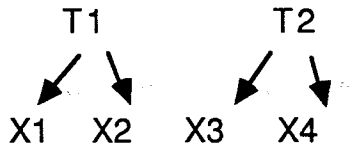
$V_{Ti} := V_{Ti} - \{Vi \text{ with lowest score}\}$.

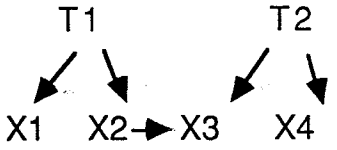
Until $(Min(Score_{Vi}) > \text{User set cutoff})$ Or $|V_{Ti}| <= 4$

Cross-Construct Foursomes

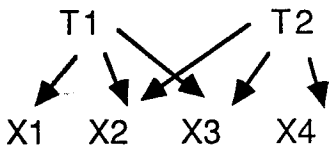
2x2 Foursomes

Since intra-construct foursomes involve only one latent variable, the causal connections among the latent variables has no effect on this part of the algorithm. Cross-construct foursomes involve two latent variables, say T1 and T2. Luckily, the causal structure between T1 and T2 has no effect on the tetrad equations applied among cross-construct foursomes involving T1 and T2.





rX1,X3 * rX2,X4 = rX1,X4 * rX2,X3



<x1,x3> or <x2,x4> or
<x1,x4> or <x2,x3>

Figure 11

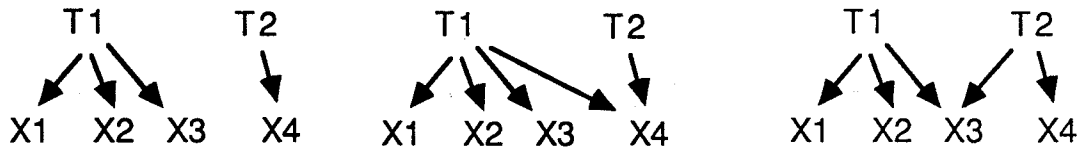
In the left side of figure 11 we show a 2x2 cross-construct foursome from a pure measurement model, and under it the tetrad equation it implies regardless of the nature of the causal connection between T1 and T2. This eqaution is not implied by a model in which any of the four possible cross-construct pairs are impure. On the right side we show two

ways in which one of the cross-construct pairs, <X2,X3>, is impure. Below the models on the right we show what we can learn from this foursome if either of these models is correct. Again, to narrow the blame we look at overlapping foursomes. The algorithm for 2x2 foursomes is as follows.

Repeat
  For each indicator Vi in **V**,
    For each 2x2 cross-construct foursome F involving Vi,
      For each tetrad equation T in F,
        If p(T) > significance level, then Score = Score + p(T),
        else $Score_{Vi}$ = $Score_{Vi}$ - (1 - p(T)).
  V:= **V** - {Vi with lowest score}, provided no $|V_{Ti}|$ <= 3.
Until (Min($Score_{Vi}$) > User set cutoff

3x1 Foursomes

After the 2x2 procedure, we will have eliminated all cross-construct impurities except for a single latent-measured impurity, if there is one. This is because a 2x2 foursome implies one tetrad equation if it is pure or if exactly one of its indicators is latent-measured impure. In the 3x1 cross-construct foursome the situation is better. Whereas the two measurement models on the left side of figure 12 imply all three tetrad equations no matter what the causal connection between T1 and T2, the model on the right implies only one. The difference is that in the middle model the indicator on the 1 side of a 3x1 (the singleton) is latent-measured impure, whereas on the right one of the indicators on the 3 side of a 3x1 is impure. In the latter case the tetrad equations involving a correlation between the impure indicator and the singleton are not implied by the model. Thus if one equation holds, we can uniquely identify the impure indicator.



rX1,X3 * rX2,X4 = rX1,X4 * rX2,X3
rX1,X4 * rX2,X3 = rX1,X2 * rX3,X4
rX1,X3 * rX2,X4 = rX1,X2 * rX3,X4

rX1,X3 * rX2,X4 = rX1,X4 * rX2,X3

<T2 -> x3>

Figure 12

Again, however, we use a strategy of redundancy to detect latent-measured impurities. The algorithm for 3x1 foursomes is as follows.

Repeat
  For each indicator Vi in **V**,
    For each 3x1 cross-construct foursome F involving Vi,
      For each tetrad equation T in F,
        If p(T) > significance level, then Score = Score + p(T),
        else $Score_{Vi}$ = $Score_{Vi}$ - (1 - p(T)).
  V:= V - {Vi with lowest score}, provided no $|V_{Ti}|$ <= 2.
Until (Min($Score_{Vi}$) > User set cutoff

After the 3x1 procedure we have a subset of **V** that constitutes a pure measurement model. We can now search for facts about the causal connection among the latent variables.

3.4 Finding the Structural Model.

If a set of measured variables **V** is causally sufficient, i.e., if every common cause of two variables in **V** is also in **V**, then V1 and V2 (in **V**) are causally adjacent if and only if V1 and V2 are dependent conditional on every subset in **V** that doesn't include V1 or V2. If we begin with an undirected graph in which every pair V1 and V2 is adjacent, as soon as we

find a conditioning set **S** such that V1 and V2 are independent conditional on **S**, we can remove the undirected edge between V1 and V2.

If a pair <V1,V2> is independent conditional on a set **S**, and the cardinality of **S** is n, we say that V1 and V2 are nth-order independent. In the PC-algorithm,[12] the strategy is to begin by removing all edges that connect pairs that are 0-order independent, ascend to sets of size 1, size 2, etc.[13] The current strategy is to use PC on the latent variables, but instead of providing with facts about conditional independence, we provide facts about 0-order and1st-order *trek-separability*, which we can determine in pure latent variable models respectively from the correlations between indicators of distinct latent variables and from the tetrad equations that hold in 1-2-1 foursomes.

3.4.1 Trek Separability.

Let a **path** between vertices X and Y in an graph G be a sequence of edges from X to Y.

Let a **trek** between X and Y be either an acyclic path from X to Y, an acyclic path from Y to X, or a pair of acylic paths from some other variable Z to X and to Y such that these paths intersect only at Z.

V1 and V2 are **nth-order trek-separated** just in case there are n vertices $V_i...V_n \neq V1,V2$, and no set smaller than $V_i...V_n$ such that each trek between V1 and V2 includes some member of $V_i...V_n$.

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[12]See (Spirtes 91).

[13]The number of nth-order independence facts PC needs to check is constrained not only by the edges removed in the n-1th order stage, but by the variables that are neighbors. For details, see (spirtes 91).

V1 and V2 are **0-order trek-separated** just in case there is no trek connecting them.

V1 and V2 are **1st-order trek-separated** just in case there is a vertex V3 such that all treks between V1 and V2 pass through V3.

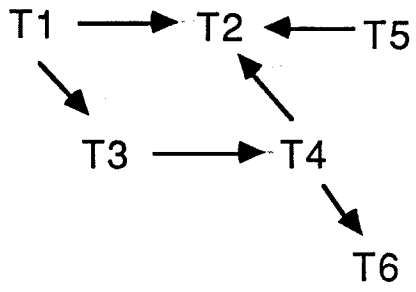


Figure 13

For example, in the model in figure 13 the following trek-separability facts hold.

0-order trek-separated:
{<T1,T5> <T3,T5> <T4,T5> <T6,T5> }

1st-order trek-separted:
{<T1,T4 by T3> <T1,T6 by T4 or T3> <T2,T6 by T4> <T3,T6 by T4> }

2nd-0rder trek-separated:
{ <T2,T3 by {T1,T4}> }

The SCALES program finds a pure latent variable model and with it determines the 0-order and 1st-order trek separability facts. We use its output as input to a slight modification of the PC algorithm:

Latent-PC

Begin with a complete undirected graph G among **T**.

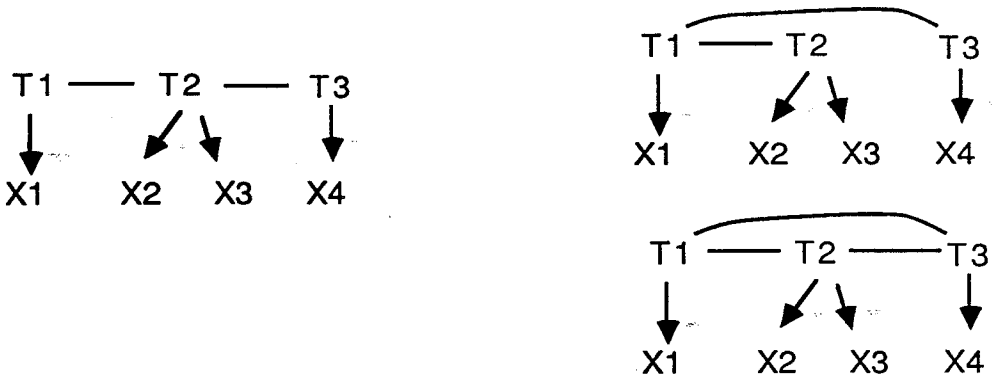
For each pair $<T_i,T_j>$ that are 0-order trek separated, remove the edge Ti-Tj in G.

For each pair T_i,T_j that still have an edge between them in G, remove T_i-T_j are 1st-order trek separated by some T_k, $k \neq i,j$.

For each triple Ti, Tj, Tk, such that Ti-Tj and Tj-Tk and not Ti-Tk, direct Ti->Tj and Tk->Tj if Ti and Tk are 0-order trek separated.

The question now reduces to how we can determine facts about 0-order and 1st-order trek-separability relations. There is an obvious method for determining 0 order trek separability facts among latent variables. Two measured variables are uncorrelated in a pure latent variable model if and only if they are effects of distinct latent variables that are not trek connected. In figure 5, T2 is 0-order trek separated from T4 because $\rho X6\ X13 = \rho X6\ X14 = \rho X7\ X13 = \rho X7\ X14 = 0$. In practice, we consider all the correlation between a indicator from one latent and an indicator from another. If the majority of these correlations are insignificant, then we judge the pair of latents to be 0-order trek separated.

Tetrad equations among 1-2-1 foursomes in a pure measurement model identify 1st order trek-separability relations. For example, on the left side of figure 14 we show a 1-2-1 foursome in which T2 1st-order trek separates T1 and T3. This structure and only this structure implies all three tetrad equations among this foursome. Assuming that T1, T2 and T3 are trek connected, which we have already determined from the 0-order trek separability procedure, any other causal arrangement among the three latents will imply only 1 tetrad equation, as we show on the right side of figure 14.



rX1,X3 * rX2,X4 = rX1,X4 * rX2,X3
rX1,X4 * rX2,X3 = rX1,X2 * rX3,X4
rX1,X3 * rX2,X4 = rX1,X2 * rX3,X4

rX1,X4 * rX2,X3 = rX1,X2 * rX3,X4

Figure 14

In order to determine whether a latent variable T2 1st-order trek separates T1 and T3, we again exploit the redundancy inherent in a multiplie-indicator model.

For each 1-2-1 foursome F in which T2 is the parent of the twosome and T1 and T3 are parents of the singeltons,
For both tetrad equations T in F implied only by a model in which T2 trek-separates T1 and T3,
If p(T) > user set significance level, then Score:= Score + p(T)
else Score:= Score - (1 - p(T)).
If Score > 0, then T2 trek separates T1 and T3.

If, for a given triple of latent variables, we discover that the score for more than one 1st-order trek separation relation is greater than 0, we take only the relation with the largest score to be true, thus we avoid inconistency.

4. Reliability

There are two issues that bear on the full algorithm's reliability. First we can consider PC-latents reliability, i.e. the full algoritym's reliability if the the facts about 0-order and 1st-order trek-separability latent-PC takes as input are correct, and second we can consider its reliability in determing these facts, i.e. the reliability of SCALES.

4.1 Latent-PC Reliability

First suppose that latent-PC receives correct 0-order and 1st-order trek separability facts. Two variables V_i and V_j are adjacent if and only if V_i is a direct cause of V_j or vice versa. The full set of trek-separability facts determine the full set of adjacencies. A subset of the trek-separability facts determine a superset of the adjacencies. Because we can now only determine 0-order and 1st-order trek separability facts, the algorithm can at best identify all the correct adjacencies, but might also identify some that do not exist. It may well be that non-quadratic constraints on correlations determine further higher order trek separability relations among the unmeasured variables, but it is now an open question.

The test for ordering we perform in latent-PC is strictly weaker than the full PC test. Thus, latent-PC might fail to order an edge that the full PC algorithm could order, but it will never order an edge that the full PC algorithm would leave undirected.

For an example of what the latent-PC algorithm could recover with the correct 0-rder and 1st-order trek separability facts, consider figure 15. Let a **pattern** be a graph in which some edges are directed and some are not. Both graphs in figure 15 are displayed as patterns. An edge is undirected in a pattern if the full PC algorithm for ordering cannot order it.[14]

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[14]See (Spirtes 91)

E.g.,

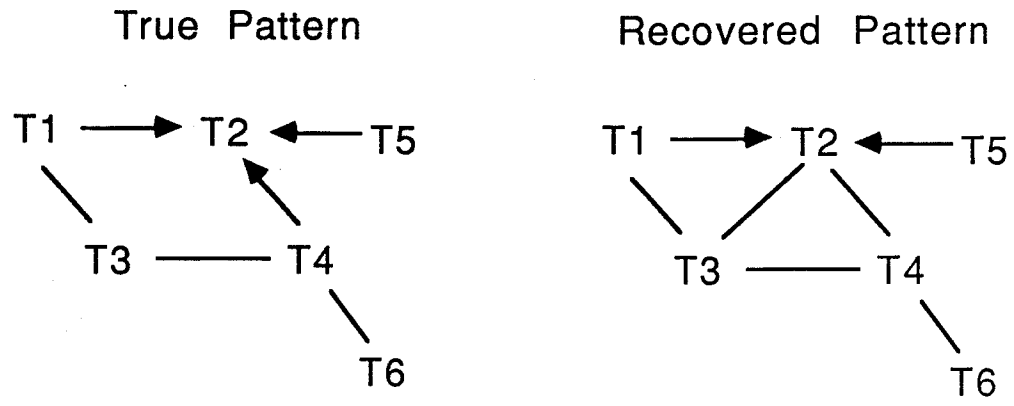


Figure 15

Latent-PC would recover all the adjacencies, but add one that does not exist in the true pattern, T2-T3. It does so because <T2,T3> are 2nd-order trek-separated by {T1,T4}, but latent-PC cannot obtain such information and would thus leave in the edge.

4.2 SCALES Reliability

The reliability of the SCALES procedure is less well understood.

If

1) the statistical assumptions are satisfied, and
2) the clustering information is correct, and
3) there is a measurement model of the true model such that there are at least two pure indicators connected to each latent, and
4) there is no sampling error,
5) no tetrad equations hold in the population that are not implied by the causal structure,

then,

there exist cutoffs such that SCALES will output the correct 0-order and 1st-order trek separability facts.

We do not know exactly what these cutoffs are, and we are currently performing monte carlo simulation tests to optimize them.

5. Complexity.

The algorithm's complexity is determined by the number of tetrad differences it must check, which is determine by how many foursomes of variables there are. If there are n measured variables the total number of foursomes is $O(n^4)$. We don't check each possible foursome, however, and the actual complexity depends on the number of latent variables and how many variables measure each latent. If there are m latent variables and s measured variables for each, then the number of foursomes is $O(m * s^4)$ Under these assumptions, the number of intra-construct foursomes is exactly m*s4. The number of 2x2 foursomes is $s-1 * \binom{s}{2} * m$. The number of 3x1 foursomes is $m* \binom{s}{3} + s*m*\binom{s-1}{2}$. Thus the order of the calculation is $m*s^4$. Since m*s = n, this is $O(n * s^3)$, which is much lower than $O(n^4)$ if $s << n$.

We have implemented the algorithm in Pascal on a Decstation 3100 and tested it on graphs with up to six latent variables and 50 measured variables. It runs in under 20 seconds on graphs of such size, and we are thus confident that it is feasible to construct models with well over 100 measured variables.

6. Simulation Studies

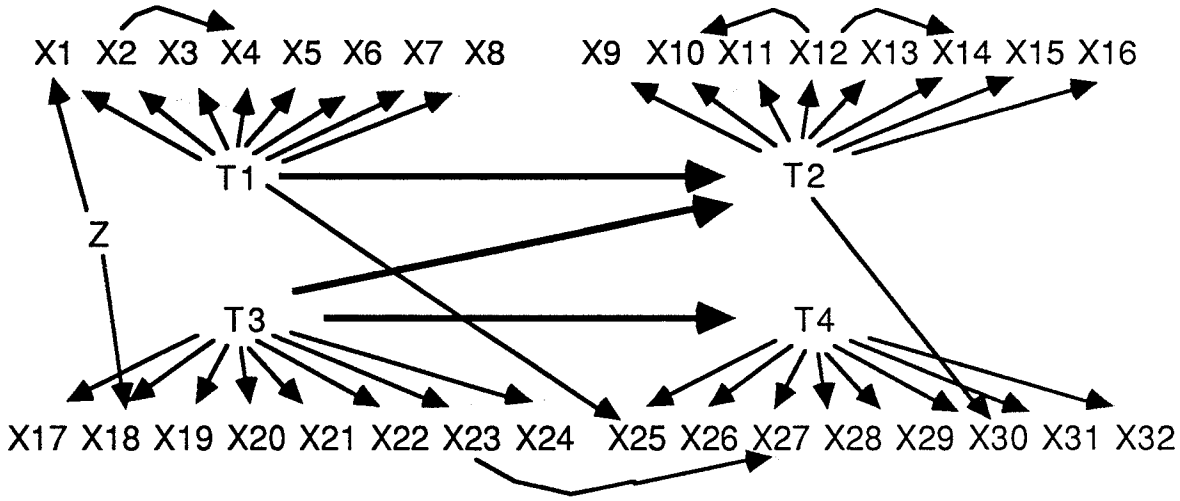
The algorithm has already been used to extract causal information from real data bases. Naval manpower researchers used an earlier version of the algorithm to build latent variable models from a data base that includes more than 200 variables and 7,000 observations. They were able to find several plausible pure latent variable models that passed a statistical goodness of fit test, even on a sample size powerful enough to detect very small failures in any of the assumptions underlying the model.

Pure latent variable models that pass a chi-square test on samples of this size are almost unheard of.

Decisions throughout the SCALES procedure rest on simultaneously testing individual tetrad equations that are not independent. Simultaneous inference is a well known statistical problem, but solutions to it that we might apply are still forthcoming. Our best evidence about its reliability on realistic data can at present only come from Monte Carlo studies.

We conducted simulation studies on the causal graph in figure 9, which has 10 impure indicators out of a possible 32.

True Causal Graph



Impure Indicators = {x1, x2, x4, x10, x12, x14, x18, x23, x25, x27, x30}

Figure 16

We set the distribution for the exogenous variables to be standard multivariate normal (all variables have mean 0 and variance 1), and chose the linear coefficients randomly to be inbetween .5 and 1.5. We generated data from this model for each unit in the sample by psuedo-randomly sampling to produce values for the exogenous variables, and then by using

the linear equations to fill in the values for the rest of the variables. We recorded only data for the measured variables, i.e. those that begin with X.

We conducted 20 trials at sample sizes of 100, 500, and 2000. We counted errors of commission and errors of ommission for 0 order and 1st order trek separability. In each case we counted how many errors the procedure could have made and how many it actually made. We also give the number of samples in which the algorithm identified the trek separability facts perfectly. At sample size 2,000, the algorithm was literally perfect.

Sample Size	0 order		1st Order		Number
	Commission	Omission	Commission	Omission	Perfect
100	2/80	0/40	7/220	1/20	13/20
500	1/80	0/40	2/220	0/20	19/20
2000	0	0	0	0	20/20

In a more substantial study, we used the base model we show in figure 17, and added causal connections to it randomly that make indicators impure.

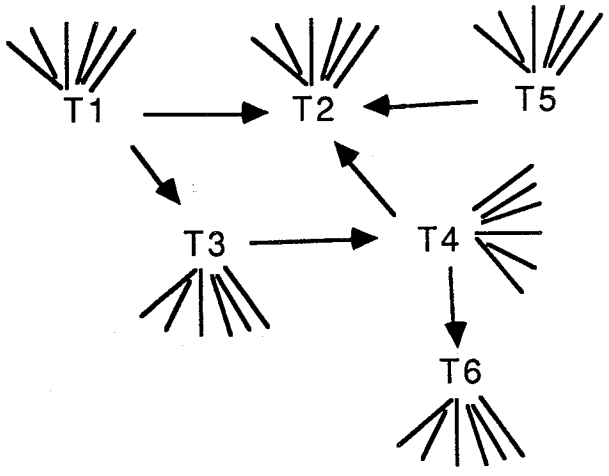
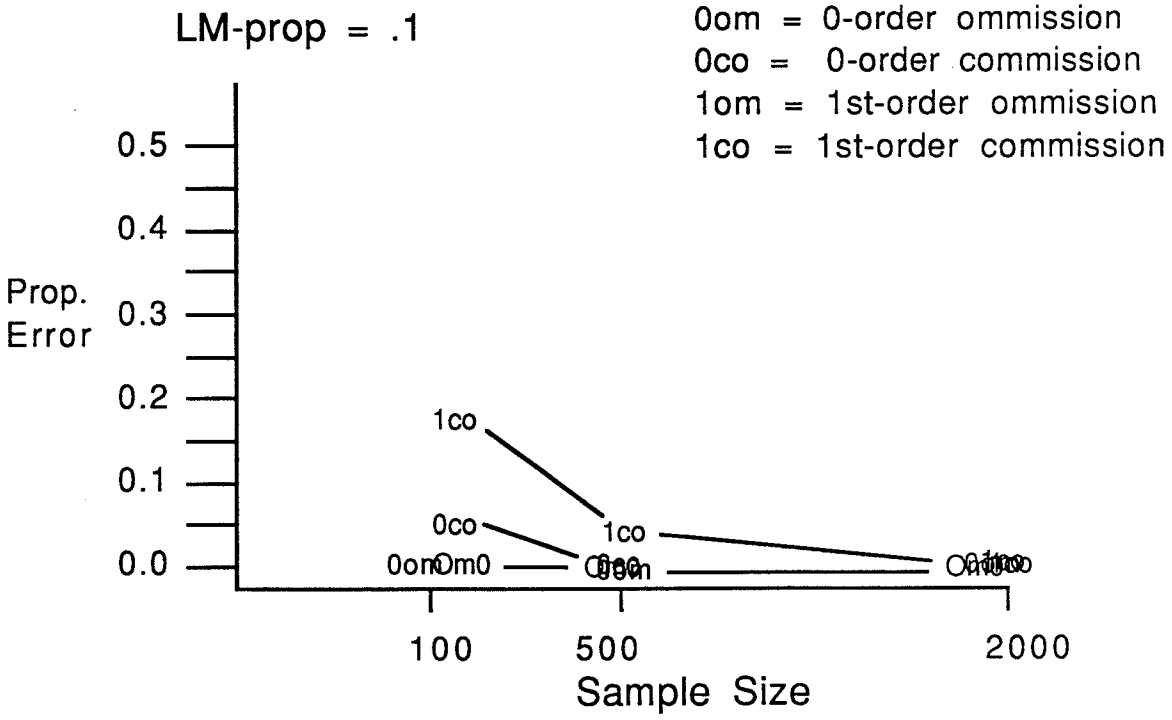


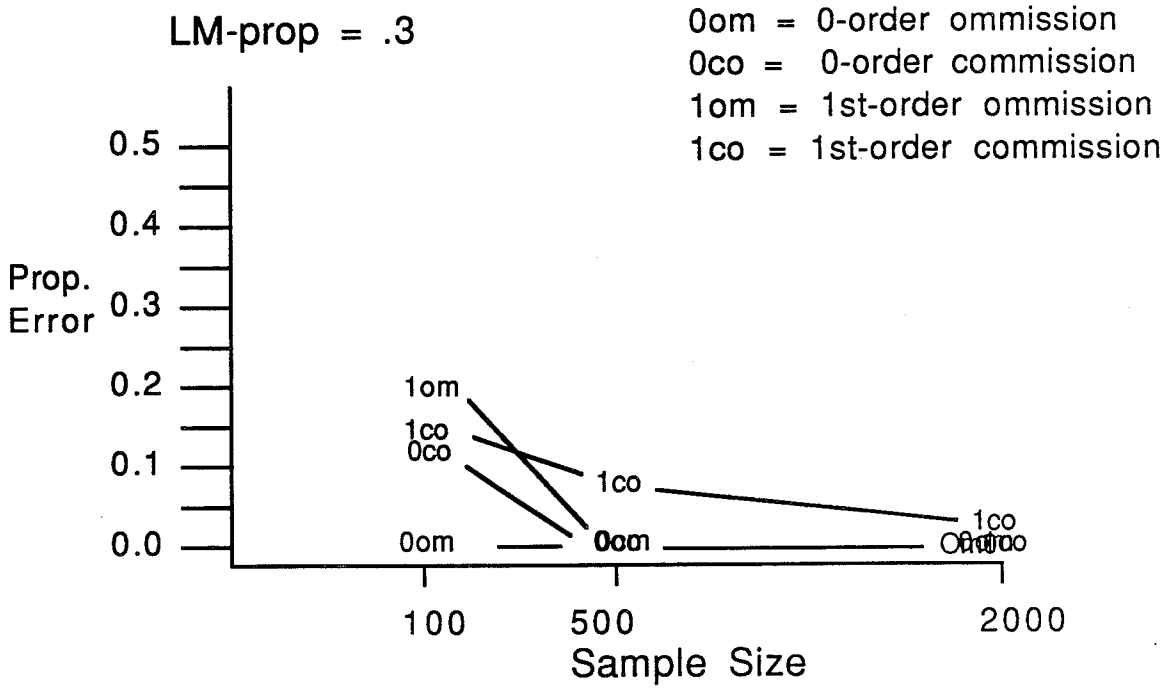
Figure 17

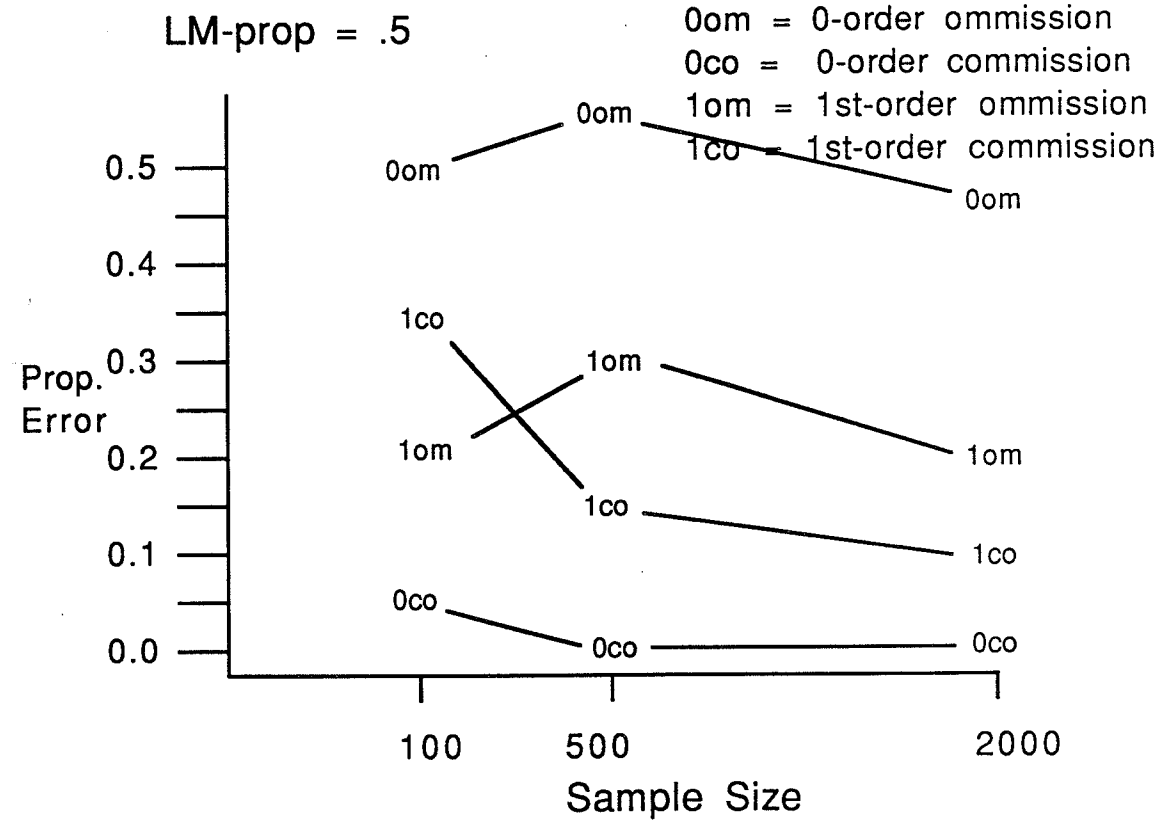
We varied three parameters, the proportion of indicators are latent-measured impure (lm-prop), the proportion of indicators that are measured-measured impure (mm-prop), and the sample size. For this study we kept the cutoff fixed arbitrarily at .25. In the first study we kept mm-prop at 0 and varied only the lm-prop and the sample size. We show the results graphically below. The behavior of the algorithm is very reliable at sample sizes above 500 and at levels of impurity below .5.

Study 1

1) LM-prop = (.1 .3 .5)
2) MM-prop = 0
3) Sample size = (100 500 2000)
4) Cutoff criterion = .25



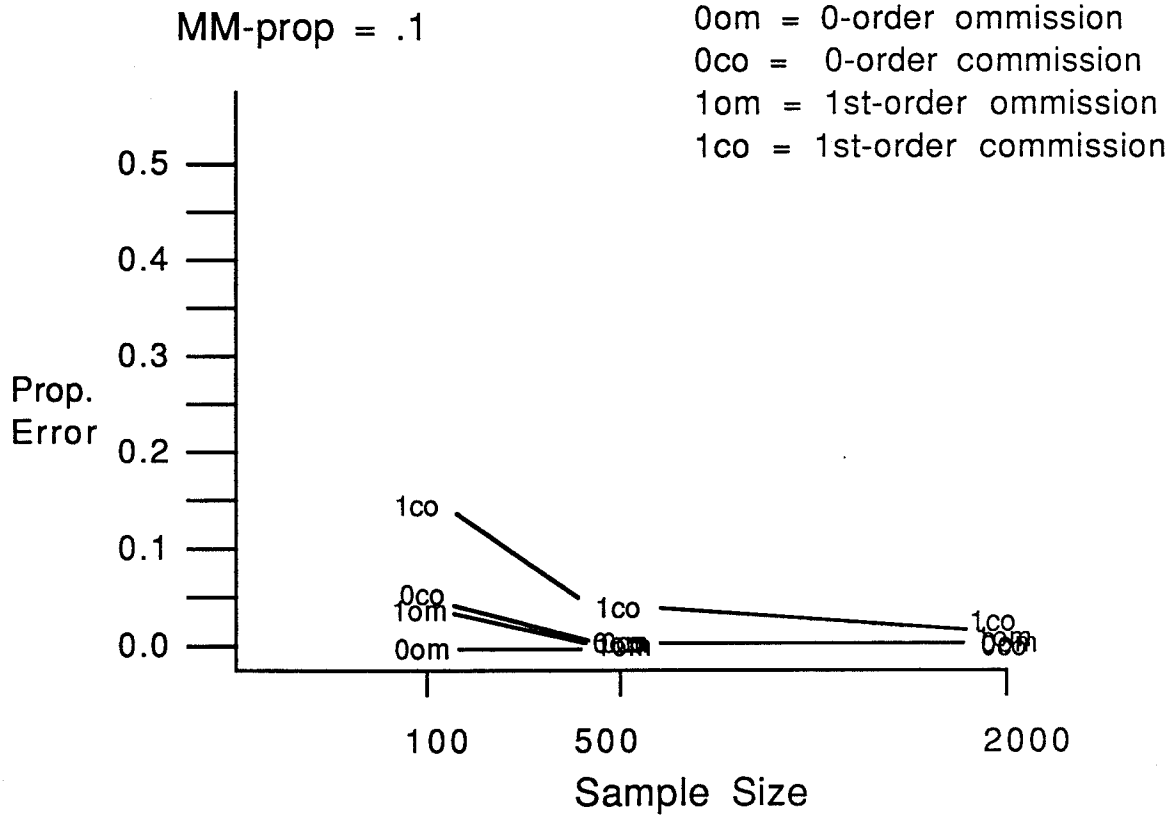

LM-prop = .3
0om = 0-order ommission
0co = 0-order commission
1om = 1st-order ommission
1co = 1st-order commission
Prop. Error
0.5
0.4
0.3
0.2
0.1
0.0
1om
1co
0co
1co
1co
0om
100
500
2000
Sample Size

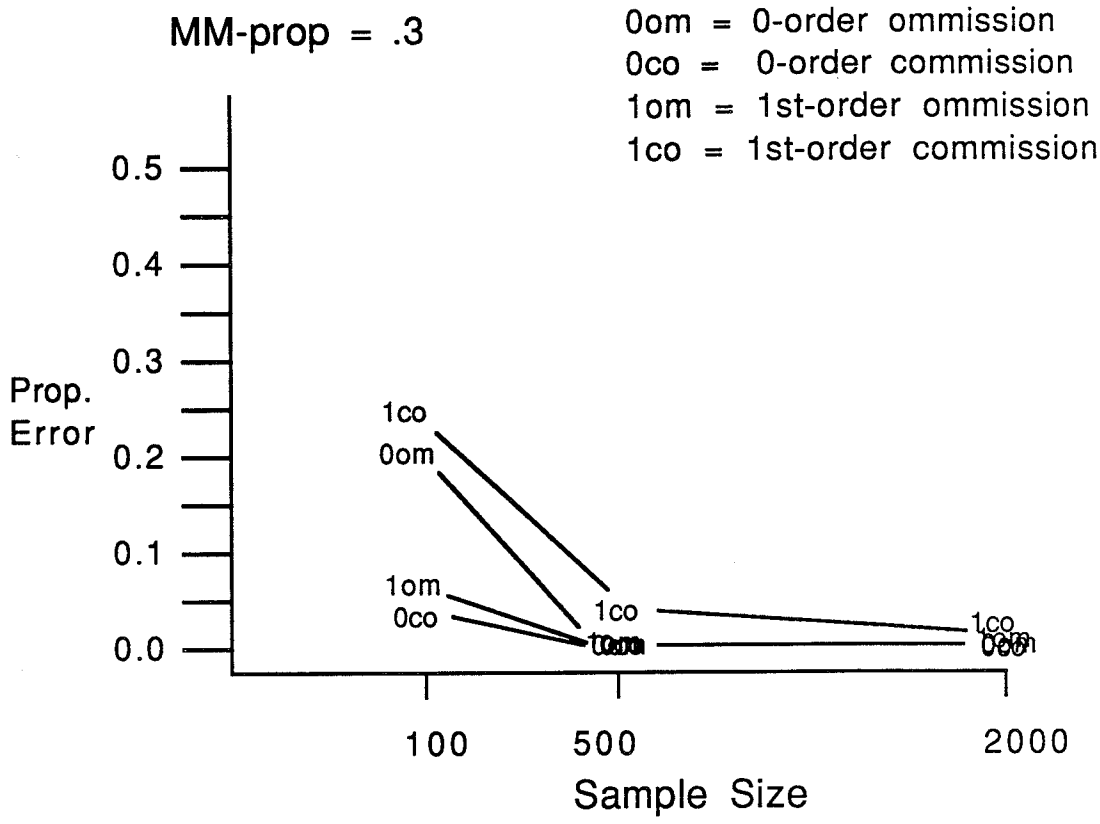

LM-prop = .5
0om = 0-order ommission
0co = 0-order commission
1om = 1st-order ommission
1co = 1st-order commission
Prop. Error
0.5
0.4
0.3
0.2
0.1
0.0
0om
0om
0om
1co
1om
1om
1om
1co
1co
0co
0co
0co
100
500
2000
Sample Size

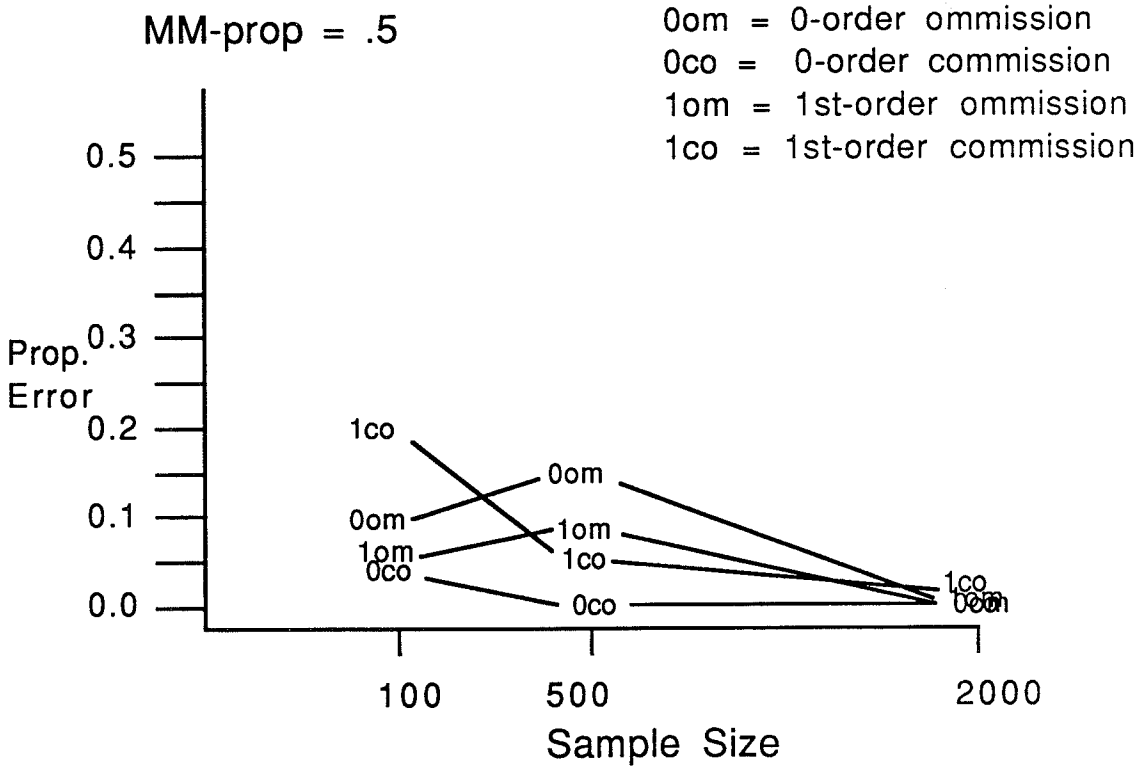
Study 2

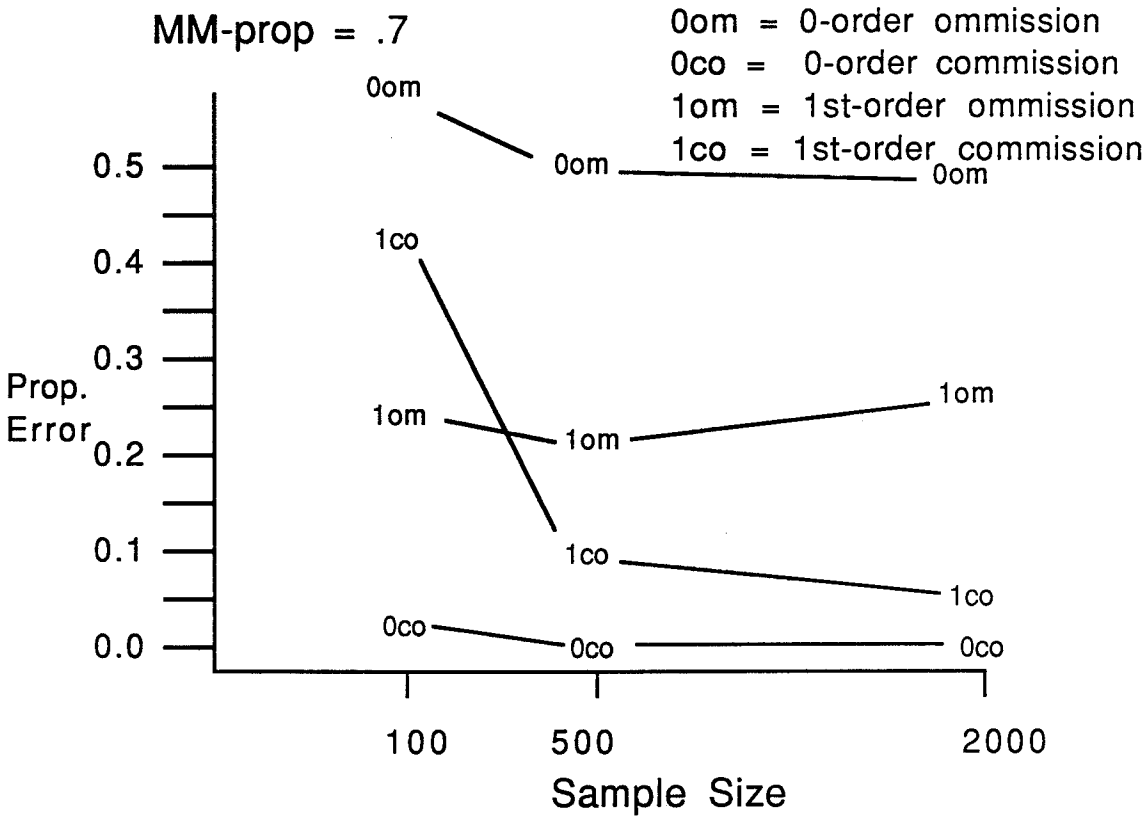
In the second study we kept lm-prop at 0 and varied mm-prop and the sample size. Again, the performance is quite good at sample sizes over 500 and at levels of mm-prop impurity below .5.

1) LM-prop = 0
2) MM-prop = (.1 .3 .5 .7)
3) Sample size = (100 500 2000)
4) Cutoff criterion = .25




MM-prop = .3
0om = 0-order ommission
0co = 0-order commission
1om = 1st-order ommission
1co = 1st-order commission
Prop. Error
0.5
0.4
0.3
0.2
0.1
0.0
1co
0om
1om
0co
100
500
2000
Sample Size


MM-prop = .5
0om = 0-order ommission
0co = 0-order commission
1om = 1st-order ommission
1co = 1st-order commission
Prop. Error
0.5
0.4
0.3
0.2
0.1
0.0
1co
0om
1om
0co
100
500
2000
Sample Size


MM-prop = .7
0om = 0-order ommission
0co = 0-order commission
1om = 1st-order ommission
1co = 1st-order commission
0om
0om
0om
1co
1om
1om
1om
1co
1co
0co
0co
0co
0.5
0.4
0.3
0.2
0.1
0.0
Prop.
Error
100
500
2000
Sample Size

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