The Turning Point and the Revolution: Philosophy of Mathematics in Logical Empiricism from *Tractatus to Logical Syntax*

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January 24, 2005

Technical Report No. CMU-PHIL-165

Philosophy

Methodology

Logic

Carnegie Mellon

Pittsburgh, Pennsylvania 15213

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Philosophy of Mathematics in Logical Empiricism from *Tractatus* to *Logical Syntax*

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> *... I am convinced that we now find ourselves at an altogether decisive turning point in philosophy. And that we are objectively justified in considering that an end has come to the fruitless conflict of systems.*
> *... Bertrand Russell and Gottlob Frege have opened up important stretches in the last decades, but Ludwig Wittgenstein (in his* Tractatus-Logico-Philosophicus, *1922) is the first to have pushed forward to the decisive turning point.*
>
> *Moritz Schlick, 1930*[1]

Its philosophy of logic and mathematics was what most characteristically distinguished logical empiricism from previous forms of empiricism or positivism. This is the aspect that gave it the name *logical* empiricism, and gave it the hope of succeeding where the nineteenth-century attempts at a scientific empiricism by such figures as Comte, Mill, and Mach had failed. The inability of these thinkers to supply a plausible account of mathematics had undermined the claim of empiricism to be the philosophical approach most adequate to modern science. It was generally agreed that Mill's empiricist account of arithmetic had not succeeded. He had portrayed the truths of arithmetic as empirical generalizations, obtained inductively from repeated experience of counting. Mach had offered a similar account of geometrical proof. Common to these nineteenth-century empiricist approaches to mathematics had been their *psychologism*, their classification of mathematical truth as a kind of empirical truth about mental processes.

From a present-day vantage point, Frege's critique of Mill and of psychologism more generally looks devastating. It is hard for us to imagine how, after that, empiricist accounts of mathematics could still have been taken seriously. But what was still missing was a *positive* account of mathematics to replace the empiricist one. The positive accounts of mathematics offered by Frege and then, more notoriously, by Russell, had not seemed persuasive to

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[1] From "Die Wende der Philosophie" ("The Turning Point in Philosophy"), which opened the first issue of *Erkenntnis*.

empiricists. Nineteenth-century mathematicians had persuasively reduced analysis to the arithmetic of the natural numbers, while Frege and Russell had reduced arithmetic to logic, and gone a long way towards showing that there are no specifically mathematical concepts or entities that are not already present in our most basic tools of thought. But for scientific empiricists, this had only pushed the problem of the status of mathematics *down* a level; it now became a problem of the status of *logic*. And on that score, neither Frege nor Russell had been helpful. They had taken opposite approaches to this problem, both still remaining within the Kantian framework. Frege had extended to arithmetic the *analytic* status of logic. Geometry remained synthetic, in his view, but he had nothing useful to say about the nature of analytic, logical truth itself, or the resulting difference between arithmetic and geometry. While Russell, accepting Kant's view of mathematics as *synthetic a priori*, took its reduction to logic as extending that status downwards to the laws of logic themselves. This was even harder for empiricists to swallow.

Moritz Schlick, for one, did not accept it, before he came to Vienna in 1922. He adhered, rather, to a pscyhologistic view along the lines of Mill or Mach. Unlike Kant, he thought the truths of arithmetic to be *analytic*.[2] And analytic sentences, he writes in one of his early papers, are *verified* by the immediate *perception* (in inner sense) that the subject and predicate (or the expressions on both sides of an equality) have the same meaning.[3] Schlick specifically rejects Russell's view (which he called the "independence theory of truth") that analytic sentences, including those of mathematics, are true *independently* of such mental verification:

> An act of judgement and the logical sentence [expressing it] are not completely separate things; above all, the logical sentence — including its truth — is nowhere to be found independently of the act of judgement. Indeed it is contained in that act and arises from it by abstraction. The logical is contained in the real act of judgement not because of a fortuitous accident by which a truth happens to be manifested [in judgement] that also

[2] He had apparently not heard of Frege at this point; in any case, Frege's logicism (unlike Russell's) still left geometry aside as a special case. For Frege, geometry retained its Kantian status of synthetic a priori, which would have been unsatisfactory to Schlick, who accepted Helmholtz's view that the question whether physical space is Euclidean or non-Euclidean was an empirical one.

[3] Schlick, M. "Das Wesen der Wahrheit nach der modernen Logik", *Vierteljahrsschrift für wissenschaftliche Philosophie und Soziologie* 34 (1910), pp. 386-477, specifically pp. 441-3.

> maintains some separate existence. The case is rather as follows: the logical meaning has its location only *in* the psychic experience and in no way exists outside it. The two are just plain impossible to separate; the judgement as a logical figure, . . with its timeless character, comes into being simply by abstracting everything individual and temporal from the real act of judgement. . . When we subtract everything that is a psychological product from our representation [*Vorstellung*] of the number 2, *nothing* is left.[4]

There could hardly be a more uncompromising statement of psychologism; the number two is *entirely* accounted for as a "psychological product". When we take that away, "*nothing* is left". Schlick's view changed significantly in the decade or so before he came to Vienna. Under the influence of Hilbert, he came to accept that axiomatically ("implicitly") defined concepts (with no psychological roots at all) play a central role in theoretical science. But he was never able to relate these axiomatic concepts convincingly to empirical concepts, and made no progress beyond Mill or Mach on the critical problem of specifying the *status* of mathematics in a way that was compatible with scientific empiricism.

Against this background it is easier to appreciate what a revelation Wittgenstein's *Tractatus* was for the Vienna Circle. It solved what had been, in their eyes, the fundamental problem of empiricism. For them, this was the "turning point in philosophy", and during the 1920s their philosophy of mathematics was rooted in that of the *Tractatus*. We describe this view in section I below. But in fact the Circle was aware from the start that this view required some drastic surgery to be useful for their purposes, and they discussed these problems, which are the subject of section II below, very openly. The logical empiricists were unable to move beyond their original framework in any fundamental way, though, until 1931, when Carnap showed the way with his new ideas that were eventually formulated as the "syntax" program in 1932-34. This transition came in two steps, which we describe in sections III and IV. These post-1931 developments were presented in a rather technical form, however, and were published just at the time that the European logical empiricists were scattered to the winds by the Nazis. So the "syntax" idea and its successors did not achieve the widespread notoriety of the earlier,

[4] Schlick "Wesen der Wahrheit", p. 405.

Wittgensteinian approach, and was often misunderstood. It was, we conclude by emphasizing (against still widespread prejudice to the contrary), a fundamental break with that earlier view.[5]

I. The *Tractatus* Conception as Turning Point

The nineteenth-century empiricists had rejected Kant's account of mathematics as synthetic a priori, for it implied that some non-trivial knowledge is not empirical. A consistent empiricism, they agreed, required the reduction of all genuine knowledge to observable facts. Kant had thought that mathematics is an exception to this. He had not really given any persuasive arguments for this view, but had essentially taken it for granted as evident. He was one of the first to notice, though, that classical Aristotelian syllogistic logic could not account for all the reasoning processes in mathematics, which must therefore, he thought, be synthetic. By this he meant that the sentences of arithmetic and geometry predicate more of their subjects than could be found in the subjects themselves; 5 + 7 = 12, he claimed, was to assert something about 5 and 7 that we could not know just by considering these two individual items in isolation. Lest one be tempted to judge Kant's philosophy by this apparently rather feeble example, he is on much stronger ground in geometry, where it really seems almost miraculous that we should be able to extract something as un-obvious as the Pythagorean theorem from the apparently obvious axioms. There must, therefore, he thought, be something in the axioms that, while obvious to us intuitively, actually goes beyond anything directly observable; the axioms, he said, are built into our perceptual system. It is impossible for us to doubt them. And the *reasoning process* of mathematics, he thought, must go beyond mere logic. Our intellects are evidently capable of a

[5] A full history of logical empiricist philosophy of mathematics would have to describe many by-ways we have to omit here. We focus largely on the Wittgensteinian "turning point" that inspired the Vienna Circle's early doctrine, and then Carnap's efforts first to work *within* that framework, later to overcome it. Our reason for this focus is that Carnap's development was what led to the "revolution" we describe in parts III and IV, which is widely regarded as the most innovative development within logical empiricist philosophy of mathematics. More foundationally oriented readers will prefer more of an emphasis on Felix Kaufmann's phenomenologically-oriented intuitionism, Hans Hahn's logicism, Schlick's notion of implicit definition, and Karl Menger's attempts to mediate between Brouwer and the logicist ideas prevalent in the Circle. Gödel, who does play a role in our story, may well have been influenced by some of these currents in the later formulation of his philosophical ideas. We omit them only in the interest of telling a coherent story in a limited space.

kind of *synthesis* that goes beyond what the trivial mechanical procedures of mere logic (by which he meant Aristotelian logic) can achieve on their own.

Nineteenth-century empiricists had a two-fold task, then, in responding to Kant's philosophy of mathematics. First, they had to show that all the *axioms* were empirical. And second, they had to show that all mathematical *reasoning processes* were not dependent on the human mind (i.e. that they were just as mechanical as traditional Aristotelian logic). The first of these tasks, they felt, had essentially been achieved by the 1870s, mainly in the work of the German physicist and physiologist Hermann von Helmholtz. The second task was a little more complicated. Certainly Frege had taken the first step in 1879, when he envisaged (and began to spell out) a purely formal or mechanical system of logic that, unlike Aristotelian logic, was fully capable of accounting exhaustively for every step in all the reasoning processes of both arithmetic and geometry (the *axioms* of geometry, for Frege, remained synthetic a priori). But mathematics was developing very rapidly during this period. By the beginning of the twentieth century Georg Cantor's set-theoretic apparatus for infinite numbers had become well established, and raised serious questions for logicism. Ernst Zermelo had responded, on behalf of the Hilbert school, with an axiomatization of set theory, which eventuated in a clear distinction, unavailable to Frege, between logic and set theory. Russell, on the other hand, had responded with his theory of types, which still made the reduction of mathematics to logic possible, at the cost of a very small number of axioms of uncertain status.[6] Russell himself, followed by certain well-known set theorists such as Fraenkel (and by the young Carnap), had argued that these axioms could (or should) be regarded as "logical", thus rescuing his reduction of mathematics to logic, as sketched in his *Principles of Mathematics* (1902) and partly carried out in his and Whitehead's *Principia Mathematica* (1910-12).

This was the situation when the *Tractatus* was published in 1922. Wittgenstein rejected Russell's arguments for the logical status of the questionable axioms, but accepted both Frege's conception of logic as a purely syntactic, mechanical system *and* Frege's account of

[6] Such as the axioms of infinity, choice, and reducibility.

mathematical reasoning as purely logical. Far more important to the Vienna Circle than these particular arguments, though, was Wittgenstein's response to Frege's and Russell's "universalist" conception of logic. Though Frege and Russell had, as we saw, taken opposite philosophical approaches to the assimilation of mathematics to logic (Frege making arithmetic analytic, while Russell at least initially made logic synthetic), they shared with Aristotle, Kant, and many other philosophers the view that the laws of logic (whatever their status, analytic or synthetic) were laws of *everything*; they governed all being — physical, mental, and otherwise. They were, according to Russell, like the laws of physics, only more general. So even if the logicist reduction of all mathematics to logic could be made to work, the *status* of this logic-and-mathematics was not really compatible, before Wittgenstein, with an empiricist view.

Wittgenstein himself was not an empiricist, but his account of logic broke with Frege's and Russell's universalism, and put forward a radically different conception of the nature of logic. Logical laws were not about anything extra-linguistic, in this conception. They were not laws of everything, pertaining to a universe of objects and expressed transparently in language along with other sorts of facts. Instead, language itself was regarded as a *medium*, as not part of the world but as representing the world to us. Though imperfect, it was still, as in Frege and Russell, a universal medium. It represented the world to us, Wittgenstein thought, by means of logical pictures that were somehow isomorphic to facts in the world. Complex facts were represented by compound sentences (pictures) constructed from sentences representing simple facts by the logical truth functions ("and", "or", "not", "if-then") which are completely determined by their truth tables. Language was governed by a system of rules, then, connected to the world by this picture theory of meaning.[7] The truths of logic were a by-product of this representational function of language. They ceased to be part of what languages describes, and became instead an artifact of the representational capacity of language. As such, logical truths became

[7] The degree to which each component of this view represents Wittgenstein's own, as opposed to one he puts forward in the early parts of the *Tractatus* only dialectically, so that we can better appreciate its self-undermining character in the later parts, is controversial; see T. Ricketts, "Pictures, Logic, and the Limits of Sense in Wittgenstein's *Tractatus*", in H. Sluga and D.G. Stern, eds. *The Cambridge Companion to Wittgenstein* (Cambridge: Cambridge University Press 1996), pp. 59-99, esp. pp. 88-94). Our discussion of the Vienna Circle's ideas is independent of this question.

tautological and empty, despite still being universal.[8] Hans Hahn, like the rest of the Vienna Circle, thought this idea of critical importance:

> If one wants to regard logic — as this has in fact been done — as the study of the most general qualities of objects, as the study of objects in general [*überhaupt*], then empiricism would in fact be confronted here with an impassable hurdle. In reality, though, logic says nothing whatever about objects. Logic is not something that is to be found in the world. Logic only arises, rather, when — by means of a symbolism — we *speak about the world*. . . The sentences of logic say nothing about the world.[9]

This was Schlick's "turning point", which so impressed and inspired the Vienna Circle. Apart from Russell himself, they were the first careful readers of the *Tractatus*. Schlick was called to the chair in the philosophy of the inductive sciences at the University of Vienna in 1922, the same year as the *Tractatus* first appeared in book form. Soon after his arrival, the first of several close, line-by-line readings of the text in the Schlick circle's weekly meetings began. By 1926, when Carnap moved to Vienna, Schlick and his assistants Feigl and Waismann were meeting personally with Wittgenstein and reporting on these conversations to the full Circle. Soon after Carnap's arrival another line-by-line reading of the *Tractatus* began, in the Circle, and eventually Carnap also joined the personal meetings with Wittgenstein — until Wittgenstein excluded him in 1929. But from about that time until Wittgenstein's departure for Cambridge, Waismann took careful notes of the conversations so as to report the master's words back to the Circle more accurately.[10]

This fascination with the *Tractatus* derives from the Circle's perception that Wittgenstein had finally solved the age-old Platonic problem of the cognitive status of logic (and thus, in their minds, mathematics), making it safe for empiricism. "It really does seem on first sight as if the very existence of mathematics must mean the failure of pure empiricism — as if we had in mathematics a knowledge about the world that doesn't come from experience, as if we had a

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[8] See the exposition by Ricketts, "Pictures", pp. 59-64.
[9] H. Hahn, "Empirismus, Mathematik, Logik" (1929), repr. in H. Hahn *Empirismus, Logik, Mathematik* (Frankfurt/Main: Suhrkamp 1988), pp. 55-65, this passage p. 56.
[10] This later became a book, *Wttgenstein und der Wiener Kreis*, edited by Friedrich Waismann (Frankfurt/Main: Suhrkamp 1967).

priori knowledge," Hans Hahn had said. "And this evident difficulty for empiricism is so plain, so brutal, that anyone who wants to hold a consistent empiricism has to face this difficulty. . .".[11] Wittgenstein had solved this problem, in the Circle's view. "Wittgenstein's book exerted a strong influence upon our Circle," Carnap later said, "we learned much by our discussions of the book, and accepted many views as far as we could assimilate them to our basic conceptions".[12] "The thinking of our Circle was strongly influenced by Wittgenstein's ideas, first because of our common reading of the *Tractatus* and later by virtue of Waismann's systematic exposition of certain conceptions of Wittgenstein's on the basis of his talks with him."[13] Carnap specifically included himself in these statements: "For me personally, Wittgenstein was perhaps the philosopher who, besides Russsell and Frege, had the greatest influence on my thinking."[14]

It is generally now taken for granted by many Wittgenstein scholars that, despite this very thorough involvement with the text of the *Tractatus* and with Wittgenstein personally, the Circle fundamentally misunderstood some of Wittgenstein's intentions. But it should be kept in mind that their priority was to *apply* Wittgenstein's ideas to their overriding project of a consistent empiricism that could account for mathematics and mathematical science, not to remain faithful to Wittgenstein's own intentions.

II. Problems with the *Tractatus* Conception

The Vienna Circe's Wittgensteinian solution to the old problems carried a high price tag. Linguistic representation was limited to truth-functions of "atomic sentences", which the Vienna Circle (and Wittgenstein as well, at least at certain points during the late 1920s and early

[11] Hahn, "Empirismus", pp. 55-6.
[12] R. Carnap "Intellectual Autobiography" in P. Schilpp, ed. *The Philosophy of Rudolf Carnap* (LaSalle, IL: Open Court 1963), p. 24.
[13] Carnap "Intellectual Autobiography", p. 28.
[14] Carnap "Intellectual Autobiography", p. 25.

1930s[15]) thought of as simplest observation statements, along the lines of Mach's "elements" (or the "elementary experiences" of Carnap's *Aufbau*). This created two headaches for the Vienna Circle. First, not even a fragment of actually existing science could be expressed as finitary truth functions of simple observation statements. Indeed, it seemed that most theoretical science could not be expressed, or had no meaning, within the *Tractatus* framework.[16] The second problem was perhaps not Wittgenstein's own, but was suggested by Wittgenstein's emphasis on the impossibility of representing the representation relation itself in language. This conception seemed to exclude the possibility of meta-linguistic discourse. This problem was much discussed in the Vienna Circle in 1930-31. Gödel, for instance, is recorded as asking on one occasion "how the discussion of logical questions could be justified, as it involves the utterance not of any meaningful sentences but only of elucidations [*Erläuterungen*]. This raises the question how admissable elucidations are to be demarcated from metaphysical pseudo-sentences."[17] Hardly a sentence in the *Tractatus* itself (or any of the Vienna Circle publications) could reasonably be construed as a truth function of atomic sentences. And Wittgenstein himself had made this self-undermining conclusion explicit in the final sentences of his book.

These consequences of the *Tractatus* were unacceptable to the Vienna Circle because they conflicted with their central project of rational reconstruction. If much of existing theoretical science fails to qualify as meaningful, and discourse about language is excluded in any case, then it becomes impossible even to compare different expressions regarding their precision or their usefulness for some practical purpose. It becomes impossible to say, for instance, that a rationally reconstructed concept is *more precise*, or *more useful*, than the concept to be reconstructed. This obstructs the Vienna Circle's practical critique of metaphysics and unclear

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[15] In a conversation of 1930-31, for instance, he says that "object" in the *Tractatus* is "used for such things as a colour, a point in visual space, etc." (D. Lee, ed. *Wittgenstein's Lectures, Cambridge 1930-1932, From the Notes of John King and Desmond Lee* (Chicago: University of Chicago Press 1980), p. 120.).

[16] If a scientific theory is a truth function of observation sentences, then it *can* only be a statement about a finite number of instances, not a universal law. This was why the picture theory, combined with the Circle's empiricism, made theoretical science as ordinarily conceived impossible.

[17] ASP/RC 081-07-11, reprinted in F. Stadler *Studien zum Wiener Kreis: Ursprung, Entwicklung und Wirkung des logischen Empirismus im Kontext* (Frankfurt/Main: Suhrkamp 1997), p. 288.

thinking, and undermines its entire Enlightenment project. It was imperative, therefore, to address these two problems.

Though the Circle discussed these problems in Wittgensteinian terms, they approached them with their own agendas firmly in mind. The constitution system of Carnap's *Aufbau*, in particular, had left a good deal of unfinished business. It had been criticized (by Reichenbach and Eino Kaila[18]) for failing to accommodate certain modes of inference required in actual science, such as empirical induction, probability, and statistical inference.

An even more serious problem was raised by axiomatic systems. The explicit definitions in which Carnap had (nominally, at least) constructed the whole of knowledge in the *Aufbau* could not accommodate "implicit definitions" of concepts in axiomatic systems. Schlick, influenced by Hilbert, had made such implicit definitions central to his treatise *General Theory of Knowledge* (1918; second edition 1925). He had contrasted them with ordinary ostensive or "concrete" definitions based on psychological abstraction, which he portrayed as the traditional form of concept formation in science. Only with the recent domination of mathematical physics (especially in the theory of relativity), he said, had axiomatic systems come to dominate the exact sciences. The problem, though, was that while concretely defined concepts had a clearly identifiable empirical content, implicitly defined concepts did not:

> . . . an implicit definition has nothing in common with reality, no connection with it; implicit definition rejects such connection purposely, and in principle; it sticks to the realm of concepts. A system of truths created with the aid of implicit definition rests nowhere on the foundation of reality, but rather floats freely. . . In general, we deal with the abstract only to apply it to the concrete. But — and this is the point to which our investigation returns again and again — the moment a conceptual relation is applied to a concrete example, exact rigor is no longer guaranteed. Given real objects, how can we

[18] Kaila's critique (*Der logische Neupositivismus: Eine kritische Studie* (Turku: Annales Universitatis Aboensis 1930) was the first book-length assessment of the Vienna Circle; it focussed its criticisms almost exclusively on Carnap's *Aufbau*. It frequently invokes Reichenbach ("Ziele und Wege der physikalischen Erkenntnis", in *Handbuch der Physik*, vol. 4 *Allgemeine Grundlagen der Physik* (Berlin: Springer 1929), Chapter 1, pp. 1-80), which argues (pp. 26ff.) in favor of realism and against positivism, though not explicitly against Carnap.

> ever know with absolute certainty whether they stand in precisely those relations to each other which are fixed in the postulates by means of which we can define our concepts?[19]

Einstein, who during the early 1920s frequently conversed with Schlick about this very issue, had famously declared that "Insofar as the sentences of mathematics refer to reality, they are not certain, and insofar as they are certain they do not refer to reality."[20] In the same vein, Schlick said that although implicit definition is a means to making concepts absolutely precise, this has required "a radical separation of the concept from intuition, of thought from reality. We relate the two spheres to each other, certainly, but they appear in no way connected; the bridges between them are demolished".[21]

Given their adherence to Wittgenstein's framework, however, the Vienna Circle could not be satisfied with this. It was imperative to bring axiomatic, implicitly defined concepts within the ambit of explicit definitions and truth-functional combinations of atomic sentences. Carnap sat down to address this problem soon after he arrived in Vienna, and from 1927 to 1930 spent most of his time on a large-scale project to reconcile axiomatic definitions with logicism, and transform implicit into explicit definitions, or, as he said, to transform "improper concepts" into "proper concepts" (so called because they were explicitly constructed, with fixed meanings to which e.g. the law of excluded middle applied).[22] The result was a large, unfinished manuscript entitled *Investigations in General Axiomatics*. In the framework set forth in this treatise, axiomatic systems are not regarded as purely syntactic, in the way Hilbert and Schlick had described them, but are given a fixed range of interpretions within a basic system, a *Grunddisziplin*, as Carnap called it, of arithmetic and set theory. This made it possible to regard axiomatic systems as having content, as long as it could be shown that the sentences of the *Grunddisziplin* itself had definite meanings. So every theorem of an axiomatic system has a definite meaning, since it is interpreted in the *Grunddisziplin*.

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[19] M. Schlick *Allgemeine Erkenntnislehre*, 2nd ed. (Berlin: Springer 1925), p. 55.
[20] A. Einstein *Geometrie und Erfahrung* (Berlin: Springer 1921), p. 3.
[21] Schlick *Allgemeine Erkenntnislehre*, p. 56.
[22] This is discussed in Carnap's paper "Eigentliche und uneigentliche Begriffe" (*Symposion* 1 (1927), pp. 355-374), which also makes very explicit the connection of the *Axiomatics* project with the *Aufbau* and the Wittgensteinian conception of meaning.

Within this framework, Carnap set out to explore the relations among the various properties of axiom systems, especially the relation among the different definitions of "completeness" of an axiom system current at the time. His main result, and the central theorem of the first part of the *Axiomatics* manuscript, proves that an axiom system is categorical if and only if it is complete (in modern terms). This theorem, Carnap hoped, could help to sort useful axiom systems determining "proper concepts" from defective ones — those to which e.g. the law of excluded middle then failed to apply. Thus, in particular, axiomatic arithmetic was inferred to be complete, as the Peano axioms were known to be categorical.[23]

Within his *Axiomatics* manuscript, though, there is no attempt to explain how the *Grunddisziplin* acquires its fixed interpretation. This task Carnap attempted in a loose sketch he wrote down in Davos in April 1929, when he was attending the "Europäische Hochschultage" where Heidegger and Cassirer debated the legacy of Kant.[24] The sketch was headed, ambitiously, "New Foundation for Logic" [*Neue Grundlegung der Logik*]. Its main idea is to erect a Hilbertian axiomatic superstructure on a Wittgensteinian basis. The atomic sentences are pictures of elementary facts, as in the *Tractatus*. But other signs, not given a definite meaning in advance, may also be added and treated just like atomic sentences, as may "inference rules" governing the transformation of given sentence forms into other sentence forms. All sentences containing the meaningless signs still have a definite meaning, Carnap argues, as they confine the total space of possibilities to certain rows of the truth-table of a complete truth-functional state-description of the world (of the kind envisaged by Wittgenstein). The only requirement of a "logic" so constructed — evidently intended as a preliminary sketch for building a *Grunddisziplin*[25] — is

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[23] This project is discussed in our paper "Carnap, Completeness, and Categoricity: The *Gabelbarkeitssatz* of 1928" *Erkenntnis* 54 (2001), pp. 145-172, where we also give a more detailed account of the importance of this theorem for Carnap's *Aufbau* project as well as the Vienna Circle's entire philosophy of logic and mathematics. Carnap's proof of it is actually correct, in his own terms, despite appearances, but fails to capture what he intended, as we discuss in detail in the above paper, and as Carnap himself realized in 1930, even before Gödel's incompleteness results later that year.

[24] On the whole background to this meeting, and its wider significance, see M. Friedman *A Parting of the Ways: Carnap, Cassirer, and Heidegger* (LaSalle, IL: Open Court 2000).

[25] Though there is no explicit provision for the quantifiers, Carnap may have intended to develop them axiomatically, as Hilbert and Ackermann (*Grundzüge der theoretischen Logik* (Berlin: Springer1928), pp. 22-23 and

that it not allow inference to any atomic sentence that is not already among the premisses. Axiom systems may then be framed within such a "logic", and all theorems resulting from them can likewise be assigned a definite meaning because they constrain the truth-table of the complete state-description of the world.[26] This is the case even if they contain signs for infinite sets. These, Carnap says, are licensed within his system, though not purely "formalistically" as in Hilbert; they have a definite meaning, even if not a complete one:

> If now, to introduce the infinite, one "adjoins ideal propositions" (Hilbert), i.e. writes down formulas that have no contentful meaning, but permit us to derive the mathematics of the infinite, then we have once again been able to determine the meaning of the signs introduced as meaningless ones, by investigating for which logical constants the formulas would become tautologies.[27]

Unlike Hilbert, Carnap admits no purely formal, uninterpreted signs. Despite this, he calls his idea "radical formalism" because it allows not only logical inferences, but any sort of scientific inference — including, relevantly, inductive inference in empirical science or statistical inference — to be employed as part of a "system of logic" in this way. All these inferences are now at the same level. In a talk at Reichenbach's seminar in late 1929, Carnap said that all such inferences could be assimilated to truth-functional inference like that described by Wittgenstein. We can regard any mode of inference, whether in mathematics or in empirical science, he said, formalistically, as a rule for transforming sentences of a certain specified form into sentences of a different form. We can even take axiomatic systems of infinitary mathematics and theoretical physics in this way.[28] In a lecture in Warsaw of December 1930 he said, along these same lines, that there is only one rule of inference in science: We can transform a sentence however we like, but the conclusion is to have no more content than the premisses; it is to constrain the range of possibly true atomic sentences no less than the premisses; i.e. no new atomic sentences are

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53-54) had for both the propositional and the predicate calculus. The terminology of the "New Foundation" coincides with Hilbert and Ackermann, where the quantifiers are introduced by "formal axioms", which are distinguished from the "inhaltliche" (material, contentful) rules of inference — the term also used by Carnap.

[26] This idea, too, seems to have been suggested by the *Tractatus*, which says "The truth or falsehood of every single sentence changes something in the general construction of the world. And the range [*Spielraum*] that is left its construction by the totality of atomic sentences is precisely that which the most general sentences delimit." (5.5262)

[27] UCLA/RC1029/Box 4/CM13, p. 62.

[28] Carnap's shorthand notes for this talk are preserved in UCLA/RC1029/Box 4/folder CM13, item 3.

recognized as true. All laws of logic, as well as all rules of inference in science, he maintains, follow from this principle.[29]

Though this idea is not thought through, and is in many ways incomplete, it indicates how Carnap was attempting to extend a truth-functional Wittgensteinian language to one usable for mathematics and science, though the kind of solution Carnap was considering saw for mathematics very much the role that Wittgenstein had envisaged for it in the *Tractatus*:

> The sentence of mathematics expresses no thought. In life it is never the mathematical sentence we need. We use the mathematical sentence *only* to derive sentences that do not belong to mathematics from other sentences that also do not belong to mathematics.[30]

In the course of 1930, however, this somewhat shaky "New Foundation for Logic" collapsed. Three developments contributed to undermine it. First, Carnap's central theorem of the *Axiomatics* fell victim to Gödel's first incompleteness theorem. As Gödel indicated in the discussion following the famous symposium on the philosophy of mathematics in Königsberg in September 1930 (at which Carnap had been the spokesperson for logicism, Heyting for intuitionism, and von Neumann for formalism), there could be true arithmetic sentences that were not provable:

> One can even (given the consistency of classical mathematics) give examples of sentences (of the kind stated by *Goldbach* or *Fermat*) that are correct in their content, but not provable in the formal system of classical mathematics. By adding the negation of such a sentence to the axioms of classical mathematics, one obtains a consistent system in which a sentence whose content is false is provable.[31]

Second, the incompleness result had an even more fundamentally devastating effect on logicism itself, which the Vienna Circle had relied on to guarantee the tautological (and thus empty) character of mathematics and indeed of all logical reasoning. The Circle had needed this to undermine the metaphysical idea that conclusions about the real world could be reached by

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[29] ASP/RC 110-07-35, p. 2.
[30] L. Wittgenstein *Tractatus Logico-Philosophicus* (London: Routledge 1922), 6.21-6.211.
[31] H. Hahn *et al.* 'Diskussion zur Grundlegung der Mathematik', *Erkenntnis* 2, pp. 135-149; this passage p. 148.

reasoning alone, without factual knowledge.[32] But now it turned out that there could be sentences of arithmetic that, despite the logicist explicit definition of the numbers, were not decidable after all.

Third and finally, the apparent incompatibility of meta-linguistic discourse with Wittgenstein's framework was, as we saw, a fundamental barrier to the Vienna Circle's larger goals, and they sought to overcome it. The new work in mathematical logic, especially by Hilbert, Gödel, and Tarski, made essential use of the distinction between a language and its meta-language. This work appeared to be rigorous, indeed more rigorous than older logical work like Russell's. It thus seemed to represent a clear counterexample to what the Vienna Circle read into Wittgenstein's final sentences. Still, there was the difficulty that "elucidations", meta-linguistic sentences of the kind Wittgenstein and the Vienna Circle themselves had used in their writings, seemed impossible to put *either* in the form of truth functions of atomic sentences *or* in the mathematical form that Gödel and Tarski were using. Thus by the end of 1930, the basis for the Vienna Circle's entire philosophical edifice was crumbling.

III. The Revolution, Step 1: Syntax

On 21 January 1931 Carnap came down with a fever. That and the problems described in the previous section kept him awake the following night. But as he tossed and turned, the solution to all his problems came to him in a flash. Or so he says in his autobiography:

> After thinking about these problems for several years, the whole theory of language structure and its possible applications in philosophy came to me like a vision during a sleepless night in January 1931, when I was ill. On the following day, still in bed with a fever, I wrote down my ideas on forty-four pages under the title "Attempt at a Metalogic". These shorthand notes were the first version of my book *Logical Syntax of Language*.[33]

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[32] R. Carnap 'Die alte und die neue Logik' *Erkenntnis* 1 (1930), pp. 12-26, specifically p. 25.

[33] R. Carnap "Intellectual Autobiography", p. 53.

These shorthand notes present a radically different perspective from the Wittgensteinian one of the "New Foundation" two years previously.[34] Carnap has here adopted the fully formal, "metalogical" viewpoint of Gödel and Tarski, according to which the logical language is a system of uninterpreted marks rather than meaningful signs. In the perspective of the "New Foundation", the atomic sentences had been pictures of atomic facts, which had given them their meaning. In the "Attempt at a Metalogic", an atomic sentence is a finite sequence of superscript dots, followed by the letter "f" with a finite sequence of subscript dots, followed by a left parenthesis, followed by the letter "a" with a finite sequence of subscript dots, followed by a right parenthesis, e.g.:

$$\cdots f_{\ldots}(a_{\ldots})$$

An atomic sentence, then, was just a finite string meeting certain conditions and consisting of instances of finitely many basic marks [*Zeichen*]. In the "New Foundation", a sentence had been a tautology because of what it says, or not, about the world. In the "Attempt at a Metalogic", being a tautology is a property of a string of marks that is defined entirely in terms of its outer form — the type and order of the marks occuring in it. No use is made of the "meaning", "designation", etc. of the marks in defining the central notions of truth-value assignment, consequence, tautology, and the like. Carnap even mentions that the undefined notion "true" might be better to avoid entirely.[35]

From the viewpoint of modern logic, this idea may not seem particularly momentous. Even at the time, it represented no technical innovation; Hilbert and others had been writing on formalism in connection with axiomatics for decades, and the methods of Gödel and Tarski were essentially that. But though Carnap's first attempt to formulate his "metalogic" was in terms of a particular formal system, his aim was not merely the mathematical study of a given formal

[34] These notes are preserved in Carnap's papers at the Young Research Library, UCLA, and we discuss them in more detail in our forthcoming paper "Carnap's Dream: Wittgenstein, Gödel, and *Logical Syntax*".

[35] In the margin of p. 3 of the manuscript, Carnap has scrawled, "Regarding the undefined concept '*true*'. It is completely different from the other concepts of metalogic. Perhaps avoidable? [Perhaps] just define which atomic sentences are the "basis" of a sentence, and how. (?)"

logical system. His new idea was precisely to *apply* the insights of Hilbert, Gödel, and Tarski to the entirety of human knowledge. As we saw above, he had previously accepted Wittgenstein's basic account of the logical language framework in which all science was to be expressed, as the basis for the project of rational reconstruction. In that context the "metalogical" perspective of regarding language purely as a system of rules, without reference to anything outside itself, was indeed a revolutionary idea.

Before Wittgenstein, language had been regarded as an essentially transparent medium for the expression of thought. The laws of logic were considered by Frege and Russell to be laws of thought, judgement, or perhaps nature — but certainly not of language. Wittgenstein had recognized that they were laws of language. He had been the first to consider the entirety of language as nothing but a system of rules. But he had arrived at this idea via a theory of representation that forced language to consist always and everywhere in a *particular system* of rules, arising necessarily from the representational function of language—the picture theory. The possibility of representation determined a particular form of linguistic intuition, so to speak. This elementary logic built into our form of representation was, like a Kantian form of intuition, an inescapable straight-jacket. The very nature of language, in Wittgenstein's view (at least as seen by the Vienna Circle), prevented us from stepping outside it. One could call this quasi-Kantian view "Wittgenstein's prison".

Under the suggestive influence of Hilbert's formalist approach and the technical work of Gödel and Tarski, Carnap was able to escape from Wittgenstein's prison by taking Wittgenstein's own idea of language as governed by a system of rules one step further. Carnap distinguished the representational or meaning function of language from its purely combinatorial one, and now took the *latter*, rather than the former, as his starting point. The metalogical methods developed in pursuit of the very mathematical results (such as the incompleteness theorem) that had led to the disintegration of his Wittgenstein-based position in the "New Foundation", it turned out, also showed a way of breaking out of Wittgenstein's prison, and making the structure of language itself the object of logical study.

As opposed to the confinement of all possible knowledge within the absolute constraints imposed by a (naturally or metaphysically) fixed structure of our means of expression, the new recognition that linguistic structure could itself be investigated opened up a whole new method for the unification and clarification of knowledge. Thus Carnap retained Wittgenstein's idea of language as a system of rules, but threw off the shackles of Wittgenstein's prison in favor of the logicians' metalogical perspective. He comprehensively and definitively turned his back on the picture theory of the *Tractatus* — and thus also on its foundationalism. Meaning was no longer built up from some basic (naturally occurring or metaphysically unavoidable) components, but was now determined entirely by rules. Rules can be determined by humans. The upshot of Carnap's dream, then, was a liberation from the manacles of a fixed structure imposed on the human mind by natural or metaphysical factors beyond human control. If Wittgenstein's insight into the formal nature of logical truth had been the turning point in philosophy, then January 1931 was the revolution it ushered in, where the Vienna Circle's voluntarism[36] could finally find its proper scope and expression. With respect to Wittgenstein's prison, this change was literally an overnight transformation from slave to master.

But there were still many obstacles to be overcome in working out the new idea. Carnap began, in the "Attempt at a Metalogic" with the notion that by keeping the logical object language free of assumptions, there would be a distinguished meta-language in which arithmetic could be read off from the dot sequences of the object language.[37] Thus the numbers are not defined as higher-order concepts in the Frege-Russell logicist style, but "purely as figures" [*rein figurell*], on the basis of the dot sequences attached to the symbols. Arithmetical properties and statements then belong to the meta-language. Thus e.g. the commutativity of addition $n + m = m + n$ was supposed to follow from the fact that n-many dots written to the left of m-many dots gives the

[36] R. Jeffrey ("Carnap's Voluntarism", in D. Prawitz, B. Skyrms, and D. Westerståhl, eds. *Logic, Methodology, and Philosophy of Science IX*, pp. 847-866) discusses Carnap's voluntarism in particular; for the Vienna Circle more widely (esp. the so-called "left" Vienna Circle), see T. Uebel "Carnap, the Left Vienna Circle, and Neopositivist Antimetaphysics", in Awodey and Klein *Carnap Brought Home*, pp. 247-278.

[37] An addition of 7 February 1931 to the manuscript says, "the syntax of the rows of dots is arithmetic" (p. 1).

same series of dots as writing them to the right of m-many dots. The question of the need for mathematical induction *in the meta-language* is considered, but dismissed with some optimism.

If arithmetic was to be formulated in the meta-language of logic, then mathematical analysis was to be formulated in its meta-meta-language. For real numbers are properties or series of natural numbers, and properties of them and statements about them properly belong one level up. Carnap may have been guided, in this idea, by Russell's suggestion, in his introduction to the *Tractatus*, that one could perhaps break out of Wittgenstein's prison by using a scheme involving a hierarchy of languages:

> These difficulties suggest to my mind some such possibility as this: that every language has, as Mr. Wittgenstein says, a structure concerning which, *in the language*, nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit.[38]

Having now found the mechanism for such a scheme in the form of "metalogic", applying it to achieve a hierarchy consisting of language, meta-language, meta-meta-language, and so on[39] must have indeed seemed rather compelling, at first sight.

IV. The Revolution, Step 2: Tolerance

Carnap soon realized that the restricted system of the "Attempt at a Metalogic" would not permit him to express certain essential meta-logical concepts, such as *provability*. The combinatorial theory required for their formulation was every bit as complicated as arithmetic itself. Thus in the late spring of 1931, Carnap decided to move to a conventional axiomatic arithmetic in the *object* language, so that the axiomatized arithmetic could then be used to express the meta-

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[38] Introduction to Wittgenstein's *Tractatus*, p. 23.

[39] The "Attempt" ends with a summary in four points: "(1) The particular *natural numbers* occur as signs of *the language itself.* (2) The so-called "*properties of natural numbers*" are not proper properties, but syntactic (Wittgenstein: internal) ones, so are to be expressed in the *metalanguage*. (3) A particular *real number* is a property or sequence of natural numbers, so is also to be expressed in the *metalanguage*. (4) The *properties of real numbers* are not real properties, but syntactic properties (with respect to the syntax of the metalanguage), and thus *to be expressed in the meta-metalanguage.*" (p. 44)

language, using Gödel's method of arithmetization. In a series of talks Carnap gave to the Vienna Circle in June and July of 1931 about the syntax idea, he explains the difference this makes to his system quite vividly: "The difference between arithmetic metalogic and the metalogic portrayed previously is this: arithmetic metalogic treats not the empirically available, but all possible configurations. Our previous metalogic is the descriptive theory of certain given configurations, it is the geography of language forms, while the arithmetized metalogic is the geometry of language forms."[40] This move had the further advantage of collapsing the entire hierarchy of languages and meta-languages into itself, at least in principle, by iterating Gödel's method of arithmetizing the metalanguage in the object language. Thus it appeared (for a time at least) that one could now get by with only a single language after all.

However well this seemed to work, there was a price to be paid for it. For the very thing that had made the "metalogical" solution possible — i.e. the precise definablity of the central metalogical notions and their expressibility in the object language — was also responsible for the essential incompleteness of the logical treatment of mathematics. The identification of the logical with the formal seemed to restrict its scope to only what can be expressed with very limited means. If, however, there were no intrinsic constraints on the sorts of formal properties of formulas that could be considered, then perhaps there could be a formal criterion for mathematical truth *different* from mere provability. Since Gödel had shown that provability was insufficient — there were "true" arithmetical statements not derivable from the axioms —the identification of such a criterion was essential. Carnap seems to have developed such a criterion sometime in the latter part of 1931, in the form of the notion of *analyticity*. This was to be a stronger sort of logical truth than provability in a formal system, but was still to be determined strictly in terms of the formal character of the symbols.

Analyticity was apparently to take the place of provability as the generalized notion of tautology or logical truth. To understand how this was intended, consider the analogy of a chess game. Think of the starting position of the pieces as the axioms, the permitted moves as the rules of

[40] ASP/RC 081-07-18; reprinted in Stadler *Studien*, p. 325.

inference, and a sequence of moves ending in checkmate as a proof of a theorem. But now observe that there are configurations of pieces on the board that constitute checkmate, but cannot be reached from the starting position by any sequence of permitted rules. Such a configuration represents an analytic sentence that has no proof. In this way, the definition of analytic sentence can be phrased entirely formally, in accordance with all the same rules of inference, and yet still be wider than provability. Thus the absolute, Wittgensteinian conception of tautology could be saved, and indeed finally extended beyond propositional logic in accordance with the Vienna Circle's original ambitions.

Such a notion of analyticity was apparently defined in the first draft of the *Logical Syntax*, entitled *Metalogik*. From the evidence available it is clear that the definition was defective. Gödel objected to it, pointing out that it will be *impossible* to give a correct definition of it in *any* meta-language that can be faithfully represented in the object language, e.g. by arithmetization. This fact has since become known as Tarski's theorem on the indefinability of truth. Thus it turns out that Carnap's single language approach will not work after all.

Carnap did eventually, with Gödel's help, work out a new definition of "analytic", but this definition, which had previously been so crucial, was not even deemed important enough to include in the first edition of *Logical Syntax of Language*; it was omitted "for reasons of space".[41] The problem with it was that the notion of analyticity it defined was not absolute, but rather in a certain sense, conventional. It gave a notion of "analytic in L", but only with respect to *another* language L', used for the interpretation of L. There might be a natural or conventional choice for L' — type theory of the next higher type, or axiomatic set theory — but it could hardly be claimed that any particular such choice is the *correct* notion of analytic for a given language. This *language relativity* of the central notions of metalogic turned out to be more important to Carnap than the particular metalogical definitions themselves.

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[41] R. Carnap *Logische Syntax der Sprache* (Vienna: Springer 1934), p. vii. For the subsequent career of the analyticity concept in Carnap's thought, see our survey paper "The Quest for Analyticity: Carnap's *Studies in Semantics*" in the *Cambridge Companion to Carnap*.

In his first publication after this exchange, we find that a new tone has suddenly entered Carnap's writing, one that was much closer in spirit to the scientific temperament of the Vienna Circle than the absolute and somewhat oracular style of Wittgenstein: "In my view the issue here is not between two conceptions that contradict each other, but rather between *two methods for constructing the language of science, which are both possible and justified.*"[42] The context for this first appearance of a new kind of pluralism is the epistemological question about the form of the observation language (or "protocol language") in science. Carnap is very explicit about his change of position:

> Not only the question whether the protocol sentences are inside or outside the syntax language, but also the further question regarding their precise specification, is to be answered, it seems to me, not by an assertion, but by a stipulation [*Festseztung*]. Though I earlier left this question open . . . I now think that the different answers are not contradictory. They are to be taken as proposals for stipulations [*Vorschläge zu Festsetzungen*]; the task is to investigate these different possible stipulations as to their consequences and assess their usefulness.[43]

In *Logical Syntax of Language*, a year or two later, Carnap formulated this new attitude as a "principle of tolerance", and announced it with some excitement in the book's preface:

> The range of possible language forms, and thus of different possible logical systems is. . . incomparably larger than the very narrow range in which modern logical investigations have so far operated. Up to now there have only been occasional small departures from the language form given by *Russell*, which has already become classical. . . The reason for not daring to depart further from this classical form would appear to lie in the widespread view that such departures must be "justified", i.e. it must be shown that the new language form is "correct", that it represents the "true logic". It is one of the main tasks of this book to eliminate this view as well as the pseudoproblems and pointless squabbles arising from it.[44]

The first attempts to escape from the "classical" forms — which themselves only went back one or two generations! — were certainly daring, he says. "But they were hobbled by a striving for

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[42] R. Carnap "Über Protokollsätze" *Erkenntnis* 2 (1932), pp. 215-228, this quotation p. 215.
[43] Carnap, "Über Protokollsätze", p. 216.
[44] Carnap *Logische Syntax*, p. v.

'correctness'." He concludes with the famous words: "But now this barrier is overcome: before us lies the open sea of free possibilities."[45]

The principle is stated, in the text of the *Logical Syntax* itself, in the context not of epistemology, as in its first application, but of philosophies of mathematics, particularly intuitionism. It is expressed as the exhortation to state meta-theoretic (or, as he now calls them, logic-of-science [*wissenschaftslogische*]) proposals in precise terms, as explicit rules or definitions, within the formation or transformation rules of a precisely defined language or calculus:

> Once it is understood that all pro- and anti-intuitionist considerations are concerned with the form of a calculus, the question will no longer be asked in the form "What *is* the case?" but rather "How do we *want* to set this up in the language being constructed?". . . And with that, the dogmatic frame of mind that often makes the discussion unfruitful is banished.[46]

This "dogmatic frame of mind" results, in Carnap's view, from the reliance on inherently vague philosophical "considerations [*Erörterungen*]" rather than on precise statements of definitions and rules. He indicates how he has tried, in Language I of the *Syntax*, to capture the philosophical concerns (expressed in various gradations of finitism or constructivism) voiced by Brouwer, Kaufmann, Wittgenstein, and others. But, he points out, there is no way of telling whether he has expressed *precisely* what they have in mind, as they have not expressed their views as proposed precise definitions and rules, but only in terms of vague *Erörterungen* that leave many specific questions open, when one gets down to the brass tacks of constructing an actual language.[47] Or they impose restrictions and requirements that appear to be normative.

Carnap's most general statement of the principle of tolerance, therefore, addresses these tendencies directly, contrasting them with his own program of precise and explicit rules:

> Our attitude to demands of this kind may be stated generally by the *principle of tolerance: we do not want to impose restrictions but to state conventions. . . In logic there*

[45] Carnap *Logische Syntax*, p. vi.
[46] Carnap *Logische Syntax*, p. 42.
[47] Carnap *Logische Syntax*, p. 44.

> *are no morals*. Everyone can construct his logic, i.e. his language form, however he wants. If we wants to discuss it with us, though, he will have to make precise how we wants to set things up. He has to give syntactic rules rather than philosophical considerations.[48]

Only by *replacing* the vague concept with a precise equivalent can the practical merits or drawbacks of a proposal be judged, for some defined purpose. And under the new regime of pluralism, where there can be no criterion of inherent "correctness", practical usefulness is the only criterion left for deciding whether a proposal should be pursued or left aside. The principle of tolerance fits well, then, into the project of "rational reconstruction" pursued by the earlier Vienna Circle, and sets the stage for the successor project of "explication", which Carnap would not formulate explicitly until after 1945.[49] And he is careful to apply the insistence on precision to his own work as well. Attention and criticism should be focussed, he repeatedly insists, not on the "inexact" informal reflections in the text, but on the precise definitions given in terms of the proposed calculi.

V. After *Logical Syntax*

In sections I and II above we discussed a position — the "turning point" initiated by the *Tractatus* — that was shared by most members of the Vienna Circle and by other logical empiricists, such as there were, until about 1930. In sections III and IV, on the other hand, we focussed more narrowly on Carnap and Gödel, leaving other developments aside. There are good reasons for this. After 1931 Carnap was no longer in Vienna; he had taken up a chair at the German University in Prague. The Nazi takeover in Germany of January 1933 not only sent all the German outposts of logical empiricism into exile or under cover, but made it evident that those remaining in Austria and elsewhere in Europe were under threat. Hans Hahn, Carnap's

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[48] Carnap *Logische Syntax*, p. 45.

[49] The classical exposition of this project is in Ch. 1 of *Logical Foundations of Probability* (Chicago: University of Chicago Press 1950); for further discussion, see H. Stein "Was Carnap Entirely Wrong, After All?" *Synthese* 93 (1992), pp. 275-95; S. Awodey and A.W. Carus "How Carnap Could Have Replied to Gödel", in S. Awodey and C. Klein *Carnap Brought Home: The View from Jena* (LaSalle, IL: Open Court 2004), pp. 203-224, section III; and A.W. Carus "Sellars, Carnap, and the Logical Space of Reasons", in Awodey and Klein *Carnap Brought Home*, pp. 317-356, section II.

main interlocutor in Vienna on questions of logic and mathematics (apart from Gödel), died that same year, and soon after that Schlick was assassinated by a deranged student. In other words, Carnap's new doctrine simply did not have *time* to become the object of general discussion within the Vienna Circle. Neurath was its only whole-hearted adherent, and he emphasized mainly epistemological aspects of the idea that were somewhat misunderstood by Schlick and others.[50]

Also, Neurath resisted Carnap's move to semantics the year after the *Logical Syntax* was published. This has often been misunderstood as a fundamental break. In fact, as most scholars now agree,[51] the shift to semantics does not represent a fundamental discontinuity in Carnap's thought. As we saw above, the original syntax idea represented, above all, a rejection of *meaning* in Wittgenstein's sense. Meaning, in this "absolutist" view, as Carnap later called it, retained a certain arbitrariness or obscurity. Certain sentences that seemed obviously "meaningful" (e.g. Newton's laws) failed to meet Wittgenstein's criteria. And what authority did those criteria claim? Only that of philosophical arguments, not that of precisely specified concepts embedded in a language framework defined by explicit rules. Before January 1931, Carnap and the Vienna Circle had been "in the grip of a picture", the picture of language deriving its meaning by truth-functional concatenation of atomic sentences representing atomic facts. But then Carnap discovered that he could retain Wittgenstein's basic insight without this picture, by extending a Hilbertian or Tarskian formalist view from logic and mathematics to the whole of knowledge.

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[50] Including Russell, who said that according to this "attempt to make the linguistic world self-sufficient", ". . . empirical truth can be determined by the police" (B. Russell 1940, pp. 147-8). A.W. Carus "Carnap, Syntax, and Truth", in J. Peregrin, ed. *Truth and its Nature (if any)*(Dordrecht: Reidel 1999) discusses these misunderstandings; Neurath's position is explained in detail by Thomas Uebel *Overcoming Logical Positivism from Within: The Emergence of Neurath's Naturalism in the Vienna Circle's Protocol Sentence Debate* (Amsterdam: Rodopi 1992).

[51] R. Creath "The Unimportance of Semantics", in *PSA 1990*, vol. 2, ed. A. Fine, M. Forbes, and L. Wessels, pp. 405-415; T. Ricketts "Carnap's Principle of Tolerance, Empiricism, and Conventionalism", in P. Clark and B. Hale, eds. *Reading Putnam* (Oxford: Blackwell 1994), pp. 176-200 and "Carnap: From Logical Syntax to Semantics" in R.N. Giere and A.W. Richardson, eds. *Origins of Logical Empiricism* (Minnesota Studies in the Philosophy of Science, vol. XVI; Minneapolis: University of Minnesota Press 1996), pp. 231-250; Awodey and Carus "The Quest for Analyticity".

It seemed entirely reasonable, at that point, to conclude that it was the Wittgensteinian theory of "meaning" that had blocked the way to this outcome. The response, accordingly, was a complete proscription of meaning: *nothing* extra-linguistic could constrain the acceptability of a precise meta-language for the logic of science. The shift away from the Wittgensteinian view had meant a corresponding shift from trying to *incorporate* "philosophical" (*wissenschaftslogische*, "elucidatory") discourse into the language of science itself to the construction of a precise *meta*-language for the language of science.[52] This was a drastic change of perspective. The criterion of empiricism, for instance, now had to be reformulated as a constraint on the scientific object language — i.e. as a set of formation and transformation rules in the meta-language, an *internal* constraint, from "above" — rather than a requirement of *meaning* — i.e. in terms of conditions for verification, an *external* constraint, from "below" (Ricketts 1994). The critique of metaphysics, above all, was similarly re-expressed, as a proscription of meaning in the meta-language.[53]

But the new syntactic view led Carnap, as we saw, directly to the principle of tolerance. The syntactic view required a definition of what it means to "follow from the rules", and Gödel had shown that the traditional, intuitive definition — provability — did not suffice. A separate and richer meta-language was required for this definition, and there is no reason to single out any

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[52] In the Vienna Circle, he says, "the philosophical problems in which we were interested ended up with problems of the logical analysis of language," and since "in our view the issue in philosophical problems concerned the language, not the world", the Circle thought that "these problems should be formulated not in the object language but in the metalanguage." It was therefore "the *chief motivation* for my development of the syntactical method," (our emphasis) to develop a "suitable metalanguage" that would "essentially contribute toward greater clarity in the formulation of philosophical problems and greater fruitfulness in their discussions" (R. Carnap "Intellectual Autobiography", p. 55).

[53] As made explicit in the famous paper (R. Carnap "Die physikalische Sprache als Universalsprache der Wissenschaft" *Erkenntnis* 2 (1932), pp. 432-465) mainly known for its advocacy of physicalism. This epistemological aspect is certainly present there. But the new syntactical doctrine, in fact, *motivates* the paper's physicalistic conclusions. After an introductory discussion about the idea that all objects and facts are of a single kind, we are told that these expressions are a concession to the customary "material" [*inhaltliche*] way of speaking. The "correct" way, Carnap says, speaks of words rather than "objects" and sentences rather than "facts", for a philosophical investigation is an analysis of *language*. In a footnote he indicates that a comprehensive, strictly formal theory of language forms, which he calls "metalogic", will soon be forthcoming, and will justify the "thesis of metalogic" here invoked, that "meaningful" [*sinnvolle*] philosophical sentences are the metalogical ones, i.e. those that speak only of the form of language. This represents a radically different basis for the critique of metaphysics from the one Carnap had previously adopted from Wittgenstein, whereby meaningful sentences were those that derived their meaning from atomic sentences by truth-functional combinations. Atomic sentences, as pictures of atomic facts, no longer play any role in distinguishing meaningful from meaningless sentences.

particular such meta-language as "correct". But the principle of tolerance and its new pluralism no longer supported the proscription of meaning, at least as made precise in Tarski's new semantic theory. Under the new pluralism, the criteria for considering a concept or language are simply: (i) specifiability by explicit rules, and (ii) practical usefulness. Under (i), the semantic definitions of "designation" and "truth" qualify; whether they qualify under (ii) cannot be decided until various possible semantic languages have been tried out and applied to the problems we want to solve.[54] The proposal in *Meaning and Necessity* to employ these definitions in the "method of extension and intension" (Carnap 1946) can be regarded as an *explication* of — a replacement for — the previously vague, somewhat obscure and arbitrary, conception of "meaning".

Carnap's rejection of meaning foundationalism in January 1931 had been all of a piece. Seen from our present perspective, though, this original syntax view can be regarded, retrospectively, as having been composed of a number of different elements that would later turn out to be separable: (a) the requirement that a language be entirely specified by explicit rules; (b) distinction between a language (a calculus, a purely syntactic symbol system) and its possible interpretations; and (c) the prohibition of interpretation or meaning in the elucidatory (*wissenschaftslogische*) meta-language.

Tolerance depends on (a) and (b), which survive unscathed and undiminished into Carnap's semantic period. (So it is rather misleading to call them "syntactic"; Carnap's original term "metalogical" might be more appropriate.) What does not survive is (c), the overreaction against Wittgensteinian "meaning" that accompanied the original insight. In distinguishing between a language and its interpretation, Carnap's first (and, as we saw, understandable) response was to reject that imprecise notion of meaning entirely. But this restriction was loosened when he saw

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[54] Carnap never claimed to have arrived at the definitive semantics; he presented his formulation as a first attempt, to get things started and as a basis for discussion: "I believe. . . semantics will be of great importance for the so-called theory of knowledge and the methodology of mathematics and empirical science. However, the form in which semantics is constructed in this book need not necessarily be the most appropriate for this purpose. This form is only a first attempt; its particular features. . . may possibly undergo fundamental changes in their further development." (R. Carnap *Introduction to Semantics* (Cambridge, MA: Harvard University Press 1942), p. xii)

that interpretation could be specified by explicit rules (governing satisfaction, designation, and truth), in accordance with component (a) of the original syntax idea.

This new position, which remained Carnap's for the rest of his career, has been subjected to many assaults since it was articulated in the 1930s. The most notorious assailant was Quine, whose famous 1951 paper "Two Dogmas of Empiricism" essentially portrayed Carnap's later position as a kind of relapse into the pre-syntactic, Wittgensteinian "verification theory of meaning" (Quine 1951, p. 41) that Carnap had overcome in January 1931.[55] In *Word and Object* (Quine 1960), Quine opened a different front, shifting attention from the Vienna Circle's preoccupation with syntax and semantics to what Carnap, by then, would have called the *pragmatic* (and partly *empirical*) question how a language can in practice convey content. Most analytic philosophers have followed Quine in these respects, though this consensus has gradually begun to fray a little.[56] The posthumous publication of Gödel's critique of Carnap in 1995[57] has recently called forth another stream of papers, re-appraising Carnap's philosophy of mathematics and that of logical empiricism more generally.[58]

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[55] W.V.O. Quine "Two Dogmas of Empiricism" (1951), repr. in *From a Logical Point of View*, 2nd ed. (Cambridge, MA: Harvard University Press 1980), p. 41.

[56] Serious dissent began with Stein "Was Carnap Entirely Wrong?" and D. Isaacson "Carnap, Quine, and Logical Truth" in D. Bell and W. Vossenkuhl, eds., *Wissenschaft und Subjektivität: Der Wiener Kreis und die Philosophie des 20. Jahrhunderts*. Berlin: Akademie Verlag, pp. 100-130, while Richard Creath's introduction to *Dear Carnap – Dear Van: The Quine-Carnap Correspondence* (Los Angeles: University of California Press1990) provided a more balanced view than had previously been available. See also 'Carnap and Quine: Internal and External Questions', *Erkenntnis* 42 (1995), pp. 41-64. and Carus "Sellars, Carnap, and the Logical Space of Reasons".

[57] K. Gödel "Is Mathematics Syntax of Language?" (two versions), in K. Gödel *Collected Works, Vol. III: Unpublished Essays and Lectures* (Oxford: Oxford University Press 1995), pp. 334-363.

[58] Much of this attention has focussed on Gödel's view that Carnap's overall framework, based on the principle of tolerance, is self-undermining (e.g. M. Friedman *Reconsidering Logical Positivism* (Cambridge: Cambridge University Press 1999), Ch. 9; M. Potter *Reason's Nearest Kin: Philosophies of Arithmetic from Kant to Carnap* (Oxford: Oxford University Press 2000), Ch. 11. Gödel's argument is accepted to varying degrees; most commentators, like W. Goldfarb and T. Ricketts ("Carnap and the Philosophy of Mathematics", in D. Bell and W. Vossenkuhl, eds. *Science and Subjectivity; The Vienna Circle and Twentieth Century Philosophy* (Berlin: Akademie-Verlag), pp. 61-78.), have held that Carnap's view can only be upheld in a weakened or diluted (and rather empty) form. We respond to these arguments, identifying an error in Gödel's argument, in S. Awodey and A.W. Carus 'Carnap vs. Gödel on Syntax and Tolerance', in P. Parrini, W.C. Salmon, and M.H. Salmon, eds. *Logical Empiricism: Historical and Contemporary Perspectives* (Pittsburgh: University of Pittsburgh Press), pp. 57-64., on behalf of Carnap's position in *Logical Syntax*, and in Awodey and Carus "How Carnap Could Have Replied to Gödel" on behalf of Carnap's later position.

Despite all these discussions, however, logical empiricism is still largely identified with the 1920s view discussed in sections I and II above. This is the view portrayed in A.J. Ayer's *Language, Truth, and Logic*, for instance, which is still regarded even by many reputable philosophers as a definitive and exemplary statement of "logical positivism" or "logical empiricism".[59] We hope to have shown, here, that this is a gross distortion. While this view was certainly the Vienna Circle's starting point, they were aware of its fundamental defects from the beginning, and worked steadily to overcome them. What is more, they *did* overcome them. In 1931 Carnap developed a *new* platform on which many of the previous logical empiricist positions could — in somewhat revised form — rest, and this new position *rejected* the previous one. The significance of the new metalogical or syntactic viewpoint, Eino Kaila agreed after discussion with Carnap in the summer of 1931, lay precisely in its "elimination of verification by comparison with facts [*Ausschaltung der Verifikation durch Vergleich mit Sachverhalten*]".[60] The significance of the new position, in other words, lay in its rejection of the previous, foundational one. The fact that this position appeared too late to be discussed while the Vienna Circle could still be considered a going concern was a quirk of fate. Seventy years later, we should stop holding the starting point of logical empiricism against it, and should look not where it came from but where it was headed.

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[59] It is so regarded, for instance, with almost no reference to any actual members of the Vienna Circle, in the recent two-volume history of analytic philosophy by Scott Soames *Philosophical Analysis in the Twentieth Century*, Vol. I: *The Dawn of Analysis* (Princeton, NJ: Princeton University Press 2003)., Ch. 12 and 13.
[60] ASP/RC [diary entry of 26 June 1931].

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