# SPACE-TIME AND SYNONYMY*

PETER SPIRTES AND CLARK GLYMOUR

*Department of History and Philosophy Science*
*University of Pittsburgh*

In "The Epistemology of Geometry" Glymour proposed a necessary structural condition for the synonymy of two space-time theories. David Zaret has recently challenged this proposal, by arguing that Newtonian gravitational theory with a flat, non-dynamic connection (FNGT) is intuitively synonymous with versions of the theory using a curved dynamical connection (CNGT), even though these two theories fail to satisfy Glymour's proposed necessary condition for synonymy.

Zaret allowed that if FNGT and CNGT were not equally well (bootstrap) tested by the relevant phenomena, the two theories would in fact not be synonymous. He argued, however, that when electrodynamic phenomena are considered, the two theories are equally well tested.

We show that it is not *FNGT* and *CNGT* which are equally well tested when the electrodynamic phenomena are considered, but only suitable extensions of FNGT and CNGT. Thus, there is good reason to consider FNGT and CNGT to be non-synonymous. We further show that the two extensions of FNGT and CNGT which are equally well tested when electrodynamic phenomena are considered (and which could be considered intuitively synonymous) not only satisfy Glymour's original proposed necessary condition for the synonymy of space-time theories, they satisfy a plausible stronger condition as well.

Philosophical accounts of the synonymy of theories must eventually be confronted with cases. For several reasons, covariant space-time theories provide especially useful tests of criteria for synonymy. While having a definite and well-understood formal structure, such theories remain reasonably close to scientific practice, and can be found presented in scientific papers, textbooks, and so forth. Thus, unlike first-order formalizations of theories, covariant space-time theories can be understood and discussed with relatively little dispute over their correct presentation or representation. Furthermore, many accounts of synonymy tie the notion to that of observational or empirical equivalence, and even with criteria of synonymy which do not analytically connect synonymy and empirical equivalence, we may naturally enough want to know the relations between the two notions in various cases. For many covariant space-time theories, the features of models of those theories which represent observational results are reasonably clear, and therefore readily studied.

Criteria for synonymy which are largely structural, and which have

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little to do with the particular content of theories, are especially useful since they may be applied virtually unambiguously. Such structural criteria can never be sufficient, but they may very well be necessary for the synonymy of theories, and in many philosophical contexts all that one requires are useful, necessary conditions. In a recent paper, David Zaret (1980) has claimed that a structural criterion for synonymy proposed by one of us is inadmissably strong, and in particular that it counts as distinct theories two versions of Newtonian Gravitational Theory which are, according to Zaret, really synonymous. We wish to examine the issues Zaret raises, and to argue that his objections are badly confused. We will show that when equivocations are sorted out, the purported counter-example is no such thing, and that the theories appealed to do not violate the structural condition, and in the most important case actually satisfy a stronger and more natural condition of the same kind. Our discussion will be informal; technical results are collected in an appendix.

Consider theories stated as systems of covariant equations, perhaps with some global constraints. Many theories have been so formulated: special and general relativity, of course, but also various versions of Newtonian Gravitational Theory, competing relativistic gravitational theories, Maxwell's electrodynamics, etc. When are two theories of this kind merely different ways of saying the same thing? Glymour (1977) has proposed the following necessary condition for the synonymy of two space-time theories *R* and *S*:

> For every manifold *M* and collection *C* of geometrical objects on *M* satisfying *R*, there are covariantly definable, from the objects in *C*, each of a collection *C′* of objects on *M* satisfying *S*, and symmetrically. (1)

Although we find (1) a perfectly acceptable, *necessary* condition for synonymy, it is easy to produce examples of pairs of space-time "theories" which, on structural grounds alone, clearly are not synonymous but whose synonymy is not excluded by condition (1).[1] A natural but stronger necessary condition for the synonymy of space-time theories suggests itself:

> There is a covariant set of equations *E*, containing only variables occurring in *R* or in *S*, and a covariant set of equations *D*, likewise containing only variables occurring in *R* or in *S*, and equations in *E* and *D* are *definitional*—that is, from values of the variables in *R*, *E* determines unique values of the variables in *S*, and from values of

[1]Let one "theory" assert merely that a (0,2) tensor field vanishes: $g_{ik} = 0$. Let another "theory" assert that a (0,2) tensor field is zero when anti-symmetrized $h_{ik} - h_{ki} = 0$. Although structurally quite different, these two "theories" satisfy principle (1).

the variables in *S*, *D* determines unique values of the variables in *R*—and every manifold and set of geometrical objects which satisfies *R* and *E* also satisfies *S* and *D*, and symmetrically. Further, *E* and *D* are conservative; that is, every model of *S* can be expanded to a model of *S* and *D*, and every model of *R* can be expanded to a model of *R* and *E*. (2)

Condition (2) is very close to requiring that the covariant theories have a common definitional extension and clearly implies condition (1).

The argument for either condition is much the same: they agree with the formal requirements of a synonymy relation—for example, they determine equivalence classes of theories; they are close analogues of intuitive conditions for formalized first-order theories; and, finally, they accord with all of the obvious and uncontroversial cases.

Covariant versions of Newtonian Gravitational Theory were first introduced to a philosophical audience by John Earman and Michael Friedman (1973) who provided a valuable discussion of the status of Newton's first law in the context of such theories. Flat Newtonian Gravitational Theory, hereafter, regrettably, FNGT, postulates an absolute time, an absolute Euclidean spatial measure on any set of simultaneous point-events, and an affine geometry on all of space-time. The affine geometry, of course, determines a class of "straight" lines in space-time, and the geodesics are understood counterfactually, as the trajectories that particles would follow were they subject to no forces whatsoever. The affine geometry is flat in a technical sense, i.e., its associated curvature is zero; it is also "rigid" or "absolute"—that is to say, entirely unaffected by the distribution of matter and energy in space-time. In addition, the theory postulates that every bit of mass generates a gravitational potential, in accordance with the covariant generalization of Poisson's equation. Finally, the theory postulates that particles moving in a gravitational field do *not* move along geodesics of the affine geometry of space-time, but instead are deflected from such paths by the gravitational forces.

An apparently different covariant version of Newtonian Gravitational Theory involves a dynamic affine connection. This theory, which we will call Curved Newtonian Gravitational Theory, abbreviated CNGT, postulates, as before, an absolute time, and an absolute Euclidean spatial measure on any set of simultaneous events. In contrast to the Flat Theory, which postulates that the gravitational force is determined by the distribution of masses in space-time, according to the Curved Theory it is the affine geometry that is determined by the distribution of masses in space-time. This dynamic affine geometry allows the Curved Theory to "geometrize away" the force of gravity. That is, according to the Curved Theory, there is no "gravitational force" exerted by massive bodies which deflects test particles from geodesic paths, as the Flat Theory postulates.

Since the affine geometry is determined by the family of all geodesics, the difference between the two geometries can be understood in terms of the difference in their geodesics. Suppose, for example, that one is given a particular distribution of masses in space-time, and that there is a test particle which, according to the Flat Theory, has only gravitational forces acting on it. According to the Flat Theory, the test particle will be deflected from the geodesic path by the gravitational force. In contrast, according to the Curved Theory, there is no gravitational force, and that same test particle has *no* forces at all acting upon it. Thus, according to the Curved Theory, the test particle will travel along a geodesic path. The two theories make the same prediction about what path the test particle will follow; they differ only on whether that path is a geodesic, and whether any forces are acting on the test particle.

According to the Curved Theory, in space-times that are not empty of all mass, the affine geometry that results is not flat in the technical sense, but curved. Curved Newtonian Gravitational Theory does not postulate any distinct quantity representing the gravitational potential. Its law of motion states, quite simply, that particles subject to no forces (a situation which would be described by the Flat Theory as particles subject only to gravitational forces) traverse geodesics of space-time.

Certain features of the relations of these two versions of Newtonian gravity are not controversial. In the first place, in all situations in which non-gravitational forces can be neglected, these theories are empirically indistinguishable: under a given distribution of sources (i.e., of masses) the two theories predict exactly the same space-time trajectory for any test particle. The manner of calculation is slightly different in the two cases. The Curved version permits one to calculate the geodesics of the connection from the given distribution of sources, and the theory predicts that a test particle will move on one of these paths. The Flat version permits one to calculate a certain *function* which depends on the flat geodesics and on the gravitational potential, and the equation of motion determines the predicted possible trajectories from this function. In the Flat version of the theory, while one can calculate from a given distribution of sources the relevant function of the geodesics (or, rather, of parameters describing the geodesics) and the potential, *neither the geodesics nor the potential are themselves determinable from such data.* Further, these quantities are not determinable even if one knows, in addition, all of the possible trajectories of freely falling bodies.[2] In the second place, these

[2]However, in the special case that all of the sources are distributed in a finite region, *if* it is assumed that the potential vanishes at infinite distances from the sources, then the potential can be separately determined. See Glymour (1977), Trautman (1964).

two theories satisfy neither principle (1) above, nor principle (2). The reason is that while the dynamical affine geometry of the Curved Theory is covariantly definable from the objects of the Flat Theory, the reverse is not the case: the Flat affine geometry of FNGT cannot be covariantly defined from the objects of Curved Gravitational Theory.

According either to principle (1) or to principle (2), FNGT and CNGT are not synonymous. Zaret claims otherwise. He is unimpressed by the general arguments for these principles, and claims that these theories do really say the same thing, and that "the choice between them is a matter of convention" (Zaret 1980, p. 474). In claiming as much, Zaret does not mean to say merely that there could be no empirical basis for such a choice, and he insists that his judgment is not founded on the claim that empirically indistinguishable theories are conventional alternatives. The claim seems to be, instead, that the two theories really say the same thing, and that this synonymy results not just from the empirical equivalence of those theories, but from that feature together with other features that are unique to the theories in question. Zaret supports his view with one negative argument against Glymour, and with one positive argument. We find neither of these arguments to be sound. Consider first the positive argument.

Zaret's positive argument rests on two analogies. Suppose there were a theory which postulated two forces, a $Y$ force and a $Y'$ force, affecting bodies in proportion to a $y$ coupling constant and a $y'$ coupling constant characteristic of each body. Suppose further that the theory claims that there is a number $d$ such that for all bodies the ratio of the $y$ constant of the body to the $y'$ constant of the body equals $d$. If this theory saves the phenomena, then one will never be able to determine either the value of the $Y$ force acting or the value of the $Y'$ force: only their sum will be determinable from the phenomena. Zaret writes:

> In this situation, it may no longer be tenable to maintain that $Y$ and $Y'$ are *different* forces. For since $Y$ and $Y'$ have the same effect on the motion of material objects, it seems to follow that we can avoid collapsing them into a single force only if we can differentiate the physically possible conditions under which they have (non-null) effects. Thus we might choose a particular well-defined set of conditions, and then stipulate: if these conditions hold, any $y$-deflection is a $Y$ effect—otherwise, any $y$-deflection is a $Y'$-effect. However, the limited conventionalist will claim that our choice of such a set of conditions is completely arbitrary (Zaret 1980, p. 484).

Again, Zaret points out that we might make a similar decomposition of any force; in particular we could claim (he says "stipulate") that there is no electrical force, really, but instead, two distinct forces—electrical$_1$

which operates in odd-numbered years, and $\text{electrical}_2$ which operates in even-numbered years. ''Of course'', he writes, ''such a stipulation is at least gratuitous and arbitrary, if not devoid of physical meaning'' (Zaret 1980, p. 484).

The analogy with the case of Newtonian Gravitational Theory is as follows. In the case of FNGT, we recall that the theory does not permit us to determine either the gravitational potential or the family of flat geodesics; instead, it permits us to determine only a function of both of them. In an arbitrary frame of reference, according to the theory, the forces acting on a body will derive from two independent features: on the one hand, from non-inertial features of the reference frame itself (e.g., from the fact that the origin of the frame does not move along an affine geodesic), and, on the other hand, from the force of gravitational attraction.[3] But just as with the $Y$ and $Y'$ forces, we cannot determine exactly how much of the body's acceleration is due to inertial forces and how much is due to gravitational forces.

What are we supposed to conclude from this analogy? Zaret's conclusion is the following:

> Furthermore, the limited conventionalist will note that inertial and gravitational forces act on all bodies in the same way. And he will argue that we can establish, by reference to examples such as that of $Y$ and $Y'$, that the assignment of the relative contributions of different forces which act on all bodies in the same way can only be made by convention. Therefore, the assignment of the relative contributions of inertial and gravitational forces can only be made by convention. The limited conventionalist will conclude that the choice between $C^*$ [i.e., CNGT] and the different versions of $F^*$ [i.e., FNGT] is, indeed, a matter of convention. In other words, he will conclude that these theories collapse into a single theory, in which the global structure of space-time is a matter of convention (Zaret 1980, pp. 488–489).

The argument seems to us both invalid and misdirected. Let us grant

[3]Both the term ''force'' and the term ''acceleration'' are ambiguous. The covariant acceleration of a curve is a frame-independent vector $V^{\alpha}_{;\beta}V^{\beta}$ where $V$ is the tangent vector to the curve. In a particular coordinate system, the $\alpha$ component of the covariant acceleration is equal to $d^2Z^{\alpha}/dt^2 + \Gamma^{\alpha}_{\beta\delta}\,(dZ^{\beta}/dt)(dZ^{\delta}/dt)$. Sometimes the quantity $d^2Z^{\alpha}/dt$ by itself is called the acceleration. Henceforth in this article we shall use the term ''covariant acceleration'' to refer to the frame-independent quantity $V^{\alpha}_{;\beta}V^{\beta}$ and the term ''acceleration'' to refer to the frame-dependent quantity $d^2Z^{\alpha}/dt^2$. Similarly, the word ''force'' may sometimes be used to refer only to the frame-independent quantity that is equal to the mass times covariant acceleration. However, sometimes the frame-dependent quantity $\Gamma^{\alpha}_{\beta\delta}$ is also considered a type of force (the so-called ''inertial'' force). The context will make it clear whether the type of force being referred to is of the frame-independent or the frame-dependent type.

the analogy between the case of $Y$ and $Y'$ forces, on the one hand, and on the other hand, the division of forces into inertial and gravitational in the context of FNGT. One may further agree that in that context an extension of FNGT which fixes the inertial frames (and thus the Flat affine geometry and the gravitational potential) is ''conventional'' or ''stipulative'' in the straightforward sense that there is no empirical or theoretical basis for one such specification rather than another. But this is an epistemological remark, not a semantic one, and it does not follow at all that two extensions of FNGT which specify the inertial frames differently are therefore literally synonymous. Further, suppose we grant even that any two extensions of FNGT which differ only in their ''conventions'' regarding which frames are inertial are in fact synonymous theories. It does not follow that CNGT is synonymous with FNGT, for CNGT is not among these theories. The inference is obtained by an equivocation. The analogy might, as we have supposed for the sake of argument, be taken to establish that *in the context of FNGT*, different divisions of the total force into ''inertial'' and ''gravitational'' parts are different ways of saying the same thing. Zaret then concludes that CNGT, in which all of the force might be termed ''inertial'', is also synonymous with FNGT, but that does not follow, for that the subdivisions made no difference was established only in the context of extensions of FNGT. Zaret describes the Curved Theory as the ''conceptual limit'' of the various Flat theories, but that seems a most opaque conception, since the ''inertial forces'' that result in the context of CNGT (from adopting a frame of reference which is not free-falling) are not the inertial forces obtained from *any* sub-division of the total force in the context of FNGT.

Zaret's location of the ''arbitrariness'' in FNGT in the division between inertial and gravitational *forces* is misleading. That division is derivative from a more fundamental feature of the theory. In Newtonian Theory, forces are indicated by *accelerations*, and accelerations are dependent on frame of reference. One and the same motion will therefore be seen as subject to different forces in different frames of reference. However, even without regard to a frame of reference, FNGT describes the trajectory of a body in a gravitational field by postulating that the *covariant acceleration* of the body is equal to a function of the covariant derivative of the gravitational potential. The covariant acceleration and the covariant derivative of the potential are entirely independent of reference frame. The equation of motion of CNGT says simply that the covariant acceleration of a body in a gravitational field is *zero*. Because of the structure of FNGT, neither the covariant acceleration nor the gravitational potential is uniquely determined: one can be changed if the other is changed appropriately to compensate, and the same trajectories will still be described. This is the fundamental arbitrariness of FNGT. In CNGT, by contrast, the covariant acceleration is uniquely determined for freely-falling bodies. It is this arbitrariness in FNGT which generates, when a frame of reference is chosen, the arbitrariness in the division between inertial forces and gravitational forces. *In a given frame of reference,* if one changes the *covariant acceleration* and the *gravitational potential,* as indicated above, one changes how much of the (*non-covariant*) acceleration is attributed to "inertial" forces. There simply is no such indeterminacy in CNGT, and the *only* arbitrariness of the forces in that theory is the arbitrariness in the choice of reference frame. We therefore find it seriously misleading to claim, as Zaret does, that CNGT represents just another arbitrary division of forces, on all fours with the arbitrariness of any extension of FNGT that specifies a particular *covariant* acceleration and gravitational potential.

Zaret's own wording suggests that his argument has been misdirected and forced away from its natural conclusion. When discussing the theory of $Y$ and $Y'$ forces, recall, he says, that "it may no longer be tenable to maintain that $Y$ and $Y'$ are different forces . . . we can avoid collapsing them into a single force only if we can differentiate the physically possible conditions under which they have their (non-null) effects" (Zaret 1980, p. 484). In the case of inertial and gravitational forces in the context of FNGT, there is no empirical or theoretical basis for the differentiation. It seems to follow that we *ought* to collapse them into a single force, and *that is exactly what CNGT does.* Again, when discussing the subdivision of electrical force into two forces, one operative in odd-numbered years and the other in even-numbered years, Zaret claims that the subdivision "is at least gratuitous and arbitrary" (Zaret 1980, p. 484), and, of course, we agree. It seems to us that the natural conclusion from these remarks is scarcely that FNGT and CNGT are synonymous theories, for the former contains a gratuitous and arbitrary subdivision, and the latter does not; if the theories were synonymous, then the subdivision the former requires might somehow be *redundant* but it could hardly contain any new arbitrariness. The natural conclusion is that the theories are not synonymous, and that the Curved Theory is the better of the two.

We turn now to Zaret's negative argument against Glymour. Glymour (1980) has developed an account of theory testing which permits two theories to be empirically equivalent, in the sense of having the same body of observational consequences, even though these consequences may test all of the hypotheses of one theory yet fail to test all of the hypotheses of the other. Applied to CNGT and to FNGT, this account of testing yields the result that all of the fundamental equations of the former, but not all of the equations of the latter, are tested by clocks, rods, and the trajectories of particles in the gravitational field. Thus, in the case at hand, Glymour's necessary condition for the synonymy of theories, and his account of theory testing, are in accord in judging the Curved and the Flat gravitational theories to be inequivalent. Certain hypotheses of the Flat Theory are untestable because in order to carry out a test of an equation, the theory must permit an appropriate determination, from observable quantities, of each of the fundamental quantities occurring in the equation to be tested. In the case of the Flat Theory, neither the covariant acceleration nor the affine geometry nor the gravitational potential can be so determined. No similar indeterminacy holds for the Curved Theory. Zaret does not object to the account of testing. Instead he argues, incorrectly we believe, that it yields the result that in an appropriately expanded context the two theories are equally well-tested. The context is that in which electromagnetic as well as gravitational phenomena are to be accounted for.

To expand either theory to include Maxwell's electromagnetism, one must first postulate a vector field, or rigging, which ties together the points of space at different moments. One must further introduce a quantity representing the electromagnetic field, and quantities representing the density and motion of charges. With these new objects, either gravitational theory can be expanded by adding covariant versions of Maxwell's equations, equations which we will denote by ''Max''. Both for the Flat and for the Curved theories, however, the equation of motion must be modified when electromagnetic phenomena are included. The new Flat equation of motion says that the covariant acceleration of a test particle equals a function of the covariant derivative of the gravitational potential plus a function of the electromagnetic field. The new Curved equation of motion says that the covariant acceleration of a test particle equals a function of the electromagnetic field. The expanded theories are not empirically equivalent because the equations which determine the electromagnetic field, Maxwell's equations, contain covariant derivatives of that field. Now the covariant derivative is determined by the affine geometry (or more properly, by the geometrical object which determines the affine geometry) and since the Curved Theory and the Flat Theory postulate quite different affine geometries, the solutions to Maxwell's equations are different in the two cases.

However, as Zaret has pointed out, it is possible to expand CNGT into a theory which is observationally equivalent to FNGT + Max. In order to do this, one must add to CNGT a theory which differs from Max by the addition of certain correction terms to compensate for the difference between the covariant derivative in FNGT and the covariant derivative of CNGT. Call the resulting theory CNGT + Zax.

Zaret's argument against Glymour now goes as follows: FNGT + Max and CNGT + Zax are observationally equivalent, and account for both gravitational and electrodynamic phenomena. In FNGT + Max, however, both the affine geometry and the gravitational potential turn out to be uniquely determinable from observational quantities. Thus in this expanded context, Zaret concludes, the grounds for claiming that FNGT is less well-tested than CNGT no longer apply, and thus Glymour's synonymy condition is not really supported by his account of theory testing (Zaret 1980, pp. 492–493).

We hold that Zaret's objection involves serious confusions, and that when they are dissolved, the case he considers fully accords with principles (1) and (2). In the first place, we wish to make two remarks, one about the structural features of the account of testing, the other about structural features of principles (1) and (2). For the first, the account of testing in question determines a three-place relation: Hypothesis *H* is tested by observational result *O* with respect to theory *T*.[4] In using this relation to compare theories, Glymour proposes that we consider such questions as "Is it the case that for each of the equations *E* in some set of equations sufficient to axiomatize theory *T*, there is an observation *O* such that *O* tests *E* with respect to *T?*" If a body of observations is such that it accords with both of two theories, but the answer to the question just put is affirmative for one theory but negative for the other, then Glymour claims that, *ceteris paribus,* the evidence would provide more reason to believe the former theory than the latter. The second point is this: According to either principles (1) or (2), it may happen that of four theories, *A, A′, B, B′*, no two satisfy the condition for synonymy, but that the conjunctions *A* & *B,* on the one hand, and *A′* & *B′* on the other hand, *do* satisfy the necessary condition for synonymy with one another.

Returning to Zaret's argument, we note first that he is correct in claiming that one can find a version of electrodynamics, Zax, such that when it is added to CNGT the result is a theory observationally equivalent to FNGT + Max. We present such an electrodynamic theory in the appendix. Of the theories CNGT, FNGT, Max, and Zax, no two are synonymous with one another according to principle (1) or (2). *But the two theories FNGT + Max and CNGT + Zax do satisfy the necessary conditions for synonymy given by principles (1) and (2).* Structurally, at least, these theories are indistinguishable, although we take no stand on whether or not they should be regarded as actually synonymous. Although FNGT + Max and CNGT + Zax are structurally indistinguishable, and could be thought of for the purposes of this discussion as a single theory, FNGT and CNGT are different sub-theories of that theory, embodying different sets of equations. *Furthermore, with respect to this expanded*

[4]This relation is understood to obtain whenever *H* is tested by *O* with respect to any sub-theory of *T*.

*theory, the equations of FNGT are not tested by the same observations as are the equations of CNGT*. Although there *are* possible observations that will test FNGT with respect to the expanded theory, they are not the same observations as will test the equations of CNGT with respect to that theory. For example, with respect to the expanded theory, clock and rod measurements, together with the trajectories of uncharged bodies, will test all of the equations of CNGT but not of FNGT.

In view of these facts, Zaret's argument seems quite without power. CNGT and FNGT do not satisfy, as a pair, either of principles (1) or (2), and the claim that they are not synonymous is buttressed by the claim that they are not equally well tested, with respect to themselves, by imaginable values of their observable quantities. The appeal to electrodynamic phenomena is bootless, for if the claim is that CNGT and FNGT are both tested by imaginable observational data with respect to CNGT + Zax (or, equivalently, FNGT + Max), it remains true that there will be imaginable observational data that test CNGT with respect to the expanded theory but not FNGT. If, instead, the claim is that CNGT + Zax and FNGT + Max are both equally and thoroughly tested by imaginable observational data, then the claim is irrelevant to the issue, since these two expanded theories satisfy both of the principles in dispute.

We suspect that Zaret simply did not distinguish what is being tested with respect to what and did not note that, when actually developed, the expanded Curved Theory plus Zax is structurally indistinguishable from the Flat Theory plus Maxwell's electrodynamics. It seems to us that the relations of the expanded theories only give further evidence of the soundness of the structural principles (1) and (2). CNGT + Zax and FNGT + Max are obviously structurally equivalent, and one would surely demand of any proposed intertranslation of the two theories that it preserve that structure. But if, as Zaret proposes, one takes FNGT and CNGT to be synonymous, there will be no natural translation of the expanded theories into one another, for the natural translation, which preserves the structure of the expanded theories, does not take FNGT into CNGT, or conversely.

## APPENDIX

There are several different theories of the global structure of Newtonian space-time. The theories described here are similar to those found in Trautman (1964) and Glymour (1980). In the Flat Space Theory (FNGT), $M$ is a four dimensional manifold homomorphic to $R^4$. On this manifold, there is a scalar field $t$, the absolute time, which is an affine parameter of every timelike geodesic; a singular 2,0 metric tensor field $g$ with signature $(0,+,+,+)$; the mass density $\rho$; the gravitational potential $U$; and an affine connection compatible with the metric. The field equations are:

$$R^{\alpha}_{\beta\gamma\delta} = 0 \tag{1}$$

$$t_{\alpha;\beta} = 0, \; (t_{\alpha} = \partial t/\partial x^{\alpha}) \tag{2}$$

$$g^{\alpha\beta}_{;\gamma} = 0 \tag{3}$$

$$g^{\alpha\beta} t_{\alpha} t_{\beta} = 0 \tag{4}$$

$$g^{\alpha\beta} U_{;\alpha;\beta} = 4\pi k\rho \tag{5}$$

$$F^{\beta} = ma^{\beta} \tag{6}$$

$$F^{\beta}_{\text{grav}} = g^{\beta\gamma} U_{;\gamma} \tag{7}$$

$a^{\beta}$ is the $\beta$ component of the covariant acceleration and $F^{\beta}$ is the $\beta$ component of the force. The semicolon signifies covariant differentiation with respect to the index following it. $R^{\alpha}_{\beta\gamma\delta}$ is the Riemannian curvature tensor, which is definable in terms of the affine connection; equation (1) simply states that the space-time is flat. (6) and (7) imply that the equation of motion of this theory is (8)

$$d^2x^{\alpha}/dt^2 + \Gamma^{\alpha}_{\beta\delta}\,(dx^{\beta}/dt)(dx^{\delta}/dt) = -g^{\alpha\beta} U_{;\beta}. \tag{8}$$

Thus, according to FNGT, particles in the presence of no forces follow geodesics.

In CNGT, another theory of Newtonian space-time, the space-time is curved. $M$ is still a four-dimensional manifold homomorphic to $R^4$. There is still a scalar field $t$ representing absolute time; a singular 2,0 metric tensor field $g$ with signature $(0,+,+,+)$; and the mass density $\rho$. However, there is no gravitational potential $U$, and the flat affine connection of FNGT is replaced by an affine connection which is curved in the presence of matter. (It is possible for both the flat connection of FNGT and the curved connection of CNGT to be compatible with $g$, since it is singular.) The field equations of CNGT are:

$$t_{[\alpha}{}^{\circ}R_{\gamma}{}^{\beta]}{}_{\delta\epsilon} = 0 \tag*{(1)*}$$

$$g^{\alpha\beta}\,{}^{\circ}R^{\gamma}_{\delta\beta\epsilon} = g^{\gamma\beta}\,{}^{\circ}R^{\alpha}_{\epsilon\beta\delta} \tag*{(2)*}$$

$$t_{\alpha/\beta} = 0 \tag*{(3)*}$$

$$g^{\alpha\beta}{}_{/\delta} = 0 \tag*{(4)*}$$

$$g^{\alpha\beta} t_{\alpha} t_{\beta} = 0 \tag*{(5)*}$$

$${}^{\circ}R_{\alpha\beta} = -4\pi\rho t_{\alpha} t_{\beta} \tag*{(6)*}$$

$$F^{\beta} = ma^{\beta} \tag*{(7)*}$$

In these equations, the brackets denote antisymmetrization with respect to the indices between them, ${}^{\circ}R^{\alpha}_{\beta\gamma\delta}$ is the Riemannian curvature tensor defined in terms of the curved connection. ${}^{\circ}R_{\alpha\beta}$ is the contraction of the Riemannian curvature tensor, and the slash represents covariant differentiation with respect to the curved connection. The equation of motion of CNGT is

$$d^2x^{\alpha}/dt^2 + {}^{\circ}\Gamma^{\alpha}_{\beta\delta}\,(dx^{\beta}/dt)(dx^{\delta}/dt) = 0 \tag*{(8)*}$$

where ${}^{\circ}\Gamma^{\alpha}_{\beta\delta}$ is the Christoffel symbol of the curved connection. (8)* is just the equation of a geodesic of CNGT; thus, according to CNGT, freely falling particles travel along geodesics.

There is a definitional extension of FNGT which implies CNGT. More specifically, FNGT $\uplus \{{}^{\circ}\Gamma^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\beta\gamma} + t_{\beta} t_{\gamma} g^{\alpha\delta} U_{;\delta}\}$ implies CNGT (where A $\uplus$ B is the set of consequences of the set theoretical union of A and B). However, there is no definitional extension of CNGT which implies FNGT.

FNGT, unlike CNGT, is not bootstrap tested relative to itself by the available evidence (provided by mechanical clocks, rigid rods, and the trajectories of freely falling bodies). Trautman (1964) has shown that neither the gravitational potential nor the affine connection of any model of FNGT is uniquely determined by $g$, $t$, $\rho$, and the family of free fall trajectories, all of which will be assumed to be "observable" or determined by "observable" evidence. Let $\langle M, g, t, \rho, U, \Gamma, H \rangle$ be a model of FNGT, where $H$ is the collection of free falls. Let $\psi$ be any scalar field, and $\Gamma'$ any connection such that:

$$t_{[\alpha(\partial\psi/\partial X\gamma);\beta]} = 0$$

$$\Gamma'^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\beta\gamma} - t_\beta t_\alpha g^{\alpha\delta}\psi_{;\delta}$$

where, as before, the semicolon signifies covariant differentiation, using $\Gamma$. Trautman has shown that it follows that $\langle M, g, t, \rho, \psi, \Gamma' H \rangle$ is also a model of FNGT. Thus, the connection and the gravitational potential are not determined by the other quantities in the theory. This, in turn, implies that the available evidence fails to test equations such as (1) and (5) relative to FNGT, since it is impossible to use the other hypotheses of FNGT to produce instances or counterinstances of (1) and (5). Thus, the available evidence does not completely test FNGT relative to itself.

On the other hand, the affine connection of CNGT is uniquely determined by $g$, $t$, $\rho$, and $H$. (This follows from the fact that an affine connection is uniquely determined by a geodesic spray and an affine parameter). Furthermore, CNGT is completely testable relative to itself by the available evidence.

It is possible to extend FNGT into a theory of electromagnetism. Let us call this extended theory FNGT + Max. In order to extend FNGT into FNGT + Max, it is necessary to introduce an "ether" or a "rigging". The function of a "rigging" is to associate with each point of a hypersurface of simultaneity a vector $V$ tangent to the world line of an observer at rest in the ether. It may be supposed that the rest frame of the ether is an inertial frame of FNGT. The vector field satisfies the following laws:

$$V^{\beta}_{;\alpha} = 0 \tag{9}$$

$$V^{\alpha} t_{\alpha} = 1 \tag{10}$$

If $G_{\alpha\beta}$ is the Maxwell tensor, then Maxwell's electrodynamic equations may be written in the form:

$$G^{\alpha\beta}_{;\beta} = -4\pi J^{\alpha} \tag{11}$$

$$G_{[\alpha\beta;\gamma]} = 0 \tag{12}$$

$$F^{\alpha}_{\text{elect}} = q(G^{\alpha}_{\beta}(dx^{\beta}/dt) - t_{\delta} G^{\delta}_{\beta}(dx^{\beta}/dt)\, V^{\alpha}) \tag{13}$$

where $J$ represents the current, $q$ represents charge, and $G^{\alpha\beta} = (g^{\alpha\gamma} - V^{\alpha}V^{\gamma}/c^2)(g^{\beta\delta} - V^{\beta}V^{\delta}/c^2)G_{\gamma\delta}$. ($c$ is the velocity of light).

CNGT can be extended into CNGT + Zax, a theory of classical electrodynamics. In order to do this it is necessary not only to introduce the rigging of space-time, it is also necessary to introduce the potential $U$. The equations for the vector field $V$ are:

$$V^{\beta}{}_{/\alpha} - t_{\delta} t_{\alpha}\, g^{\beta\gamma}\, U_{/\gamma}\, V^{\delta} = 0 \tag{9*}$$

$$V^{\alpha} t_{\alpha} = 1 \tag{10*}$$

The field equations, and new equation of motion are:

$$G^{\alpha\beta}_{/\beta} - (g^{\alpha\delta}G^{\epsilon\beta} + g^{\epsilon\delta}G^{\alpha\epsilon})U_{/\delta}t_{\epsilon}t_{\beta} = -4\pi J^{\alpha} \tag{11*}$$

$$G_{[\alpha\beta/\gamma]} = 0 \tag{12)*}$$

$$F^{\alpha}_{\text{elect}} = q\left(G^{\alpha}_{\beta}\left(\frac{dx^{\beta}}{dt}\right) - t_{\gamma}G^{\gamma}_{\beta}\left(\frac{dx^{\beta}}{dt}\right)V^{\alpha}\right) \tag{13)*}$$

The observable evidence relevant to FNGT + Max and CNGT + Zax includes not only the behavior of solid rods and mechanical clocks, and the trajectories of freely falling bodies, but also the trajectories of bodies moving under the influence of electromagnetic forces and the behavior of light rays.

Consider the following two equations.

$$^{\circ}\Gamma^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\beta\gamma} + g^{\alpha\delta}U_{;\delta}t_{\beta}t_{\gamma} \tag{14}$$

$$\Gamma^{\alpha}_{\beta\gamma} = {}^{\circ}\Gamma^{\alpha}_{\beta\gamma} - g^{\alpha\delta}U_{/\delta}t_{\beta}t_{\gamma} \tag{14)*}$$

We will now use these two equations to demonstrate that FNGT ⊎ Max and CNGT ⊎ Zax are intertranslatable, by showing that they have a common definitional extension. In particular, FNGT ⊎ Max ⊎ {14} = CNGT ⊎ Zax ⊎ {14*}.

It has already been pointed out that FNGT ⊎ {14} implies CNGT. It is also the case that CNGT ⊎ {14*} implies FNGT. (Of course, when 14* is added to CNGT, the resulting theory is not a *definitional* extension of CNGT, since the gravitational potential $U$ does not occur in CNGT, nor is it definable in CNGT. However, when 14* is added to CNGT ⊎ Zax, the resulting theory is a definitional extension of CNGT ⊎ Zax, since the gravitational potential U does occur in the equations of Zax). Thus, in order to show that FNGT ⊎ Max ⊎ {14} = CNGT ⊎ Zax ⊎ {14*}, it is only necessary to show that FNGT ⊎ Max ⊎ {14} implies CNGT ⊎ {14*}, and that CNGT ⊎ Zax ⊎ {14*} implies FNGT ⊎ {14*}.

First, we will show that FNGT ⊎ Max ⊎ {14} implies CNGT ⊎ {14*} (equations 9*–14*). Equation 9 of Max is:

$$V^{\beta}_{;\alpha} = 0 \tag{9}$$

When this equation is written in its non-abbreviated form, it is:

$V^{\beta}_{;\alpha} = V^{\beta}_{,\alpha} + \Gamma^{\beta}_{\delta\alpha}V^{\delta} = 0$, (where "," represents coordinate differentiation).

When $^{\circ}\Gamma^{\alpha}_{\beta\gamma} - g^{\alpha\delta}U_{\delta}t_{\beta}t_{\gamma}$ is substituted for each occurrence of $\Gamma^{\alpha}_{\beta\gamma}$ in the latter equation, the result is:

$$V^{\beta}_{,\alpha} + ({}^{\circ}\Gamma^{\beta}_{\delta\alpha} - t_{\delta}t_{\alpha}g^{\beta\gamma}U_{/\gamma})V^{\delta} = V^{\beta}_{/\alpha} - t_{\delta}t_{\alpha}g^{\beta\gamma}U_{/\gamma}V^{\delta} = 0$$

But this latter equation is just 9*.

10* obviously follows from FNGT ⊎ Max ⊎ {14}, since it is identical to 10.

If $^{\circ}\Gamma^{\alpha}_{\beta\gamma} - g^{\alpha\delta}U_{/\delta}t_{\beta}t_{\gamma}$ is substituted in for $\Gamma^{\alpha}_{\beta\gamma}$ in 11, the result is 11*.

$$G^{\alpha\beta}_{;\beta} = -4\pi J^{\alpha} =$$

$$G^{\alpha\beta}_{,\beta} + \Gamma^{\alpha}_{\delta\beta}G^{\delta\beta} + \Gamma^{\beta}_{\delta\beta}G^{\alpha\delta} =$$

$$G^{\alpha\beta}_{,\beta} + ({}^{\circ}\Gamma^{\alpha}_{\delta\beta} - t_{\delta}t_{\beta}g^{\alpha\gamma}U_{/\gamma})G^{\delta\beta} + ({}^{\circ}\Gamma^{\beta}_{\delta\beta} - t_{\delta}t_{\beta}g^{\beta\gamma}U_{/\gamma})G^{\alpha\delta} =$$

$$G^{\alpha\beta}_{/\beta} - (g^{\alpha\gamma}G^{\delta\beta} + g^{\beta\gamma}G^{\alpha\delta})U_{/\gamma}t_{\delta}t_{\beta} = -4\pi J^{\alpha}. \tag{11)*}$$

Similarly, if $^{\circ}\Gamma^{\alpha}_{\beta\gamma} - g^{\alpha\delta}U_{/\delta}t_{\beta}t_{\gamma}$ is substituted in for $\Gamma^{\alpha}_{\beta\gamma}$ in 12, the result is 12*.

$$G_{[\alpha\beta;\gamma]} = 0 = 1/3!\,[G_{\alpha\beta;\gamma} + G_{\beta\gamma;\alpha} + G_{\gamma\alpha;\beta} - G_{\beta\alpha;\gamma} - G_{\gamma\beta;\alpha} - G_{\alpha\gamma;\beta}]$$

$$= 2/3!\ [G_{\alpha\beta;\gamma} + G_{\beta\gamma;\alpha} + G_{\gamma\delta;\beta}] =$$

$$2/3!\ [(G_{\alpha\beta,\gamma} - \Gamma^{\delta}_{\alpha\gamma}G_{\delta\beta} - \Gamma^{\delta}_{\beta\gamma}G_{\alpha\delta}) + (G_{\beta\gamma,\alpha} - \Gamma^{\delta}_{\beta\alpha}G_{\delta\gamma} - \Gamma^{\delta}_{\gamma\alpha}G_{\beta\delta})$$

$$+ (G_{\gamma\alpha,\beta} - \Gamma^{\delta}_{\gamma\beta}G_{\delta\alpha} - \Gamma^{\delta}_{\alpha\beta}G_{\gamma\delta})] =$$

$$2/3!\ [(G_{\alpha\beta,\gamma} - ({}^{\circ}\Gamma^{\delta}_{\alpha\gamma} - g^{\delta\epsilon}U_{/\epsilon}t_{\alpha}t_{\gamma})G_{\delta\beta} - ({}^{\circ}\Gamma^{\delta}_{\beta\gamma} - g^{\delta\epsilon}U_{/\epsilon}t_{\beta}t_{\gamma})G_{\alpha\delta})$$

$$+ (G_{\beta\gamma,\alpha} - ({}^{\circ}\Gamma^{\delta}_{\beta\alpha} - g^{\delta\epsilon}U_{/\epsilon}t_{\beta}t_{\alpha})G_{\delta\gamma} - ({}^{\circ}\Gamma^{\delta}_{\gamma\alpha} - g^{\delta\epsilon}U_{/\epsilon}t_{\gamma}t_{\alpha})G_{\beta\delta})$$

$$+ (G_{\gamma\alpha,\beta} - ({}^{\circ}\Gamma^{\delta}_{\gamma\beta} - g^{\delta\epsilon}U_{/\epsilon}t_{\gamma}t_{\beta})G_{\delta\alpha} - ({}^{\circ}\Gamma^{\delta}_{\alpha\beta} - g^{\delta\epsilon}U_{/\epsilon}t_{\alpha}t_{\beta})G_{\gamma\delta})] =$$

$$G_{[\alpha\beta/\gamma]} + U_{/\epsilon}g^{\delta\epsilon}(t_{\alpha}t_{\gamma}G_{\delta\beta} + t_{\beta}t_{\gamma}G_{\alpha\delta} + t_{\beta}t_{\alpha}G_{\delta\gamma} + t_{\gamma}t_{\alpha}G_{\beta\delta}$$

$$+ t_{\gamma}t_{\beta}G_{\delta\alpha} + t_{\alpha}t_{\beta}G_{\gamma\delta}) =$$

$$G_{[\alpha\beta/\gamma]} = 0. \qquad (12)^{*}$$

Finally, 13* is identical to 13, and 14* obviously follows from 14 by simple arithmetic. Thus, FNGT ⊍ Max ⊍ {14} implies Zax ⊍ {14*}.

It is easy to see from an examination of the derivation of Zax from FNGT ⊍ Max ⊍ {14} that the equations of Max can be derived from CNGT ⊍ Zax ⊍ {14*} by substituting $\Gamma^{\alpha}_{\beta\gamma} + g^{\alpha\delta}U_{;\delta}t_{\beta}t_{\gamma}$ for each occurrence of ${}^{\circ}\Gamma^{\alpha}_{\beta\gamma}$ in the equations of Zax. Thus, Max can be derived from CNGT ⊍ {14*}.

It follows that FNGT ⊍ Max ⊍ {14} = CNGT ⊍ Zax ⊍ {14*}, and that FNGT ⊍ Max, and CNGT ⊍ Zax are formally intertranslatable.

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